International Parity Relationships Between Germany and the United States: A Joint Modelling Approach

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Abstract

This paper examines the interrelations between purchasing power parity, uncovered interest parity, the term structure of interest rates and the Fisher real interest rate parity using cointegration analysis. Dynamic adjustment and feed-back effects are estimated jointly in a full system of equations. An important finding is that the very slow, though significant, price adjustment towards sustainable levels of real exchange rates, has been compensated by corresponding changes in the spread of long-term bond rates. Related to this is the strong empirical support for the weak exogeneity of long-term bond rates, signifying the importance of the large US trade deficits (i.e. the low levels of US savings) and, hence, their linkage to international finance. Altogether, the results suggest that the transmission mechanisms over the post Bretton Woods period have been significantly different from standard theoretical assumptions.

JEL Classifications: E31, E43, F31, F32.

Keywords: PPP, UIP, Fisher parity, Term structure.
1 Introduction\(^1\)

Parity conditions are central to international finance and, more specifically, to many open economy macro-models, such as the celebrated Dornbusch (1976) overshooting model. Although international parity conditions, such as purchasing power parity (\textit{PPP}) and uncovered interest rate parity (\textit{UIP}), have received considerable empirical scrutiny, very little empirical research has focussed on modelling such conditions jointly (exceptions are Johansen and Juselius (1992), Juselius (1991,1995) and MacDonald and Marsh (1997,1999)). This perhaps seems surprising since such parity conditions can be shown to be closely linked through interest rates and expected inflation. By modelling international parity conditions jointly, extra information may brought to bear on each individual parity condition, thereby increasing the likelihood of establishing well-defined results. In this paper we attempt to push this nascent literature further by jointly modelling \textit{PPP} and \textit{UIP} with the term spread (\textit{TS}), or yield gap, for Germany against the United States, over the period 1975 to 1998. In addition to shedding light on the interaction of these parity conditions, we hope to address a number of unresolved issues.

One important issue concerns the persistence in real exchange rates. For example, a number of studies have demonstrated that for the recent floating experience real exchange rates are \textit{I}(1) processes (see Froot and Rogoff (1995) and MacDonald (1995) for surveys). The modelling approach adopted in this paper shows that although this non-stationarity may be removed using inflation and interest differentials, it, in turn, is an important determinant of interest differentials and inflation. A second issue, which is essentially a corollary of the first, concerns the extent to which German (European) or US variables are the driving variables in the system. For much of the post-war period, particularly during the Bretton Woods period, the US has been seen as the 'locomotive' economy. But with increased integration and convergence in Europe it may be expected that European variables, represented here by Germany, will be as important in international financial linkages as US variables. A third issue we seek to address is the extent to which 'implicit' parity conditions - namely the Fisher conditions and real interest rate parity

\(^1\)A special thank goes to Charles Lai Tong who found some mistakes in a previous version of this paper. In addition the paper has benefitted from valuable comments from, Frederique Bec, David Hendry, Hans Christian Kongsted, Grayham Mizon, Adrian Pagan, Hans-Martin Krolzig, and Soren Johansen. Financial support from the Danish Social Sciences Research Council is gratefully acknowledged.
- hold for our sample period. Thus although the linkage between nominal interest rates, as in UIP, describes capital mobility between financial centres, it is the lock between real interest rates which governs the efficiency with which savings and investment are allocated internationally. To what extent does the joint modelling of UIP, PPP and the TS shed light on this issue?

The outline of the remainder of this paper is as follows. In the next section we provide a motivational discussion of a number of parity conditions used in this paper. In Section 3 a visual interpretation of the parities is presented, while in Section 4 our model specification is detailed. Section 5 contains the cointegration and weak exogeneity properties of our system. A fully identified long-run structure is detailed in Section 6, while the short-run adjustment structure is contained in Section 7. The estimated long-run impacts of shocks to the system are reported in section 8. The final section of the paper contains summary results and conclusions.

2 International Parity Conditions.

Purchasing power parity (PPP), one of the most important parity condition in international finance, is defined as:

\[ p = p^* + e, \] (1)

where \( p \) is the log of the domestic price level, \( p^* \) is the log of the foreign price level, and \( e \) denotes the log of the spot exchange rate (home currency price of a unit of foreign currency). Thus, the departure at time \( t \) from (1) is given by:

\[ ppp_t = p_t - p_t^* - e_t. \] (2)

The strong form of PPP requires \( ppp_t^2 \) to be stationary.

The nature of the empirical support for PPP is very dependent on the sample period chosen in the following sense: if the time it takes for \( ppp \) to return to its steady-state value is very long, say ten years, then we need a

\footnote{Note that the \( ppp \) term is also the (logarithm) of the real exchange rate. We prefer to use the label \( ppp \) in this paper because we are adopting a parity perspective and also because we do not model the real exchange rate in terms of so-called real fundamentals.}
long sample to get statistically significant mean reversion effects. Over century long historical data spans, there is mounting evidence that a version of the strong-form PPP is valid, but with a very small adjustment coefficient (see, for example, Froot and Rogoff (1995) and MacDonald (1995)). For the recent floating experience the sample period is too short for such a small adjustment coefficient to be statistically significant and, thus, econometrically $ppp_t$ behaves as an $I(1)$ process.

Though there are many potential reasons why the adjustment to strong-form $PPP$ is so slow we will here primarily pursue the idea that the persistence in $ppp_t$ is due to the existence of important real factors working through the current account, such as productivity differences, net foreign asset positions and fiscal imbalances. This hypothesis has received some empirical support by researchers who have explicitly modelled the real determinants of real exchange rates (see the papers contained in MacDonald and Stein (1999)).

However, through the balance of payments constraint we know that any current account imbalance generated by such movements has to be financed through the capital account. The implication of this is that $PPP$ condition is likely to be strongly related with another parity condition, namely uncovered interest rate parity ($UIP$) (see Johansen and Juselius (1992), Juselius (1991,1995) and MacDonald and Marsh (1997,1999)). Therefore, by combining the two parity conditions we may pick up the influence of the real factors on $PPP$ indirectly.

The condition of $UIP$ may be stated as:

$$E_t^e(\Delta_m e_{t+m})/m - (i_t^m + i_t^{m*}) = 0,$$

where $i_t^m$ denotes a bond yield with maturity $t + m$, $m = l, s$ where $l$ and $s$ denote a long and short maturity, respectively, and $E_t^e$ denotes an economic expectation on the basis of time $t$ information. A number of researchers (see, for example, Cumby and Obstfeld (1981) ) have tested this version of $UIP$ and essentially find that $\varepsilon_t$ is non-stationary. However, when the $UIP$ condition is modelled jointly with $PPP$ more satisfactory results have been obtained in the sense that deviations from the conditions are stationary and the sign of the coefficients conform with priors. Nevertheless, the empirical

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3See Juselius (1999) for a discussion of the statistical versus economic interpretation of unit root econometrics.
evidence strongly suggests that the assumption of market clearing underlying (3) would have to be replaced by an assumption of price adjustment.

This takes us to two further parity conditions, related to PPP and UIP, which are needed for a full understanding of some of the puzzles noted in the introduction. The first relates interest rates of different maturities, based on the expectations model of the term structure. In this model it is assumed that a long rate is a weighted average of current and expected rates of shorter interest rates, and short rates are predicted to ‘drive’ long rates. An implication of the standard expectations model of the term structure is that the term spread ($TS$) should be stationary (Campbell and Shiller, 1987). The $TS$ is defined as:

\[ i^l_t - i^s_t = v_t, \]  

(4)

where $i^s_t$ denotes the yield on a short maturity bond, $i^l_t$ on a long maturity, and $v_t$ denotes a generic random error term which under the expectations hypothesis should be stationary. However, based on a variety of empirical tests (see Campbell, 1995) $v_t$ has often been found to be non-stationary.

It is conventional to think of nominal interest rates being decomposed into real and expected inflation components using the Fisher decomposition:

\[ i^m_t = r^m_t + E_t^e (\Delta_m p_{t+m}) / m, \]  

(5)

where $r$ denotes the real interest rate. Combining (4) and (5) gives:

\[ i^l_t - i^s_t = r^l_t - r^s_t + E_t^e \left\{ \frac{(\Delta_{l-s} p_{t+1})}{(l-s)} \right\} + v_t, \]

showing that a nonstationary interest rate spread is logically consistent with expected inflation rate being a nonstationary variable. Since actual inflation is frequently found to be nonstationary variable this seems to be a plausible explanation to the finding that interest rate spreads are nonstationary.

The final parity condition to consider here is that of real interest rate parity ($RIP$):

\[ r^m_t - r^m_{t^*} = (i^m_t - i^m_{t^*}) - (E_t (\Delta_m p_m - \Delta_m p_{m^*}) / m) = v_t \]  

(6)
where \( m \) is the maturity of the underlying asset. The empirical literature on RIP usually focuses on testing if the restrictions necessary to move from (3) and (5) to (6) actually hold in the data. The majority of such studies find that RIP is strongly rejected for most country pairings (see, for example, the overview in Hallwood and MacDonald (1999)). By combining (3), (5) and (4) we get:

\[
(i_t^d - i_t^*d) - (i_t^s - i_t^*s) = E_t(\Delta_l e_{t+l} - \Delta_s e_{t+s})/(l - s)
\]

which shows that if the spread between expected domestic and foreign inflation from \( t + s \) to \( t + l \) is nonstationary, then the spread between domestic and foreign yield gap would also have to be nonstationary. Since inflation is found to be nonstationary in itself this seems very plausible. In this view (5), (4) and (7) are likely to be non-stationary, or \( I(1) \).

We now draw out the implications for the modelling of PPP, UIP and TS under the assumption that the simple parity conditions are nonstationary and that the very slow adjustment to sustainable real exchange rates is the basic reason for this nonstationarity. We formulate the following hypothetical adjustment relations for the spot exchange rate:

\[
E_t^e \Delta_l e_{t+l} = \omega_1 E_t^e \Delta_l(p_{t+l} - p_{t+l}^*) + \omega_2 E_t^e \text{ppp}_{t+l} + v_t,
\]

where the expected depreciation can be related to the expected inflation differential and to the expected real depreciation rate, with the weights \( \omega_1 \) and \( \omega_2 \). If the expected exchange rate in (3) is formed using (8) we can derive a relationship by combining the PPP and the UIP conditions:

\[
(i_t^d - i_t^*d) - (i_t^s - i_t^*s) = \omega_1 E_t(\Delta_l s - \Delta_l s^*)/(l - s)_{t+l} + \omega_2 E_t^e \text{ppp}_{t+l} + v_t.
\]

Even if expectations are generally not observable the cointegration results will be unaffected when replacing expectations with actual values under the following two conditions: (i) the difference between \( E_t(x_{t+l}) \) and \( x_{t+l} \) is stationary or, preferably, white noise (i.e. agents do not make systematic forecast errors), (ii) the differenced process \( (x_{t+l} - x_t) \) is stationary. Under these
two assumptions we can derive an empirical relationship between the interest rate spreads, the inflation spread and the real exchange rate:

\[ i_t^i - i_t^s = \omega_1 (\Delta p - \Delta p^*)_t + \omega_2 (i_s - i_s^*)_t + \omega_3 ppp_t + v_t. \] (10)

Thus, we note that implicit in (10) is all of the parity relationships discussed above: the two Fisher conditions, international real interest rate parity condition, the \textit{ppp} condition, and the term structure relationship. For example, (10) becomes the real long-term interest parity relationship for \( \omega_1 = 1 \) and \( \omega_2 = 0, \omega_3 = 0 \). By modelling these relationships jointly we can test the stationarity of the simple parity conditions as special cases of (10). If these are rejected we can test whether combinations of the parity relationships become stationary.

3 An ocular analysis of the parities

In this section we offer a first pass at how closely the various parity conditions considered above hold. We also introduce some of the relevant institutional background which will have a bearing on our econometric results.

The salient feature of the graphs in Figures 1, 2, and 3\(^4\) is the slow adjustment back to the parities. Figure 1, upper panel, shows clearly that the spot exchange rate does not closely mirror the price differential between Germany and the USA, although there seems to be a tendency for it to follow the same (very) long-run movements. The much greater variation in the spot exchange rate as compared to the price differential is quite striking\(^5\). In particular, the period between 1980 and 1985 (showing up here as a depreciation of the mark) is notable. Lothian (1997), for example, has argued that the behavior of the dollar in this period is likely to confound any test of \textit{PPP} for the recent floating period when the US dollar is used as the numeraire currency. Given the importance of this episode for the kinds of tests conducted in this paper, we believe it merits a brief discussion here.

The dollar appreciation was kick-started by the effects on interest rates of the so-called 'Reagan Experiment' of increasing the US fiscal deficit. However, the prolonged nature of the appreciation would seem to be unwarranted

\(^{4}\)The measurements of the variables discussed in this section are defined in Section 5.

\(^{5}\)See for example Krugman (1993) for an economic explanation.
solely in terms of a real interest differential or, indeed, other fundamentals which were extant at the time, such as portfolio balance or 'safe-haven' effects (see MacDonald (1988)). The consensus view seems to be that in large part it was a speculative bubble, unrelated to economic fundamentals, which drove the currency to such stratospheric levels. However, whatever the actual cause of the dollar’s rise we believe that ultimately it could not have behaved as it did if it was not accepted as the key reserve currency in the international monetary system. The role of the dollar as a reserve currency is an important element in how we interpret our results.

The lower panel of Figure 1, shows that the long movements of the ppp cannot directly be related to an adjustment of the inflation rates; the inflation spread appears too small to facilitate a long-run adjustment towards a stationary level of real exchange rates. Figure 2 relates the ppp term to the bond rate spread in the upper panel and to the Treasury bill rate spread in the lower panel. There is a quite remarkable co-movement in the long-run
behavior of the real exchange rate and the long bond differential. However, there is not the same close correspondence with respect to the short-term Treasury bill rates. This in large measure reflects the nature of these two yields. The latter are driven by short term policy considerations, whereas the former are market determined and have a term to maturity which more closely matches the long persistence in the real exchange rate (we discuss the importance of relative interest rates further below). Figure 3 demonstrates the large variation in real bond rates over this period. This is particularly so for the US real bond rate, which has varied between -7% and +15%. These are huge variations considering that theoretically it is usually assumed to be constant!

Finally, Figure 4 compares the spread of the bond rates and of the Treasury bill rates in the upper panel, and the inflation rate spread with treasury bill rate spread (middle panel) and the bond rate spread (lower panel), respectively. There are clearly periods in which both spreads mirror relative inflation quite closely and periods in which they diverge and the real interest
Figure 3: Real yearly bond rates (in 0.01%) for Germany (upper panel) and USA (lower panel) together with ±6 months moving averages.

rate spreads open up. The extent to which such real interest rate spreads are consistent with real interest rate parity is something we investigate formally in Sections 6 and 7.

The graphical inspection demonstrated a fair degree of persistence both in the spreads and the parities which is inconsistent with the stationarity assumption of the simple parities. Econometrically, we will treat these persistencies as stochastic trends and use cointegration analysis to find out how they are related. This is based on the simple idea that a persistent imbalance in one place should create a corresponding imbalance in another. The purpose is to use the econometric analysis to suggest reasons why these simple parity relationships are inadequate on their own and how they could be modified to describe the variation in the data.
Figure 4: The monthly bond rate spread and Treasury bill rate spread (upper panel), the inflation rate spread relative to the treasury bill rate spread (middle panel) and to the bond rate spread (lower panel).

4 Model specification

All test and estimation results are based on the VAR model with a constant term, \( \mu \), seasonal dummies, \( S_t \), and intervention dummies, \( D_t \), given by:

\[
\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Gamma \Delta x_{t-2} + \mu + \Phi_1 S_t + \Phi_2 D_t + \varepsilon_t,
\]

where \( x_t \) is a vector of the following monthly variables:

\[
[ppp, \Delta p_t, \Delta p_t^*, i_t^d, i_t^s, i_t^r, i_t^{rs}] \sim I(1)
\]

observed for \( t = 1975:7-1998:1 \). The set of variables is defined by

- \( ppp_t = p_t - p_t^* - e_t \), where
- \( p_t \) = the German, or ‘home’, price index,
- \( p_t^* \) = the US, or ‘foreign’, price index,
- \( e_t \) = the spot exchange rate, defined as DM/$,}

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\( i_t^l \) = the German long bond yield,
\( i_t^{l, *} \) = the US long bond yield,
\( i_t^s \) = the German 3 month Treasury bill rate,
\( i_t^{s,*} \) = the US 3 month Treasury bill rate.

The deviations from constant \( ppp \) are very large in absolute terms compared to the remaining variables in (12). Therefore, the \( ppp \) term has been divided by 100 to avoid getting very small coefficients in absolute magnitude. Nevertheless, the interpretation of the results are for the original \( ppp \) term.

All of the data used in this study have been extracted from the International Monetary Funds CD-Rom disc (December 1998). Both price series are Consumer Price Indices (line 64), the long interest rates are 10 year bond yields (line 61), the short rates are Treasury bill rates (60c), and the exchange rate is the end of period rate (line ae). All variables, apart from the interest rates (which appear as fractions) are in natural logarithms. The graphs of the variables in levels and in differences are given in the Appendix.

It appears from the graphs of the differenced variables in Appendix II that the multivariate normality assumption underlying (11) is not likely to be satisfied. Many of the marginal processes exhibit extraordinarily large observations inconsistent with the normality assumption. This is particularly so for the short-term treasury bill rates, signifying the high volatility of short-term interest rates in 1980-1983, the period of M3 targeting. To secure valid statistical inference we need to control for the largest of these observations by dummy variables or leave out the most volatile years from our sample. Since the volatile years could potentially be informative about agents' behavior we choose the former alternative and use a dummy when a residual is larger than \( |3.5\sigma| \). The implications of this criteria is that most observations in 1979-1982 were classified as 'too large' and dummied out. Thus, the impact of this period is more or less annihilated in the results. This is consistent with the findings in Hansen and Johansen (1999) that this period defines a structurally different regime.

The following dummy variables where used in the analysis:

\[ D_t' = [D_{78.09}, D_{78.10}, D_{79.12}, D_{79.11}, D_{80.02}, D_{80.03}, D_{80.05}, D_{80.07}, D_{80.11}, D_{81.01}, D_{81.02}, D_{81.03}, D_{81.05}, D_{81.10}, D_{81.11}, D_{82.01}, D_{82.08}, D_{82.10}, D_{84.12}, D_{88.08}, D_{89.02}, D_{91}, D_{s91.03}, \Delta D_{s91.03}]_t, \]

where, \( D_{xx.yy} \) is 1 at 19xx:yy, 0 otherwise, \( D_{xx.yy}t \) is 1 at 19xx:yy, -1 at 19xx:yy+1, and 0 otherwise, \( D_{91} \) is a variable measuring the effect on German prices of various excise taxes to pay for the German reunification, and \( D_{s91.03} \) is 0 for \( t = 1975:7 - 1991:03 \) and 1 otherwise. \( D_{s91.03} \) is restricted
Table 1: Misspecification tests and cointegration rank

<table>
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<td>Residual autocorr. $LM_1$</td>
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<th>$\Delta \tilde{v}_t$</th>
<th>$\Delta v_t^{***}$</th>
<th>$\Delta i_t^*$</th>
<th>$\Delta i_t^{**}$</th>
<th>$\Delta ppp_t$</th>
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<td>2.28</td>
<td>9.11</td>
<td>2.06</td>
<td>2.42</td>
<td>8.34</td>
<td>4.85</td>
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<td>Jarq.Bera(2)</td>
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<td><strong>8.12</strong></td>
<td><strong>6.72</strong></td>
<td><strong>7.36</strong></td>
<td><strong>48.26</strong></td>
<td><strong>2.90</strong></td>
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<td>Skewness</td>
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<td>0.01</td>
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<td>0.24</td>
<td>-0.09</td>
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<td>3.79</td>
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<td>3.69</td>
<td>3.73</td>
<td>5.52</td>
<td>3.40</td>
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<td>$\hat{\sigma}_e \times 0.01$</td>
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<td>0.15</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
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The trace test and the characteristic roots of the process:

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<th>$p - r$</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
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<td>$Q_{95}$</td>
<td>132</td>
<td>102</td>
<td>76</td>
<td>53</td>
<td>35</td>
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<td>9</td>
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<td>$\lambda$</td>
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<td>0.08</td>
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<td>0.01</td>
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<td>183</td>
<td>85</td>
<td><strong>43</strong></td>
<td>22</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

Modulus of 7 largest roots

$r = 4$

| 1.0 | 1.0 | 1.0 | **0.96** | 0.87 | 0.54 | 0.35 |

$r = 3$

| 1.0 | 1.0 | 1.0 | 1.0 | **0.78** | 0.61 | 0.39 |
to lie in the cointegration relations to avoid a broken linear trend effect in the model. By controlling for these extraordinary shocks the residuals of the VAR model became reasonably well-behaved as seen from Table 1, where a significant test statistic is given in bold face. The multivariate LM test for first order residual autocorrelations is borderline significant, whereas multivariate normality is clearly rejected due to excess kurtosis. Furthermore, the ARCH(2) tests for second order autoregressive heteroscedasticity and is rejected for the German bond rate and the US treasury bill rate. Since cointegration results have been found quite robust to ARCH and excess kurtosis (Gonzalo, 1994) we regard the present model specification to be acceptable.

In the lower part of Table 1 we report the estimated eigenvalues and trace statistics associated with this system. The trace test suggests four common stochastic trends and, consequently, three cointegration relations. However, the trace statistic for \( p - r = 4 \) is quite close to the 95% quantile, which might suggest that the theoretically more acceptable case \( p - r = 3 \) might be acceptable. To check the sensitivity of the model to the choice of \( r \) we have also calculated the roots of the characteristic polynomial. There are approximately four 'near unit roots' in the unrestricted system, the choice of \( r = 3 \) removes all large roots, whereas \( r = 4 \) leaves a near unit root in the model. We conclude that \( r = 3 \) is the appropriate choice and, hence, that the treasury bill rates have been subject to permanent shocks (disturbances) which are not shared by the other variables of the system. Therefore, the fourth stochastic trend is likely to describe the cumulative impact of monetary intervention shocks.

5 Cointegration properties and weak exogeneity

The hypotheses reported in Table 2 have the form \( H_i : \beta_i = \{H_i\phi_i, \psi_i\} \), that is they test whether a single restricted relation is in \( sp(\beta) \) leaving the other two relations unrestricted. Only the restricted vectors \( H_i\phi_i, i = 1, \ldots, 25 \), are reported in the table. If the hypothetical relations exist empirically, then this procedure will maximize the chance of finding them. For a technical derivation of the test procedures, see Johansen and Juselius (1992).

\( H_1 \) to \( H_7 \) are hypotheses tests on pairs of variables, such as relative inflation \( (H_1) \), relative interest rates \( (H_2 \text{ and } H_3) \), Fisher parity conditions
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<th>$\mathcal{H}$</th>
<th>$\Delta p$</th>
<th>$\Delta p^*$</th>
<th>$i^*$</th>
<th>$i^*$</th>
<th>$i^{**}$</th>
<th>$ppp^{1)}$</th>
<th>$\chi^2(v)$</th>
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<td>1</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>25.9 (3)</td>
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<td>$\mathcal{H}_2$</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>0</td>
<td>30.3 (4)</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>28.4 (4)</td>
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<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7.5 (4)</td>
<td>0.11</td>
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<td>$\mathcal{H}_5$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>24.8 (4)</td>
<td>0.00</td>
</tr>
<tr>
<td>$\mathcal{H}_6$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>28.6 (4)</td>
<td>0.00</td>
</tr>
<tr>
<td>$\mathcal{H}_7$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>23.3 (4)</td>
<td>0.00</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>35.1 (4)</td>
<td>0.00</td>
</tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>20.2 (4)</td>
<td>0.00</td>
</tr>
<tr>
<td>$\mathcal{H}_{10}$</td>
<td>1</td>
<td>-1</td>
<td>0.51</td>
<td>-0.51</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-0.09</td>
<td>0.09</td>
<td>12.6 (3)</td>
<td>0.01</td>
</tr>
<tr>
<td>$\mathcal{H}_{12}$</td>
<td>1</td>
<td>-0.21</td>
<td>-1</td>
<td>0.21</td>
<td>0</td>
<td>0</td>
<td>2.7 (3)</td>
<td>0.44</td>
</tr>
<tr>
<td>$\mathcal{H}_{13}$</td>
<td>0.14</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-0.14</td>
<td>-1</td>
<td>23.3 (3)</td>
<td>0.00</td>
</tr>
<tr>
<td>$\mathcal{H}_{14}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>-0.68</td>
<td>0.68</td>
<td>21.1 (3)</td>
<td>0.00</td>
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<tr>
<td>$\mathcal{H}_{15}$</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6.2 (3)</td>
<td>0.08</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>16.7 (3)</td>
<td>0.00</td>
</tr>
<tr>
<td>$\mathcal{H}_{17}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>25.2 (3)</td>
<td>0.00</td>
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<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2.8 (3)</td>
<td>0.43</td>
</tr>
<tr>
<td>$\mathcal{H}_{19}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>2.9 (3)</td>
<td>0.41</td>
</tr>
<tr>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>27.7 (3)</td>
<td>0.00</td>
</tr>
<tr>
<td>$\mathcal{H}_{21}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>4.6 (3)</td>
<td>0.21</td>
</tr>
<tr>
<td>$\mathcal{H}_{22}$</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2.6 (3)</td>
<td>0.46</td>
</tr>
<tr>
<td>$\mathcal{H}_{23}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>-0.41</td>
<td>0.41</td>
<td>0.37</td>
<td>0.29 (2)</td>
</tr>
<tr>
<td>$\mathcal{H}_{24}$</td>
<td>1</td>
<td>-1</td>
<td>1.58</td>
<td>-1.58</td>
<td>-1</td>
<td>1</td>
<td>0.8 (3)</td>
<td>0.84</td>
</tr>
<tr>
<td>$\mathcal{H}_{25}$</td>
<td>1</td>
<td>-0.34</td>
<td>-0.66</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.15 (3)</td>
<td>0.87</td>
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</table>

$W.E.$ | 110 | 55 | 0.2 | 3.2 | 16.6 | 21.9 | 9.5 |
<table>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(.00)</td>
<td>(.00)</td>
<td>(.98)</td>
<td>(.36)</td>
<td>(.00)</td>
<td>(.00)</td>
<td>(.02)</td>
</tr>
</tbody>
</table>

*Note 1:* The $ppp$ term has been divided by 100

*Note 2:* All relations are estimated with a constant and the 1991 shift dummy
for long and short interest rates ($\mathcal{H}_4$ to $\mathcal{H}_7$) and yield gap relationships ($\mathcal{H}_8$ and $\mathcal{H}_9$). These tests therefore seek to determine if some of the key parity conditions introduced in Section 2 are empirically verifiable on their own. Since all, apart from one, of the p-values are less than the 5% critical value, these tests offer little support for the parity conditions on their own. The remaining hypotheses tests in Table 3 involve combining parity conditions without the $ppp$ term ($\mathcal{H}_{10}$ to $\mathcal{H}_{14}$ and $\mathcal{H}_{24}$ and $\mathcal{H}_{25}$), and combining parity conditions with the $ppp$ term ($\mathcal{H}_{15}$ to $\mathcal{H}_{23}$).

$\mathcal{H}_{10}$ to $\mathcal{H}_{13}$ are tests of variants of real interest rate parity in which full proportionality has not been imposed. Restricting the two inflation rates to have unitary coefficients and the nominal interest rates to have equal and opposite signs ($\mathcal{H}_{10}$ and $\mathcal{H}_{11}$) is rejected. Relating the $ex$ post German real long-term interest rate with $ex$ post US real long-term interest rate ($\mathcal{H}_{12}$) gives a stationary relation with a p-value of 0.44, but with a very small coefficient to the US rate. A similar test for the $ex$ post real short-term interest rates is rejected ($\mathcal{H}_{13}$). Testing a form of the relative term structure relationship ($\mathcal{H}_{14}$) is clearly rejected. Therefore, combinations of parity conditions which do not include the $ppp$ term are not very successful.

Hypothesis tests $\mathcal{H}_{15}$ to $\mathcal{H}_{23}$ involve joint tests of parity conditions which include the $ppp$ term. With these tests there is now a high strike record of the joint parity conditions producing stationary relationships. For example, in $\mathcal{H}_{18}$ and $\mathcal{H}_{19}$ we note that the strong form of the Fisher condition (that is with proportionality imposed) goes through for long rates when the $ppp$ term is in the conditioning information set. The usefulness of including the $ppp$ term in these kind of tests is underscored in $\mathcal{H}_{22}$ in which $ex$ post real interest rates are equalized across countries. This result, which does not receive much support in the extant empirical literature, implies that a strict form of real interest rate parity is likely to be found in periods of a stationary $ppp$ exchange rate. It is also interesting to note that the long interest differential seems to play a similar role to the $ppp$ term since its inclusion with the short rates and inflation rates produces a strong form of real interest parity for the short rates ($\mathcal{H}_{24}$). Including the $ppp$ term in the relative interest rate relationships does not, however, produce stationary relationships ($\mathcal{H}_{16}$ and $\mathcal{H}_{17}$, respectively) and only weak support for stationarity for $ppp$ and relative inflation rates ($\mathcal{H}_{15}$). Finally, $\mathcal{H}_{25}$ describes a homogeneous relationship (i.e., the coefficients sum to zero) between German inflation, US inflation, and the German bond rate.

The test of long-run weak exogeneity (Johansen and Juselius, 1990) in-
vestigates the absence of long-run levels feedback and is formulated as a zero row of \( \alpha \), i.e. \( H^i_\alpha : \alpha_{ij} = 0, j = 1, ..., r \), where \( H^i_\alpha \) is a hypothesis that the variable \( x_i, i = 1, ..., p \), does not adjust to the equilibrium errors \( \beta'_i x_t, i = 1, ..., r \). If accepted, the variable in question can be considered a driving variable in the system: it `pushes' the system, but is not being `pushed' by it.

The last row of Table 2 reports the LR test results of weak exogeneity. Both of the long-term bond rates were found to be weakly exogenous. The joint test of weak exogeneity was accepted with a p-value of 0.70. This result, together with the rejection of weak exogeneity of the short-term interest rates, suggests that it is the shocks to long-term interest rates, rather than to the short-term interest rates which are driving the variables of this system.

The rejection of weak exogeneity for the inflation rates, (similarly rejected in Juselius and MacDonald (2000b)) suggests that prices have adjusted to deviations from parity conditions. This is a surprising result as the theoretical prediction of a floating DM/USD rate and price stickiness suggest the opposite result. However, this finding seems to be consistent with the result in Frydman and Goldberg (2002) which shows that with imperfect information expectations, traders' behavior is likely to push exchange rates away from the PPP benchmark level, even if expectations are based on macroeconomic fundamentals. Thus, it seems more likely that they have been generated by traders' behavior in the foreign exchange market.

6 A fully identified long-run structure

Relying on the test results reported in Table 2 we tested the following joint hypothesis on the full cointegration structure:

\[
H_{26} : \quad \beta = \{H_1 \varphi_1, H_2 \varphi_2, H_3 \varphi_3\},
\]

(13)

where \( H_1 \) corresponds to a homogeneous relation between German price inflation, US price inflation and German bond rate \( (H_{25}) \), \( H_2 \) to a real interest rate parity relation between Germany and USA and the \( ppp \) term \( (H_{22}) \), and \( H_3 \) to a relation between the real German and US t-bill spread and the \( ppp \) term \( (H_{24}) \). The nine overidentifying restrictions were tested with the LR
test procedure in Johansen and Juselius (1994) and accepted with a p-value of 0.85. The joint test of the structure (13) together with the weak exogeneity of the two bond rates (six zero coefficients) produced the same p-value of 0.85. Table 4 reports the estimated results based on the latter case. All \( \beta \) coefficients are strongly significant implying that the estimated structure is both formally and empirically identified. Figure 5 shows the graphs of the three equilibrium error correction mechanisms, \( \beta_i x_t, \ i = 1, 2, 3 \), all of which appear very stationary.

\[
\Delta p_t = 0.30 \Delta p^*_t + 0.70 i^*_t + 0.001 D91.3_t - 0.003 + \text{stat.error.} \quad (14)
\]

The interpretation is that German inflation is related both to the US inflation rate (an imported inflation effect) and to the domestic long-term bond rate. The shift dummy is consistent with a small increase in German inflation after the reunification and the constant term shows that German inflation, on average, is lower than the implied value as given by the determinants. The short-run adjustment to (14) occurs primarily through the changes in the German inflation rate, signifying its importance as a German relation-
Table 3: A structural representation of the cointegrating space.

<table>
<thead>
<tr>
<th>Var</th>
<th>$\hat{\beta}_1$</th>
<th>$\hat{\beta}_2$</th>
<th>$\hat{\beta}_3$</th>
<th>$\hat{\alpha}_1$</th>
<th>$\hat{\alpha}_2$</th>
<th>$\hat{\alpha}_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta p_t$</td>
<td>1.0</td>
<td>-1.0</td>
<td>1.0</td>
<td>-1.02</td>
<td>-0.11</td>
<td>0.04</td>
</tr>
<tr>
<td>$\Delta p_t^*$</td>
<td>-0.30</td>
<td>1.0</td>
<td>-1.0</td>
<td>-0.56</td>
<td>-0.56</td>
<td>0.16</td>
</tr>
<tr>
<td>$i_l$</td>
<td>-0.70</td>
<td>1.0</td>
<td>1.55</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$i_l^*$</td>
<td>0</td>
<td>-1.0</td>
<td>-1.55</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$i_s$</td>
<td>0</td>
<td>0</td>
<td>-1.0</td>
<td>0.01</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>$i_s^*$</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>-0.01</td>
<td>-0.05</td>
<td>-0.04</td>
</tr>
<tr>
<td>$\text{ppp}^{1)}$</td>
<td>0</td>
<td>-0.94</td>
<td>0</td>
<td>$\Delta \text{ppp}_t$</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>$Ds91.03$</td>
<td>-0.001</td>
<td>0.002</td>
<td>-0.002</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{const.}$</td>
<td>0.003</td>
<td>-0.003</td>
<td>0.003</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^{1)}$ The ppp term has been divided by 100

The ppp term has been divided by 100

ship. However, US inflation has reacted similarly, i.e. negatively, though less strongly so, to positive deviations from this relation.

The second cointegrating relationship, representing international real interest rate parity, is given by:

$$(i_l^* - \Delta p_t^*) = (i_l^* - \Delta p_t) - 0.01 \text{ppp}_t + 0.002 Ds91.03_t - 0.003 + \text{stat. error}. \quad (15)$$

The short-run adjustment to (15) occurs primarily through changes in US inflation rate, signifying its importance for the US economy. The interpretation is that the US real interest rate increases relative to the German rate when the ppp term is negative; i.e. when US prices are above German prices measured in the same currency. However, US real interest rate is on average lower (0.003) than the German real interest rate given the ppp effect (we interpret this as a safe haven effect). Considering the large variations in real bond rates over this period, illustrated by the graphs of Figure 3, the fact that we have been able to recover a strong-form version of real interest rate parity seems quite remarkable.
The third vector is a function of real short-term interest rates and the bond spread and can be written as:

\[ i_t - \Delta p_t = (i_t - \Delta p_t) + 1.51(i_t - i_t^*) + 0.002D91.3 + 0.003. \]

This relationship is interesting since it suggests that short-term real interest rate parity would be satisfied as a stationary relation if the long-term bond spread would become stationary. However, the nonstationarity of the bond spread is likely to be related to the nonstationary deviations from the steady-state value of the \( ppp \) rate. Empirically, this means that only in periods when the \( ppp \) rate has returned to its steady-state path and the bond yield differential has become stationary is it possible to find evidence of real interest rate parities, as the stationary relations theory would predict.

Thus our analysis suggests that empirical support for the theoretical parities might very well be found in the data, but as long as the economies stay away from their fundamental steady-state positions, direct evidence is unlikely to be found. In that sense the cointegrating relationships which we have established could be said to contain the 'theoretical' parities as a special case. For example, in the hypothetical situation where real exchange rates have returned to their steady-state path, the \( ppp \) term should be stationary and so the current account should also be balanced. With no need to finance the current account, the spread between bond yields should be stationary and the other parities would also be individually stationary.

It is noteworthy that the long-term bond rates show no evidence of adjusting to any of the long-run relations, whereas the two treasury bill rates are strongly adjusting to the last two steady-state relations. This seems to be against the expectation's hypothesis which predicts that short-term interest rates should act as exogenous variables and, hence, drive long-term rates. The significant adjustment of the US short rate to the cointegrating vectors reflects its role as a money market determined interest rate, but the lack of adjustment in the long-term bond rates seems to suggest that the transmission of the money market effects to the long-rates is not there, or only weakly so. This will be further investigated in the next section.

The \( ppp_t \) is adjusting to all three cointegrating relations although very slowly so. Therefore, the finding in Table 2 that the \( ppp_t \) is quite close to being weakly exogenous does not imply that future real exchange rates can drift away without any bounds, but only that there is a lot of inertia in the...
movements back to its fundamental value. Although the predictive value of \( ppp_t \) for one-step-ahead predictions may not be very high, when it comes to predictions over longer period it is likely to increase substantially. This interpretation is strongly supported by the results of the long-run impact analysis in Section 8.

7 A short-run adjustment structure

Using the identified cointegration relations reported in Table 3 we first estimated a multivariate dynamic equilibrium error correction model for the system.\(^6\) Because the US bond rate was found to be strongly exogenous we re-estimated the system conditional on the marginal model for this rate. By first removing insignificant lagged variables from the system, based on an F-test, and then removing insignificant coefficients from the equations, based on a Likelihood Ratio test, we arrived at the parsimonious model presented in Table 4. The column heading in the top half of the table indicates the dependent variable in each of the model equations, while the row headings indicate the conditioning variables.

Except for a negative correlation between the shocks to the German interest rates (-0.51) the residual cross correlations are essentially zero. Note that the standard deviation of the residuals from the monthly changes in CPI inflation rates is approximately 0.2%. The estimated coefficients of the included dummy variables are presented in Appendix I. The LR test of overidentifying restrictions, distributed as \( \chi^2(136) \), was 156.7 and the restrictions were accepted with a p-value of 0.11. Of the 136 exclusion restrictions only 12 are related to the system variables. The latter were accepted with a p-value of 0.71. The remaining restrictions are associated with the many intervention dummies needed to account for the turbulent movements in US treasury bill rate during the period of monetary targeting in the beginning of the eighties. In addition the monthly seasonal dummy variables are only included in the US and German inflation rate equations.

In terms of the contemporaneous effects, we note that the weakly exogenous US bond rate is the only significant and it has a pervasive effect, appearing in all equations, whereas a change in the German bond rate only has an immediate effect on the German treasury bill rate. The effect of lagged changes to the system variables can be seen to be very modest.

\(^6\)All calculations have been performed in PcFiml (see Doornick and Hendry (1998)).
Table 4: A multivariate equilibrium-correction model

<table>
<thead>
<tr>
<th>Eq.</th>
<th>$\Delta^2 p_t$</th>
<th>$\Delta^2 p_t^*$</th>
<th>$\Delta i_t^*$</th>
<th>$\Delta i_t^{**}$</th>
<th>$\Delta i_s^*$</th>
<th>$\Delta i_s^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta i_t^*$</td>
<td>1.11 (2.7)</td>
<td>1.29 (3.5)</td>
<td>0.26 (8.2)</td>
<td>-0.22 (3.8)</td>
<td>0.52 (10.1)</td>
<td>-0.20 (3.0)</td>
</tr>
<tr>
<td>$\Delta i_{t-1}^*$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.16 (7.3)</td>
<td>-</td>
<td>0.14 (2.2)</td>
</tr>
<tr>
<td>$\Delta i_{t-1}^*$</td>
<td>-</td>
<td>-</td>
<td>0.34 (6.6)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta i_{t-1}^*$</td>
<td>-</td>
<td>1.57 (3.2)</td>
<td>-</td>
<td>0.11 (2.4)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta i_{t-1}^*$</td>
<td>-</td>
<td>-</td>
<td>0.03 (2.0)</td>
<td>-</td>
<td>0.34 (13.0)</td>
<td>-</td>
</tr>
</tbody>
</table>

The standardized residual covariance matrix (standard errors in diag)

| $\Delta^2 p_t$ | (0.00193) |
| $\Delta^2 p_t^*$ | (0.00169) |
| $\Delta i_t^*$ | 0.04 (0.00014) |
| $\Delta i_{t-1}^*$ | -0.12 (0.00018) |
| $\Delta i_{t-1}^*$ | -0.01 (0.00020) |
| $\Delta i_{t-1}^*$ | -0.01 (0.00030) |
The significant adjustment effects of the error correction terms are notable. Both inflation rates are strongly adjusting to $ecm_1$, the German inflation relation, and $ecm_2$, the long-term real interest parity relation, but German inflation adjusts much more strongly to $ecm_1$ and US inflation more strongly to $ecm_2$. The adjustment coefficients to the two $ecm$ terms are negative, both in the German and US inflation equations which seems surprising. To be able to interpret this result we have calculated the underlying steady-state relation, being a combination of the significant $ecm$’s weighted by the adjustment coefficients, for each of the two inflation rates.

For the German inflation rate the combined effects became:

$$\Delta p = 0.18 \Delta p^* + 0.59i^d + 0.23i^d - 0.003 ppp,$$

and for the US inflation:

$$\Delta p^* = 0.32 \Delta p + 0.68i^d + 0.65(i^d - i^d) + 0.010 ppp.$$  

It appears that German inflation has adjusted homogeneously to German and US bond rates and to US inflation. US inflation has similarly adjusted homogeneously to the German inflation rate and the US bond rate, and, additionally, also to the bond rate spread. Altogether, the results seem to indicate the long-term interest rates play a very fundamental role for the determination of inflation rates implying that the cost of long-term financing has an important effect on prices. Furthermore, US inflation is equilibrium error correcting to the $ppp$ term (though not sufficiently fast to restore fundamental equilibrium exchange rates), whereas the $ppp$ effect on German inflation is negligible.

The German bond rate is only very weakly reacting to $ecm_1$, i.e. to ‘excess’ German inflation, consistent with the previous finding that it is essentially weakly exogenous. The two treasury bill rates and the $ppp$ term adjust similarly to $ecm_2$ and $ecm_3$, i.e. to deviations from the long-term and short-term real interest rate parity conditions. To facilitate interpretation of these relationships we also derive the combined steady state relations for these variables. The combined steady-state relation for the German treasury bill rate became:

$$i^* - i^{**} = 3.5(i^d - i^d) + (\Delta p^* - \Delta p) - 0.02 ppp.$$
and for US treasury bill rate:

\[ i^{*} - i^s = 2.5(i^l - i^l) + 0.01ppp. \]

and, finally, for the ppp term:

\[ ppp = 0.35(\Delta p^* - \Delta p) - 2(i^{*l} - i^l) + 0.65(i^{*} - i^s). \]

Thus, the treasury bill rates adjust strongly to the long-term bond spread, but also to the deviation from the ppp term and the inflation rate differential.

The ppp term adjusts homogeneously to the inflation spread and the short-term interest spread and shows a strong negative effect from the US-German long-term bond spread. The results confirm the crucial role of the long-term interest rate, but also the short-term interest rates for the development of the real exchange rates in this period. It is quite interesting that an increase in the spread between US and German bond rates is associated with an appreciation of the dollar, whereas the opposite is the case with an increase in the short spread and the inflation rate differential.

Altogether the results seem to suggest that the reserve currency (safe haven) effect of the dollar has indeed prevented the adjustment towards equilibrium exchange rates and resulted in the overvalued dollar. The need to finance the low US savings rate drives up the US bond rate relative to the German rate and the increase in the bond yield results in the US$ appreciating, making the adjustment towards stationary real exchange rates very slow.

### 8 The long-run impact of shocks

We noted above that the German and US long bond yields are weakly exogenous for the long-run parameters, \( \beta \), implying that they act as driving variables (a common stochastic trend) in the system. By inverting the VAR subject to the reduced rank restriction \( \Pi = \alpha \beta \) we get the so called moving average representation:

\[ x_t = C \sum_{1}^{t} \varepsilon_i + C \Phi_1 \sum_{1}^{t} D_i + C \Phi_2 \sum_{1}^{t} S_i + C^*(L)(\varepsilon_t + \mu + \Phi_1 S_t + \Phi_2 D_t) + Z_0 \]

(16)
Table 5: The estimates of the long-run impact matrix $C$

<table>
<thead>
<tr>
<th></th>
<th>$\Sigma \varepsilon_{\Delta p}$</th>
<th>$\Sigma \varepsilon_{\Delta p^*}$</th>
<th>$\Sigma \varepsilon_{\Delta \pi}$</th>
<th>$\Sigma \varepsilon_{\Delta \pi^*}$</th>
<th>$\Sigma \varepsilon_{\pi^*}$</th>
<th>$\Sigma \varepsilon_{\pi}$</th>
<th>$\Sigma \varepsilon_{pp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta p_t$</td>
<td>0.00 (0.3)</td>
<td>0.01 (0.6)</td>
<td>0.68 (3.7)</td>
<td>0.17 (1.2)</td>
<td>0.10 (1.1)</td>
<td>0.25 (2.6)</td>
<td>0.29 (5.3)</td>
</tr>
<tr>
<td>$\Delta p_t^*$</td>
<td>0.01 (0.5)</td>
<td>0.02 (1.1)</td>
<td>-0.29 (-0.8)</td>
<td>0.28 (1.1)</td>
<td>0.15 (0.9)</td>
<td>0.73 (4.1)</td>
<td>0.97 (9.4)</td>
</tr>
<tr>
<td>$\epsilon_t^l$</td>
<td>0.00 (0.0)</td>
<td>0.00 (0.0)</td>
<td>1.33 (5.6)</td>
<td>0.26 (1.4)</td>
<td>0.00 (0.2)</td>
<td>0.02 (0.2)</td>
<td>0.03 (0.4)</td>
</tr>
<tr>
<td>$\epsilon_t^l^*$</td>
<td>-0.00 (-0.0)</td>
<td>0.00 (0.0)</td>
<td>0.08 (0.3)</td>
<td>1.21 (5.4)</td>
<td>0.00 (0.0)</td>
<td>-0.00 (-0.1)</td>
<td>-0.01 (-0.1)</td>
</tr>
<tr>
<td>$\epsilon_t^s$</td>
<td>0.00 (0.0)</td>
<td>0.00 (0.0)</td>
<td>1.22 (3.9)</td>
<td>-0.33 (-1.4)</td>
<td>0.91 (6.6)</td>
<td>0.41 (-2.5)</td>
<td>-0.26 (-2.8)</td>
</tr>
<tr>
<td>$\epsilon_t^s^*$</td>
<td>0.00 (0.2)</td>
<td>0.01 (0.1)</td>
<td>-1.27 (-2.9)</td>
<td>1.49 (4.5)</td>
<td>0.53 (2.8)</td>
<td>0.61 (2.7)</td>
<td>0.42 (3.3)</td>
</tr>
<tr>
<td>$ppp_t$</td>
<td>0.01 (0.5)</td>
<td>0.02 (0.8)</td>
<td>0.44 (1.5)</td>
<td>-0.70 (-2.3)</td>
<td>-0.29 (-3.1)</td>
<td>0.35 (2.2)</td>
<td>0.82 (9.1)</td>
</tr>
</tbody>
</table>

where $C = \beta_1 (\alpha'_1 \Gamma \beta_1)^{-1} \alpha'_1$, $C^*(L)$ is an infinite polynomial in the lag operator $L$, and $Z_0$ is a function of the initial values. Based on (16) it is possible to calculate the impulse responses of a shock to one variable and how it is transmitted over time within the system. Instead of reporting the impulse response functions for a unitary change of $\hat{\varepsilon}_{it}$, we report only the final impact matrix, $C$ in Table 5.

The estimates of the columns of the $C$ matrix in Table 5 measures the total impact of permanent shocks to each of the variables on all other variables of the system. A row of the $C$ matrix gives an indication of which variables have been particularly important for the stochastic trend behavior of the variable in that specific row. The t-ratios in parenthesis are based on the asymptotic standard errors suggested by Paruolo (1997).

These results reinforce our previous findings from the analysis of the long-run relations. We note that cumulative shocks to the inflation rates have no significant long-run impact on any of the variables, accentuating our previous findings that inflation rates are solely adjusting in this system, but not pushing. We also note that the two long term bond yields have significant cumulative impacts on short term interest rate yields, the $ppp$ term and to some extent also on inflation rates, whereas shocks to the short-term interest rates have no long-run impact on the bond rates. The latter result is again in conflict with the basic premise of the expectations hypothesis of the term structure. Furthermore, permanent shocks to the short-term US treasury bill rate do have a permanent positive impact on inflation rates. Thus, in-
creases in the US short-term interest rate tend to increase inflation and not the other way around. Permanent shocks to the \textit{ppp} term are also important as they have a significant long-run impact on inflation rates and short term bill yields.

Therefore, the results strongly suggest that the developments in 'world' financial markets, as measured by the dominant rate yields - the US and German long rates and the treasury bill rates - are driving this system and inflation rates are essentially adjusting. This latter finding reinforces the point made earlier that the Fisher conditions do not seem to work in the predicted manner.

9 The role of short-term interest rates

To gain a further perspective on the role of the short- relative to the long-term interest rates we report, in Table 6, a comparative analysis of the combined effects, as measured by $\hat{\alpha}_r\hat{\beta}_r = \hat{\Pi}_r$, where the subscript $r$ stands for the restricted estimates as reported in Table 3, for the full system, including both long and short interest rates and for the smaller system without short rates (as reported in Juselius and MacDonald (2000a)).

It appears that German inflation is essentially unaffected by the inclusion of the treasury bill rates into the analysis. It is, as in the small system, determined by the long bond rate and US inflation.

The results for US inflation show that the US short-term treasury bill rate has now replaced the long-term bond rates in the small system. However, the results for the US treasury bill rate show significant reaction from the bond yield spread. Thus, it seems likely that the short-run effects go from bond rates influencing treasury bill rates, influencing inflation rates. Indeed, the results in Table 5 show that the long-run impact on US inflation derives from permanent shocks to the short-term treasury bill rates, the effect is positive rather than negative sign \footnote{This is a frequent empirical finding, the so called ”price puzzle".} that would be expected form conventional theory.

Consistent with the weak exogeneity results of Table 3, the equations for the German and US bond rate exhibit hardly any significant effects. The \textit{ppp} term is significantly affected by the bond and the short-term spread, such that the US$ appreciates with an increasing bond spread and depreciates with an increasing treasury bill spread.
Table 6: The combined long-run effects

<table>
<thead>
<tr>
<th>Eq.</th>
<th>$\Delta^* p_t$</th>
<th>$\Delta^*_t$</th>
<th>$i_t^l$</th>
<th>$i_t^{ls}$</th>
<th>ppp$_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{Eq.}$</td>
<td>$\Delta p_t$</td>
<td>$\Delta p_t^*$</td>
<td>$\Delta i_t^{ls}$</td>
<td>$i_t^{ls}$</td>
</tr>
<tr>
<td>$\Delta^* p_t$</td>
<td>$-0.88$</td>
<td>$0.19$</td>
<td>$0.58$</td>
<td>$-0.12$</td>
<td>$0.15$</td>
</tr>
<tr>
<td></td>
<td>(-12.8)</td>
<td>(2.9)</td>
<td>(12.0)</td>
<td>(-1.5)</td>
<td>(3.8)</td>
</tr>
<tr>
<td>$\Delta^2 p_t^*$</td>
<td>$0.08$</td>
<td>$-0.47$</td>
<td>$-0.13$</td>
<td>$0.44$</td>
<td>$0.58$</td>
</tr>
<tr>
<td></td>
<td>(1.3)</td>
<td>(-7.3)</td>
<td>(-2.6)</td>
<td>(7.5)</td>
<td>(7.5)</td>
</tr>
<tr>
<td>$\Delta i_t^l$</td>
<td>$0.01$</td>
<td>$-0.00$</td>
<td>$-0.00$</td>
<td>$0.00$</td>
<td>$0.00$</td>
</tr>
<tr>
<td></td>
<td>(0.8)</td>
<td>(-0.3)</td>
<td>(-0.8)</td>
<td>(0.0)</td>
<td>(0.0)</td>
</tr>
<tr>
<td>$\Delta i_t^{ls}$</td>
<td>$-0.01$</td>
<td>$0.01$</td>
<td>$0.01$</td>
<td>$-0.02$</td>
<td>$-0.00$</td>
</tr>
<tr>
<td></td>
<td>(-1.6)</td>
<td>(0.7)</td>
<td>(1.5)</td>
<td>(-0.2)</td>
<td>(-0.2)</td>
</tr>
<tr>
<td>$\Delta ppp_t$</td>
<td>$0.006$</td>
<td>$0.012$</td>
<td>$-0.002$</td>
<td>$-0.014$</td>
<td>$-0.02$</td>
</tr>
<tr>
<td></td>
<td>(0.5)</td>
<td>(1.1)</td>
<td>(-0.2)</td>
<td>(-1.5)</td>
<td>(1.5)</td>
</tr>
</tbody>
</table>

The results for the short-term treasury bill equations show strong adjustment to essentially all determinants except for inflation rates! The lack of significant inflationary effects in all four interest rate equations is very pronounced. This is to be contrasted with the significant interest rate effects in the inflation rate equations. Similar results have also been found in Danish, Spanish, and Italian data (Juselius, 1992, Juselius and Toro, 1999, Juselius and Gennari, 1999).

10 Summary and conclusions

This paper has empirically examined the joint determination of a number of key parity conditions for Germany and the US using monthly data from the recent experience with floating exchange rates. The vector of variables con-
sidered in this paper, consisted of the German Mark-US dollar exchange rate, prices, short term interest rates and long term interest rates. We used the cointegrated VAR model to define long-run stationary relationships as well as common stochastic trends, and a general-to-specific approach to produce parsimonious dynamic short-run equations. We now summarize our main findings.

Our results strongly rejected the stationarity hypothesis of the 'pure' parity conditions. However, by allowing them to be interdependent, stationarity was recovered. The important finding was that the nonstationarity of the 'simple' parity relationships was primarily related to the nonstationarity of the $ppp$ exchange rate and the long-term bond rate differential. An obvious interpretation of the results was that the lack of empirical support for the simple parity conditions was due to the lack of (or very, very slow) adjustment to a stationary $ppp$ steady state and increasing long-term bond spreads as a plausible consequence of the latter. Thus, the theoretical assumption of stationary parity conditions appeared to be a special case of a more general formulation allowing for persistent deviations from steady-state and, hence, market failure in a simple model framework.

Therefore, the theoretical assumption of two common driving trends had to be replaced by the empirical finding of four common trends, hypothetized as: (1) a nominal price trend driving the goods market, (2) a trend describing relative national savings behavior, (3) a 'safe haven' trend capturing the role of the dollar as a world reserve currency, and (4) a short-term capital market trend describing central bank policy behavior.

Not surprisingly, the empirical modification of the original parity conditions as a result of the above 'market failure' trends, produced a number of new results related to the dynamics of the international transmission mechanism. Some of the major (empirically strong) findings were the following:

1. In our system of inflation rates, $ppp$ exchange rates, 10 year bond rates and 3 months treasury bill rates, US and German long-term bond rates proved to be the main driving forces and not the short-term interest rates.

2. US and German inflation rates were strongly adjusting to the other variables of the system, primarily to the bond rates and the real $PPP$ exchange rates, but they were not affecting the other variables, in particular, they did not push nominal interest rates.
3. The nonstationary movements in the bond and inflation rate differential were closely related to the nonstationary movements in the \textit{ppp} exchange rate.

4. The short-term interest rates (the 3 months treasury bill rates) were important for the determination of the \textit{ppp} exchange rate both in the short and the long run. They had essentially no impact on the bond rates and the inflation rates, with the caveat that the US short rate had a positive (cost push) effect on US inflation.

5. Permanent shocks to long-term, as well as to short-term, interest rates had a \textit{positive} long-run impact on inflation, signifying the cost effect of interest rates on capital stock.

The above findings were shown to be remarkably robust (empirically as well as econometrically) over a period of fundamental changes and therefore cannot be discarded as sample dependent results. Our findings seem to suggest that:

1. The role of the dollar as a reserve currency (the ‘safe haven’ effect) has facilitated relatively cheap financing of the large US current account deficits in this period. This might explain one of the ‘market failure’ puzzles: why an adequate adjustment toward purchasing power parity between the USA and Germany has not taken place.

2. The large differences between national savings rates seemed to be an important reason why the long-term bond rates were found to be so crucial in this system.

3. Though the role of central bank policy for stabilizing the short-term capital market has evidently been crucial as the turbulent years of monetary targeting in the eighties demonstrated, its role for controlling inflation seemed much more modest than is usually believed.

However, although the non-stationary of the parities will disappear with the disappearance of other disequilibria in the economy, in the presence of free capital movements we do not believe that the parity reversals in the term structure and Fisher relationships, will disappear. The latter finding would appear to have important policy implications.
Finally, by joint modelling of the parities we have managed not only to recover stationary parity conditions, but also to describe the variation of the data with a remarkable degree of precision as evidenced by the very small residual standard errors. Hence, the results should be used as a benchmark against which the results of other models, possibly with more theory content, could be evaluated.

11 References


Lothian, J. (1997), 'Multi-country evidence on the behaviour of purchasing power parity under the current float', *Journal of International Money and Finance*.


Appendix I: The data

Figure A.1: The graphs of German and US CPI inflation and real exchange rate in levels (l.h.s. panels) and differences (r.h.s. panels).
Figure A.2: The graphs of the German and US bond rate and 3 months t-bill rates in levels (r.h.s. panels) and differences (l.h.s. panels).