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Publication date:
2004

Document Version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
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December 2004

Abstract

In several European merger cases competition authorities have demanded that the merging firm auctions off virtual capacity. The buyer of virtual capacity receives an option on an amount of output at a prespecified price, typically equal to marginal cost. This output is sold in the market in competition with the merging firm.

The paper compares sale of physical and virtual capacity by the merging firm and shows that virtual capacity makes tacit collusion easier. The reason is that the auction price on virtual capacity increases, when the merging firm reduces production in order to increase the output price. This reduces its temptation to deviate.

Keywords: Virtual Capacity, Tacit Collusion, Anti-trust, Mergers, Competition Policy

JEL: L40, L41, D44

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1 Introduction

It is common in merger cases that the competition authority requires dominant firms to sell off capacity, so that its market share does not grow (too much). Recently, several European merger cases in electricity markets have resulted in the sale of virtual capacity in the form of so called Virtual Power Plants (VVP). The aim of this paper is to investigate, whether the competitive effects of these requirements are as good as the selling of real physical capacity. Tacit collusion - or coordinated interaction - is a major worry in merger cases; see for instance the horizontal merger guidelines published by Department of Justice (1997). We are interested in whether the possibility of tacit collusion is affected by the fact that the capacity is virtual and not in the form of ownership of the capacity.

Virtual capacity is an option to buy products (e.g. electricity) at a predetermined price per unit (typically equal to marginal cost), which the buyer then sells in the final product market in competition with the producer. In the European electricity examples, auctions are held at regular intervals (several times a year).

When the virtual capacity is auctioned, the recipient of the revenue is the large merging firm. If the auction is efficient, the price for the virtual capacity will equal the expected profit - suitably adjusted for risk etc. - from having access to the capacity. So although the merging firm meets competition in the market from the virtual competitor, it will pocket the profits made by the competitor in the auction. This potentially has effects on the incentives to collude. The paper investigates this.

The paper provides a simple model of a market with one producing firm. The producing firm first auctions off virtual capacity and then the firm and the virtual producer competes in the market. For simplicity we consider a
Cournot model, where the firms choose production and the price is set in
the market. We first consider a static market. There the competitive effects
of introducing the virtual producer are equivalent to those of introducing an
independent producer who owns his capacity. The reason is simple. Once
the auction is held, the payment in the auction is sunk and everything is as
if there are two independent firms in the market. In line with the European
examples alluded to, we consider the case where the virtual capacity is small
relative to the market, so that the virtual producer wants to market his whole
capacity. In principle one could conceive of large virtual producers, but this
has not been demanded by competitive authorities yet and is left for future
research.

Secondly, we consider tacit collusion. Since the virtual capacity typi-
cally is auctioned off for relatively short periods and there are many bidders,
we first consider the case where the virtual producers do not participate in
tacit collusion. They are assumed to be short run players, who just market
whatever capacity they have. Clearly, this makes it more difficult to main-
tain tacit collusion. In the standard case with two independent firms, tacit
collusion cannot be implemented without the cooperation of both firms. If
one firm plays a best response, the best the other can do is to play best
response and this results in the Cournot equilibrium. With virtual capacity
this logic breaks down. Since the big (merging) firm sells the virtual capacity
it will pocket the expected profit to be earned on the virtual capacity. If it
acts moderately in the market and lowers production, the price will rise. In
fact it can induce the monopoly price by selling the monopoly output minus
the virtual capacity it has sold off. Then the market price will equal the
monopoly price, and the virtual producer will net the monopoly price minus
marginal (virtual) cost times virtual capacity. In a repeated game where this
happens in each period, the participants in the auction will realize this and the revenue of the big selling firm will equal the high earnings of the virtual producer. In this way the big producer can realize the whole monopoly profit. Of course, it has an incentive to deviate in a given period. When it has auctioned off the virtual capacity in a period, there is an incentive to produce more than the low level giving rise to the monopoly price. However, there will be a future punishment, as future bidders in the auction will realize that monopoly profits cannot be earned in the market; they will only bid the virtual producer’s Cournot profit. We show that this punishment is sufficient for the big producer to restrain production and maintain monopoly profits if the discount factor is sufficiently high. This is a very general result, which just assumes that the monopoly profit exceeds the Cournot profit.

We also show that if the discount factor is lower than the crucial value, which allows the firm to reap the monopoly profit, the highest obtainable price and profit are increasing in the discount factor.

For given time preference, the relevant discount factor in the model depends on the duration of the contracting period for the virtual capacity. The longer the duration, the smaller is the relevant discount factor. It therefore follows that a shorter duration of contract for virtual capacity facilitates tacit collusion. The reason is intuitive: Tacit collusion implies that the virtual capacity is sold for a high price in the auction in the expectation that total market production will be low, so that the product price as well as profit will be high. The big producer is tempted to increase production in order to increase the share of current profits. The punishment is that such behavior will be expected in the future so the auction price for virtual capacity will fall. This punishment first commences in the next auction. The farther in the future it is, the less this punishment hurts.
If the small firm is an independent producer, who owns his own capacity, and the firm is short sighted, tacit collusion is impossible. It therefore follows that virtual capacity in itself facilitates tacit collusion if the small firm is a short run player.

Of course, one may argue that it is not reasonable to assume that the small firm will be a short run player, who will not participate in tacit collusion, if the firm owns its own capacity. In this case it will be in the market for many periods, and be interested in future profits. A non-trivial question here is how the firms share production - and profits - when they collude on the monopoly outcome. We assume that they split the market in the same proportion as they do in the Cournot equilibrium. This would for instance be the case in a split the surplus bargain where the Cournot outcome is the threat point. This complicates the model somewhat, and we resort to a linear specification. For this case we show that the minimum discount factor allowing collusion for monopoly profits is higher if the small firm is independent than if it is a virtual producer. So in this comparison, virtual capacity also facilitates collusion.

In the linear specification it is, however, also true that the minimal discount factor necessary for tacit collusion on the monopoly profit is higher when the small producer is a short run player with virtual capacity than when he is a long run player with own capacity. Hence, an auction format which ensures a lot of competition among the potential bidders, so they become short-sighted, is important for promoting the competitive effects of introducing virtual capacity.

As stated above, virtual capacity has been introduced in a number of recent European merger cases. In relation to Electricité de France’s (EDF) purchase of 34,5% of the shares in the German utility EnBW, EDF agreed
to make 6,000 MW of virtual capacity available in France by November 2003 in order to increase competition in the market. EDF was at the time selling to around 90% of the so-called free customers in the French market. The virtual capacity is to be auctioned to companies who will act as sellers in the French power market. The contracts for virtual capacity have durations of 3, 6, 12, 24 and 36 months. The first auction for 1,200 MW took place in September 2001. As of April 2004 11 auctions have been held. The auctions are organized as ascending clock auctions. Around 30 energy traders and suppliers competed in previous auctions, which were conducted over the Internet, with approximately 20 bidders emerging as successful purchasers.

According to agreement with the European Commission, EDF shall provide virtual capacity for a period of five years. The French electricity market is then expected to have developed so that sufficient competition will be present without the Virtual Capacity (see Electricité de France, 2004).

Due to the Electricity Supply Board’s (ESB) dominance in the Irish power market, the Irish government has initiated the Virtual Independent Power Producer Auction (VIPP), a form of virtual capacity auction as in France. The auctions - where independent suppliers can bid for 600 MW out of a total of 4,500 MW - are intended to reduce ESB’s market power until more independent suppliers enter the market (see European Commission, Madrid Forum, 2002).

In 2003, the Belgian Competition Council approved that a subsidiary of Electrabel became the default supplier for the customers of several inter-municipal distribution companies. As Electrabel has a very large market share in Belgium it was agreed that Electrabel should offer, via auctions, up to a maximum of 1,200 MW of virtual power plant (VPP) capacity in Belgium. The terms are to a large extent similar to the French, in particular
capacity shall be offered for a period of five years (see Konkurrencestyrelsen, 2004).

The Dutch electricity producer Nuon agreed with competition authorities that it would auction 900 MW virtual capacity in order to be allowed to buy Reliant and its 3500 MW capacity. Again there is a five year limit on the requirement. The Dutch market size is around 20,000 MW, (Konkurrencestyrelsen, 2004)

In March 2004 the large Danish producer Elsam agreed to auction off 600 MW virtual capacity in order to be allowed to make an indirect purchase of 36% of the shares in the other big Danish producer E2, see Konkurrencestyrelsen (2004). The total Danish market size is about 7000 MW. As in the other countries auctions are to be held regularly, and for varying durations all below three years. The Danish rules specify that a single buyer at most must acquire 300 MW. The agreement with the competition authorities stipulates that the virtual producer can buy electricity at the lowest marginal cost obtainable in the different plants owned by Elsam. Contrary to the previously mentioned cases, the Danish competition authorities required that the virtual capacity should be provided indefinitely. This makes worries about tacit collusion potentially more important as there will be no end game effects.

There is a long literature on tacit collusion, see Tirole (1991) for an overview. The detection of tacit collusion in electricity markets have been the subject of a number of papers including Green and Newbury (1992), von der Fehr and Harbord (1993), Borenstein and Bushnell (1999), Wolfram (1999), and Fabra and Toro (2004). To the best of my knowledge the issue of virtual capacity and tacit collusion has not been considered in the literature.

The organization of the paper is as follows. Section 2 describes the basic model and derives the static solution. Tacit collusion with virtual capacity
and short run players is considered in section 3. Section 4 contains the case where the small producer is independent. Section 5 offers some concluding remarks.

2 Basics: The static market

We consider a market with two firms, a big firm 1, who sells some virtual capacity $q_2$. The buyer of the capacity, firm 2, is also called the virtual firm. Both firms sell in a final market. The amount sold by firm 1 in the final market is denoted $q_1$. The price in the final market is given by the inverse demand curve $p(Q)$ where $Q$ is total production. For some - but not all - of the results we will rely on the linear specification

$$p(Q) = a - bQ,$$

where $a$ and $b$ are two positive parameters. Although Theorem 1 below is valid under general concavity assumptions, we introduce the linear specification already from the start and give the results for the linear model as we go along in order to shorten on the presentation. We will state explicitly, when a result depends on the linear specification.

In this section we consider a single period. In a period, the timeline is as follows. First virtual capacity in the amount $q_2$ is sold in an auction. After the auction, the big firm, firm 1, and the buyer of the virtual capacity, firm 2, competes ala Cournot in the final market. We assume that there are sufficiently many potential bidders and the auction format is such that the price of the virtual capacity equals the profit which can be earned in the final market with the virtual capacity. This is for instance the case if the auction is an open English auction with at least two independent bidders.
The virtual capacity allows the buyer to request up to $q_2$ units at firm 1’s marginal cost $c \geq 0$.

We assume that the amount of virtual capacity, $q_2$, is so small that firm 2 is capacity constrained and wants to utilize all capacity. This assumption is motivated by the examples discussed in the Introduction, where the virtual producers indeed are small.

We solve the static model for the subgame perfect equilibrium, as usual by solving backwards. After the auction, firm 2 possesses virtual capacity $q_2$ and it sells all $q_2$ units in the final market. We will verify below that this is indeed optimal. Given firm 2 sells $q_2$ units, the problem of firm 1 is

$$\max_{q_1} (p(q_1 + q_2) - c)q_1.$$ 

The best reply is (using superscript “c” for Cournot),

$$q_1^c(q_2) = \frac{a - c}{2b} - \frac{1}{2}q_2; \quad (1)$$

and the total production is therefore

$$q_1^c(q_2) + q_2 = \frac{a - c}{2b} + \frac{1}{2}q_2.$$ 

The final price in the market is

$$p^c = \frac{a + c}{2} - \frac{b}{2}q_2; \quad (2)$$

Equation (2) clearly shows that the introduction of virtual capacity lowers the market price.

The profit to each firm is

$$\pi_1^c = \frac{(a - c - b q_2)^2}{4b} \quad \text{and} \quad \pi_2^c = \frac{(a - c - b q_2) q_2}{2}$$

As mentioned above, we assume that $q_2$ is so low that the virtual firm is capacity constrained, i.e. that $q_2$ is less than firm 2’s best reply to $q_1^c(q_2)$. 


The best reply to $q_1^c(q_2)$ is given by (1) with $q_1^c(q_2)$ inserted for $q_2$ on the right hand side. We therefore get that 2 is capacity constrained if

$$q_2 < \frac{a - c}{2b} - \frac{1}{2} \left(\frac{a - c}{2b} - \frac{1}{2} q_2\right)$$

or

$$q_2 < \frac{1}{3} \frac{a - c}{b}$$

This condition says that $q_2$ should be less than the production level of each firm in the symmetric Cournot equilibrium. With two firms in the market this implies that $q_2$ should be less than 50% of the market. This is clearly fulfilled in the examples discussed in the Introduction. In the model, we only include one successful bidder in the auction, whereas in the examples discussed in the introduction, there typically were many. As long as the virtual producers use all capacity, the results derived here would not change if we introduced more virtual producers with total capacity $q_2$. This would just lead to a more cumbersome notation, so we refrain from that.

Now we look at the auction stage. The prospective buyers are rational and foresee that the Cournot equilibrium will arise and that the profit, which can be earned from the virtual capacity is $\pi_2^c$. Under the assumption that the auction is competitive, the price of the capacity will equal this profit.

The total profit to firm 1 from own sales and the sale of the virtual capacity therefore equals

$$\pi_1^c + \pi_2^c = \frac{(a - c)^2 - (bq_2)^2}{4b}$$

We see that the larger the virtual capacity, the lower is the total profit of firm 1. This is of course just a mirror of the lower price. Virtual capacity enhances the competitiveness of the static market.
3 Tacit collusion with virtual capacity

In this section we consider tacit collusion when there is virtual capacity. There are infinitely many periods \( t = 0, ..., \infty \).

At first we will assume that the auction format and the many participants in the auction means that the winner of the virtual capacity has a short horizon. She can not be sure to win the next auction and will therefore not be willing to reduce supply in order to raise the price. She will seek to gain as much as possible and for \( q_2 \) sufficiently small, this means that she will wish to supply \( q_2 \). In short, the owner of the virtual capacity is not willing to collude, she is a short run player in the language of Fudenberg, Kreps and Maskin (1990).

Firm 1, however, is a long run player and has an incentive to keep prices high in the market. We assume that firm 1 discounts future profits with the discount factor \( \delta \), where \( 0 < \delta < 1 \), and the firm is interested in the sum of discounted future profits.

The participants in the auction has an expectation about the market price and therefore of the profit, which can be earned using the virtual capacity. The participants observe previous prices and as time passes, the expectations for period \( t \) may depend on these previous prices. At time \( t \) the expectation about prices for period \( t \) is a function of previous prices. In equilibrium, these expectations are rational, which in this non-stochastic model means that they are correct\(^1\).

A subgame perfect, rational expectations equilibrium of the repeated

\(^1\)In principle the expectation for period \( t \)'s price can depend on the whole history of the game (productions, profits, and prices of all previous periods). As will be clear, the more simple formulation chosen here just simplifies the exposition and does not affect the results.
game consists of an expectation function for the participants in the auction, which is correct for all possible histories - including out of equilibrium histories - and a strategy for the big producer which is sequentially rational in all subgames.

First, we will find the condition under which an equilibrium, where firm 1 earns monopoly profits in the market, exists.

Suppose that the auction participants have the following expectations function

\[
p_t = \begin{cases} 
  p^m & \text{if } p_{t'} = p^m \forall t' < t \text{ or } t = 0 \\
  p^c & \text{otherwise}
\end{cases}
\]  

(3)

where

\[
p^m = \frac{a + c}{2}
\]

is the monopoly price.

The auction participants expect that the price will be the monopoly price as long as this has been the case in the past (or it is the very first period). If they ever see another price, they expect the Cournot price, \(p^c\), in all future. These are trigger expectations, which punishes firm 1 if it ever floods the market and makes the price go below \(p^m\). If the participants in the auction are unable to collude on bidding zero, this is the hardest punishment available. If they can collude on bidding zero for the virtual capacity an even harder punishment is available\(^2\). We will assume that the number of participants in the auction is sufficiently large that such collusion is not possible. Notice, however, that if such collusion is possible, and firm 1 thus can be punished even harder than assumed here, this would just make tacit collusion easier. In this sense our assumption stacks the deck against tacit collusion.

\(^2\)As is well-known the harder the punishment, the better equilibrium can be sustained. See Abreu (1988).
Given the expectations, firm 1 essentially has two options if the monopoly price \( p^m \) is expected for a period. Either it can choose \( q_1^m \equiv Q^m - q_2 \), where

\[
Q^m = \frac{1}{2} \frac{a - c}{b},
\]

is the monopoly output and get the advantage that the price expectation for the next period will be high. Alternatively firm 1 can deviate to the best possible production, which equals \( q_1^c (q_2) \). Then the price will fall, and price expectations for the future periods will be \( p^c \). If firm 1 chooses \( q_1^m \), and the price becomes \( p^m \), the profit of the virtual firm becomes

\[
\pi_2^m = (p^m - c) q_2 = \frac{a - c}{2} q_2.
\]

This will be the price of the virtual capacity in a period where \( p^m \) is expected. If firm 1 chooses \( q_1^m \) its total profit, from own production and selling the virtual capacity, becomes the total monopoly profit

\[
\pi^m = \pi_1^m + \pi_2^m = \frac{1}{4} \frac{(a - c)^2}{b}.
\]

If firm 1 chooses \( q_1^m \) in each period, it will then realize the monopoly profit in each period. If firm 1 deviates to \( q_1^c (q_2) \), total production in the period will be total Cournot production \( q_1^c (q_2) + q_2 \) and the firm’s total profit from the period will be \( \pi_1^c + \pi_2^m \), as the virtual producer expected \( p^m \) and the winning bid in the auction has been \( \pi_2^m \). In the next period, however, expectations will be that \( p = p^c \), so the winning bid will be \( \pi_2^c \) and the profit of firm 1 from that period and onwards will be \( \pi_1^c + \pi_2^c \). The no-deviation constraint for firm 1 therefore reads

\[
\frac{1}{1 - \delta} \pi^m \geq \pi_1^c + \pi_2^m + \frac{\delta}{1 - \delta} (\pi_1^c + \pi_2^c),
\]

(5)
which requires that firm 1’s discount factor \( \delta \) is no less than

\[
\hat{\delta} = \frac{\pi_1^c - \pi_1^m}{\pi_2^m - \pi_2^c} > 0.
\]  

(6)

(Recall that when firm one produces \( q_1^m \) it restricts output below \( q_1^c \), which is the best reply, so \( \pi_1^m < \pi_1^c \)).

In general, as long as the monopoly profit exceeds the sum of Cournot profits, i.e. as long as

\[
\pi_1^c + \pi_2^c < \pi_1^m + \pi_2^m,
\]

we have that

\[
\hat{\delta} = \frac{\pi_1^c - \pi_1^m}{\pi_2^m - \pi_2^c} < 1.
\]

So if the discount factor \( \delta \geq \hat{\delta} \), it is possible to realize the monopoly profit through tacit collusion. Notice, importantly, that this result does not depend on demand and cost being linear. It holds true under general concavity assumptions ensuring existence of Cournot equilibrium and optimum for the monopolist, as long as monopoly profit exceeds Cournot profit.

Finally, notice that if the discount factor is high and firm 1 chooses \( q_1^m \), the expectation that \( p = p^m \) in each period is correct. If in the past another price has been observed and the expectation becomes \( p = p^c \), then the big firm can only earn \( \pi_2^c \) in all future auctions regardless of its action in a period \( t \). The optimal level of production is therefore \( q_1^c (q_2) \) and the price becomes \( p^c \) as expected. Hence the price expectation is rational also off the equilibrium path. Summarizing the result.

**Theorem 1** For any demand and cost function, such that the monopoly profit exceeds the total profit of the Cournot equilibrium the following is true:
If the virtual firm is a short run player, there exists a subgame perfect equilibrium, where the firm 1’s total earnings equals the monopoly profit in each period, if the discount factor $\delta$ exceeds $\hat{\delta}$ given in equation (6).

The duration of a period equals the duration of the contract for the virtual capacity. For given time preferences, discount rate $r$, and duration of contract for virtual capacity, $\Delta t$, the relevant discount factor is

$$\delta = \exp(-r\Delta t).$$

This is smaller, the longer duration of the contract. A longer contract therefore makes it more difficult to fulfill the requirement that the discount factor exceeds the crucial discount factor $\hat{\delta}$. In this sense a longer contract makes tacit collusion more difficult.

In the linear model, we can get an explicit expression for $\hat{\delta}$. Inserting the relevant values gives

$$\hat{\delta} = \frac{(a-c-bq_2)^2}{4b} - \left(\frac{1}{4} \frac{(a-c)^2}{b} - \frac{a-c}{2} q_2\right) = \frac{1}{2} \left(\frac{a-c}{2} q_2 - \frac{(a-c-bq_2)q_2}{2}\right) = 1$$

(7)

This crucial discount factor does not depend on the amount of virtual capacity, $q_2$. Hence, in the linear model, if full collusion on the monopoly price is feasible, a larger amount of virtual capacity does not make tacit collusion more difficult. While interesting in itself, it is not a general result, it depends on the linear specification.

Suppose that the discount factor is less than $\hat{\delta}$, so that collusion on the monopoly price is impossible. The firm must then settle for partial collusion. The best profit, which can be realized, fulfills the incentive constraint (5) with equality. This is equivalent to

$$\pi_1 + \delta\pi_2 = \pi_1^c + \delta\pi_2^c$$
In order to proceed, we focus on the linear model. Then, we get the condition
\[(a - b(q_1 + q_2) - c)(q_1 + \delta q_2) = ((a - b(q_1^c(q_2) + q_2) - c)(q_1^c(q_2) + \delta q_2))\]
which has two solutions, \(q_1 = q_1^c(q_2)\) and
\[q_1 = \frac{a - c}{b} - q_2 - \delta q_2 - q_1^c(q_2)\]
Inserting into the profit functions and manipulating a bit, we get that the total profit to firm 1 from partial collusion is
\[\pi_1 + \pi_2 = \frac{(a - c)^2 - (bq_2 (1 - 2\delta))^2}{4b}\] (8)
For \(\delta = \frac{1}{2}\), this gives the monopoly profit as it should. It is increasing in \(\delta\) and decreasing in \(q_2\) for \(\delta < \frac{1}{2}\). For \(\delta = 0\), it equals the sum of Cournot profits.
The price is
\[p = \frac{a + c}{2} - \left(\frac{1}{2} - \delta\right) bq_2\] (9)
which is lower than the monopoly price and decreasing in \(q_2\) for \(\delta < \frac{1}{2}\) and decreasing in \(\delta\). Thus for discount factors below \(\frac{1}{2}\), larger virtual capacity is pro-competitive, it lowers the market price and it lowers the total profits of the firms.

**Theorem 2** In the linear model where the virtual producer is a short run player, the highest profit the firm 1 can obtain from partial collusion when \(\delta < \hat{\delta} = \frac{1}{2}\) is given by (8) and the price is given by (9). Both is increasing in the discount factor, \(\delta\), and decreasing in the amount of virtual capacity, \(q_2\). Larger virtual capacity is therefore pro-competitive.

We conclude that when the discount factor is so low that it is impossible to realize the monopoly profit, and the firm has to practise partial collusion,
then the market becomes more competitive, the larger the virtual capacity is and the lower the discount factor is. As discussed above the relevant discount factor is lower, the longer is the duration of the contract for the virtual capacity.

Suppose that firm 2 does not buy virtual capacity from firm 1, either because it owns capacity itself, or because, it buys it from somewhere else. It could, for instance, import the capacity from a neighboring country, whose market is separated from the one we consider here. When firm 2 is a short run player it will produce \( q_2 \) in each period. Firm 1 only receives profit from its own sale in the market and its non-deviation constraint (5) modifies to

\[
\frac{1}{1-\delta} \pi^m_1 \geq \pi^c_1 + \frac{\delta}{1-\delta} \pi^c_1,
\]

which is equivalent with the condition

\[
\pi^m_1 \geq \pi^c_1.
\]

As firm 2 is a short run player, who produces with all its capacity, \( q_2 \), we have that \( \pi^m_1 < \pi^c_1 \), so the non-deviation constraint (10) cannot be fulfilled for any \( \delta \). Tacit collusion is impossible. This is not surprising. In the Cournot equilibrium, firm 1 is playing best reply to \( q_2 \). Playing something less, must give less profit. If this does not make firm 2 reduce its output or firm 1 receives other revenues, as is the case in the auction for the virtual capacity, this can never be optimal.

It thus follows directly, that tacit collusion is easier when firm 2 relies on virtual capacity sold by firm 1, than when firm 2 either owns its own capacity or gets it from somewhere else.
4 Tacit collusion when both firms are long run players

The assumption that player 2 is a short run player may appear questionable if indeed firm 2 owns its own capacity and is in the market indefinitely. We therefore now assume that both firms are long run players, and compare the cases where firm two has virtual capacity and owns its own capacity respectively. As we assume that the firms are not symmetric (firm 2 is the small firm) it is an important question how the firms share the monopoly profit in the collusive phase. A natural benchmark is when the firms share the market in the collusive phase in the same way as they do in the absence of collusion. This would be the case if one conceives of equal split bargaining taking as the threat point the non-collusive profits. We will focus on this case below. In this case the market share of firm 1 is

\[ s_1 = \frac{q_1^c (q_2)}{q_1^c (q_2) + q_2}, \]

and the production of each firm in the collusive-monopolistic phase is \( q_1^{ml} = s_1 Q^m; q_2^{ml} = (1 - s_1) Q^m \). The profits are correspondingly \( \pi_1^{ml} = s_1 \pi^m; \pi_2^{ml} = (1 - s_1) \pi^m \).

As the formulas become a little heavy-handed, we focus on the linear model in the special case where \( a = 1, b = 1 \) and \( c = 0 \). Then

\[ q_1^c (q_2) = \frac{1 - q_2}{2}; \text{ and } Q^m = \frac{1}{2}, \]

and

\[ \pi_1^c (q_2) = \frac{(1 - q_2)^2}{4}; \pi_2^c (q_2) = \frac{(1 - q_2) q_2}{2}. \]

The productions in the collusive phase are

\[ q_1^{ml} = \frac{1 - q_2}{1 + q_2} \frac{1}{2} \text{ and } q_2^{ml} = \frac{q_2}{1 + q_2}. \]
If a firm decides to deviate from the collusive phase, the best reply is found using (1)

\[ q_1(q_{ml}^2) = \frac{1}{2(1 + q_2)}; \quad \text{and} \quad q_2(q_{ml}^1) = q_2, \]

and the associated profits are

\[ \pi^{dl}_1 = \frac{1}{4(1 + q_2)^2}; \quad \text{and} \quad \pi^{dl}_2 = \frac{1}{2} \left(1 + q_2 - 2q_2^2\right) \frac{q_2}{1 + q_2}. \]

If firm 2 has own capacity, the non-deviation constraint of firm 1 is

\[ \frac{1}{1 - \delta} s_1 \pi^m \geq \pi^{dl}_1 \frac{1}{s_1} + \frac{\delta}{1 - \delta} s_1 \pi^c, \]

which we can rewrite

\[ \frac{1}{1 - \delta} \pi^m \geq \pi^{dl}_1 \frac{1}{s_1} + \frac{\delta}{1 - \delta} \pi^c, \]

while that of firm 2 is

\[ \frac{1}{1 - \delta} \pi^m \geq \pi^{dl}_2 \frac{1}{s_2} + \frac{\delta}{1 - \delta} \pi^c. \]

As

\[ \frac{\pi^{dl}_1 \frac{1}{s_1}}{\pi^{dl}_2 \frac{1}{s_2}} = \frac{1}{(1 - q_2)^2 (1 + q_2 - 2q_2^2)} > 1, \]

we have that

\[ \pi^{dl}_1 \frac{1}{s_1} > \pi^{dl}_2 \frac{1}{s_2}, \]

so that the constraint for firm 1 is the most binding constraint.

Inserting the relevant values we get

\[ \frac{1}{1 - \delta} \frac{1 - q_2}{1 + q_2} \frac{1}{4} \geq \frac{1}{4(1 + q_2)^2} + \frac{\delta}{1 - \delta} \frac{1 - q_2}{1 + q_2} \frac{(1 - q_2)^2}{4}, \]

which is fulfilled if \( \delta \) exceeds

\[ \delta_i = \frac{q_2}{2 - 2q_2^2 + q_2^3}. \]
It can be verified (see figure 1 below) that for $q_2 \in [0, 1/3]$, $\delta_i \in [0, 1]$. So if firm 2 is independent - has its own plant - and participates in the collusion, collusion on the monopoly price is possible if $\delta$ is sufficiently high.

Finally, let us consider the case where there is virtual capacity and firm two participates in the collusion. The everything is as just described, except firm 1 gets the profits earned by firm 2 back in the auction. In the normal phase, firm 1 reaps the whole monopoly profit, $\pi^m$. The relevant non-deviation constraint is therefore

$$\frac{1}{1 - \delta} \pi^m \geq \pi_1^{dl} + s_2 \pi^m + \frac{\delta}{1 - \delta} \left( \pi_1^c + \pi_2^c \right),$$

and we get

$$\frac{1}{1 - \delta} \frac{1}{4} \geq \frac{1}{4(1 + q_2)^2} + \frac{2q_2}{1 + q_2} \frac{1}{4} + \frac{\delta}{1 - \delta} \frac{(1 - q_2)^2}{4},$$

which is fulfilled for $\delta$ exceeding.

$$\delta_v = \frac{q_2}{2 + 4q_2 - q_3^2}, \quad (12)$$

which clearly belongs to $[0,1[$.

Comparing the two discount factors it is easily checked that for $q_2 \in [0, 1]$, $\delta_i > \delta_v$. Hence tacit collusion is again easier with virtual capacity than without.

**Theorem 3** Consider the linear model with $a = b = 1$, and $c = 0$. If firm 2 is a long run player, tacit collusion on the monopoly price takes that the discount factor exceeds $\delta_i$ given by (11) if firm 2 owns its own plant and that it exceeds $\delta_v$ given by (12) if it is a virtual producer. As $\delta_i > \delta_v$, the requirement to the discount factor is less when capacity is virtual.

The two crucial discount factors are plotted in the figure 1 below.
We see from the Figure that the two crucial discount factors $\delta_i$ and $\delta_v$ are below the value for $\hat{\delta}$ in the linear model, where $\hat{\delta} = \frac{1}{2}$. In this sense tacit collusion is easier when firm 2 is a long run player, whether it relies on virtual capacity or not. This points to that it is important that the auction is competitive and that the format is such that it induces shortsightedness on the part of the buyer if one is interested in promoting the competitive effects of virtual capacity.

If the discount factor is so low that collusion on the monopoly profit is impossible, the firms can collude partially.

In this case the non-deviation constraint is fulfilled with equality. For firm 1 it reads

$$\frac{1}{1 - \delta} s_1 (\pi_1 + \pi_2) = \pi^d_1 + \frac{\delta}{1 - \delta} s_1 (\pi^c_1 + \pi^c_2).$$

When total production in the collusive phase is $Q$, firm 1’s production is $s_1 Q$, while firm 2’s is $(1 - s_1) Q$. The best reply for firm 1, when 2 produces $(1 - s_1) Q$ is $\frac{1 - (1 - s_1) Q}{2}$. Inserting this into the non-deviation constraint gives
two solutions: the total Cournot production and
\[ Q = \frac{1}{2 (1 - \delta q_2^2)} (1 + q_2 - \delta q_2 (2 + q_2 - q_2^2)). \]
The total profit is therefore
\[ \pi_i = (1 - Q) Q = \frac{1}{4} \frac{(1 - q_2^2)(1 + \delta q_2^2 (2 - 4 \delta + \delta q_2^2))}{(1 - \delta q_2^2)^2}. \]
For \( q_2 = 0 \), this gives the monopoly profit, as it should. Then consider the case where there is virtual capacity and firm two participates in the collusion. The non-deviation constraint is
\[ \frac{1}{1 - \delta} (\pi_1 + \pi_2) = \pi_1^d + s_2 \pi_2 + \frac{\delta}{1 - \delta} (\pi_1^c + \pi_2^c) \]
Inserting the relevant values we again find two solutions in \( Q \), the Cournot production and
\[ Q = \frac{1 + q_2 - \delta q_2^2 - \delta q_2^3}{2 + 4 \delta q_2 + 2 \delta q_2^2} \]
The total profit is thus
\[ \pi_v = (1 - Q) Q = \frac{1}{4} \frac{(4 \delta q_2 - q_2 + 3 \delta q_2^2 + \delta q_2^3 + 1) (1 - \delta q_2^2) (q_2 + 1)}{(2 \delta q_2 + \delta q_2^2 + 1)^2}. \]
For \( q_2 = 0 \), this again gives the monopoly profit - as it should.

The expressions for the profits are unfortunately a little heavy-handed. Tedious manipulations however reveal that indeed \( \pi_v > \pi_i \) for the relevant parameter-values \( q_2 \in [0, 1/3] \) and \( \delta \in [0, \frac{1}{10}] \). Given the results above, this is by now unsurprising. Numerical examples also show that these profits are decreasing in \( q_2 \) and increasing in \( \delta \) as expected.

5 Concluding remarks

Virtual capacity has been a new and interesting feature in major merger cases. Competition authorities have tried to mitigate the anti-competitive
effects of mergers by requiring that the merging firm auctions off virtual capacity to prospective competitors in the final market. We have shown that indeed in a static setting this has the expected competitive effects. As regards tacit collusion the picture is more blurred. The fact that the merging firm pockets the profit of the competitor through the initial auction, gives incentives to moderate production so that the price is kept high, the value of the virtual capacity is enhanced and so is auction revenue. This is a further incentive to tacit collusion, which is not present when the competitor is an independent producer.

References


