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Browning, Martin; Chiappori, Pierre-André; Lewbel, Arthur

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Martin Browning
Pierre-André Chiappori
Arthur Lewbel

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Estimating Consumption Economies of Scale, Adult Equivalence Scales, and Household Bargaining Power*

Martin Browning
CAM, Institute of Economics,
University of Copenhagen

Pierre-André Chiappori
Department of Economics,
University of Chicago

Arthur Lewbel
Department of Economics,
Boston College

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Abstract

How much income would a woman living alone require to attain the same standard of living that she would have if she were married? What percentage of a married couple’s expenditures are controlled by the husband? How much money does a couple save on consumption goods by living together versus living apart? We propose and estimate a collective model of household behavior that permits identification and estimation of concepts such as these. We model households in terms of the utility functions of its members, a bargaining or social welfare function, and a consumption technology function. We demonstrate generic nonparametric identification of the model, and hence of equivalence scales, consumption economies of scale, household members’ bargaining power and other related concepts.

1 Introduction

On average, how much income would a woman living alone require to attain the same standard of living that she would have if she were married? What percentage of a married couple’s expenditures benefit the husband? How much money does a couple save on consumption goods by living together versus living apart? The goal of this paper is to propose a collective model of household behavior aimed at answering questions such as these.1

Equivalence scales revisited Questions like these are traditionally addressed using equivalence scales. Our approach is crucially different. Equivalence scales seek to answer the question, “how much money does a household need to spend to be as well off as a single person living alone?” The equivalence scale itself is then the expenditures of the household divided by the expenditures of the single

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person that enjoys the same “standard of living” as the household.\textsuperscript{2} The early equivalence scale literature attempted to define this ratio of costs of living directly in terms of measurable quantities such as the costs of acquiring a required number of calories, but this was soon replaced by the concept of defining the household and the single person as equally well off to mean that they attain an equal level of utility (see, e.g., Jorgenson and Slesnick 1987, and for a survey, Lewbel 1997). Just as a true cost of living price index measures the ratio of costs of attaining the same utility level under different price regimes, equivalence scales were supposed to measure the ratio of costs of attaining the same utility level under different household compositions.

Unfortunately, unlike true cost of living indexes, equivalence scales defined in this way can never be identified from revealed preference data (that is, from the observed expenditures of households under different price and income regimes). The reason is that defining a household to have the same utility level as a single individual requires that the utility functions of the household and of the single individual be comparable. We cannot avoid this problem by defining the household and the single to be equally well off when they attain the same indifference curve, analogous to the construction of true cost of living indices, because the household and the single have different preferences and hence do not possess the same indifference curves. Pollak and Wales (1979, 1992) describe these identification problems in detail, while Blundell and Lewbel (1991) prove that only changes in traditional equivalence scales, but not their levels, can be identified by revealed preference.\textsuperscript{3}

We argue that the source of these identification problems is that the standard equivalence scale question is badly posed, for two reasons. First, by definition any comparison between the preferences of two distinct decision units entails interper-

\textsuperscript{2}Equivalence scales have many practical applications. They are commonly used for generating poverty lines for households of various compositions given a poverty line for single males. Income inequality measures have been applied to equivalence scaled income rather than observed income to adjust for household composition.(see, e.g., Jorgenson 1997). Calculation of appropriate levels for alimony or life insurance also require comparing costs of living for couples versus those of singles.

\textsuperscript{3}Many previous attempts to identify equivalence scales are based on assumptions such as Independence of Base (IB) or Equivalence Scale Exactness (see, e.g., Lewbel (1989)). Jorgenson and Slesnick’s (1987) model is a special case of an IB assumption. IB imposes testable restrictions on demands, but it also requires untestable (that is, cardinal or comparable utility type) restrictions. IB is an example of the paradigm of comparing an individual’s utility to that of a household, which is intrinsically impossible without some cardinalization or interpersonal comparability of utility.
sonal utility comparisons. Second, and perhaps more fundamental, the notion of a household utility is potentially flawed. Individuals have utility, not households. What is relevant is not the ‘preferences’ of a given household, but rather the preferences of the individuals that compose it.

We propose therefore that meaningful comparisons must be undertaken at the individual level, and that the appropriate question to ask is, “how much income would an individual living alone need to attain the same indifference curve over goods that the individual attains as a member of the household?” This latter question avoids issues of interpersonal comparability and differences in indifference curves (it only depends on ordinal preferences), and hence is at least in principle answerable from revealed preference data. Consequently, in sharp contrast with the existing equivalence scale literature, our framework does not assume the existence of a unique household utility function, nor does it require comparability of utility between individuals and groups (such as the household). Instead, following the basic ideas of the collective approach to household behavior, we assume that each individual is characterized by his/her own utility function, so the only comparisons we make is between the same person’s welfare in different living arrangements.

A New Collective Model An obvious practical obstacle to implementing our new approach to equivalence scales is the difficulty of observing individual consumptions within a household and the intra-household allocation of resources. In general, only household’s total purchases are observed, and not their distribution and use among members. This raises three questions. First, one has to identify individual preferences. Secondly, since the distribution of resources within the household is not recorded, it has to be identified from the aggregate household demand - a standard problem of the collective literature. Finally, household consumption entails shared consumption, and hence economies of scale and scope in consumption. As a result, in a typical multiperson household individual consumptions add up to ‘more’ than total purchases. This sharing and jointness of consumption is not directly observed, and must also be identified.

The solution to this problem obviously depends on the nature of available data. For instance, if each individual household member’s consumption were observed, identification of each member’s indifference curves could proceed along standard lines. In this paper we assume, as usual, that only the household’s aggregate consumption of various goods (and their prices) are observable.

For both identification and empirical tractability, we characterize the jointness
of consumption and the allocation of resources within a household by what we call the consumption technology function and the sharing rule. The idea of the consumption technology function is that features of household consumption such as economies of scale or scope, joint use of resources, etc., can be defined as a technology that describes the set of options for the joint consumption of goods that are available to household members. The distinction between public and private goods within a household in the existing collective models literature (see, e.g., Vermeulen 2000) is a special case of our consumption technology function (see Section 5 for details). The consumption technology function is similar to Becker’s (1965) notion of household production, with differences being that we extend it to a collective framework, and that what is produced (by sharing and joint consumption) is effectively an increase in the quantities of purchased goods. We argue that this extension significantly changes the status of the household production concept, and in particular its identifiability. While the household production function is typically unidentifiable in a unitary framework, we show a general identification result for the technology function in the collective setting. The consumption technology function is also closely related to the motivations behind Barten (1964) scales and Gorman’s (1976) general linear technologies.

The sharing rule describes the allocation of resources within the household. The concept of a sharing rule is borrowed directly from the collective approach (see Chiappori 1992, Browning and Chiappori 1998, and Vermeulen 2000 for a survey). Specifically our sharing rule is derived from the assumption that household decisions are Pareto efficient and that individual consumptions (after conversion from purchased quantities by the consumption technology function) are private. Considering the household as a small, open economy, by the second welfare theorem any efficient outcome can be decentralized as an equilibrium, possibly after lump sum transfers between members. The sharing rule summarizes those transfers, and can thus be seen as a reduced form of the decision process regarding the distribution of resources within the household.

Our collective based equivalence scales do not impose particular assumptions on this intra-household distribution of resources. In particular, we do not assume that all household members are equally well off. Household resources could result from bargaining models or more generally from any household allocation process.

\footnote{Like all the other concepts we define, including our version of equivalence scales, the sharing rule and the consumption technology function are purely ordinal, and so do not depend on any chosen cardinalization of individual utility functions.}

\footnote{The privateness assumption can be relaxed by a generalization of the household technology. See Section 5 for a precise discussion.}
that does not violate efficiency.

In our model, aggregate household demand functions are determined by the sharing rule, the consumption technology function, and the preferences of the individual household members. All of these functions must be identified to correctly evaluate consumption economies of scale, household bargaining power, and appropriately defined equivalence scales.

To facilitate this identification we assume, at least initially, that individual ordinal preferences do not change after marriage\(^6\). This allows us to use the demand data of people living alone to identify individual preferences, thereby leaving to household data the job of identifying the consumption technology and sharing rule. Discussion of how this assumption may be relaxed within our methodology is deferred to section 5\(^7\). This assumption is not as restrictive as it seems, specifically, it does not imply that household consumption equals the simple sum of members demands, because of the consumption technology function.

The collective approach to equivalence scales The consumption technology function transforms the \(n\) vector of purchased quantities of goods and services \(z\) into an \(n\) vector of private good equivalents \(x\). The greater the degree of economies of scale in consumption of a component of \(z\), the larger is the corresponding component of \(x\). These private good equivalents are then divided up between the household members via a household welfare or bargaining function, with each member deriving utility from consuming their share of \(x\). The sharing rule summarizes this division of \(x\) among household members.

In this framework, we define a collective based equivalent income (or expenditure) to be the income or total expenditure level \(y^i^*\) required by an individual \(i\), when living alone, to be as well off materially as he or she would be when living with others in a household that has joint income \(y\). Member \(i\)’s collective based equivalence scale is then \(y/y^i^*\), the ratio of the household’s total expenditures to member \(i\)’s collective based equivalent total expenditures.

To see the usefulness of collective based equivalent income, consider the ques-

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\(^6\)Of course, the utility level may be shifted by marriage; we simply assume that the individual’s indifference curves are unchanged.

\(^7\)What we require is some method of identifying the indifference curves describing the preferences over goods of individuals within households. The simplest solution is to assume they are the same as for singles living alone. More complicated solutions will require either explicit models of changes in preferences resulting from marriage, or some combination of stronger modeling assumptions and richer data sets from which the indifference curves of individual household members may be directly recovered.
tion of determining an appropriate level of life insurance for a spouse. If the couple spends \( y \) dollars per year then for the wife to maintain the same standard of living after the husband dies, she will need an insurance policy that pays enough to permit spending \( y / x \) dollars per year. Similarly, in cases of wrongful death, juries are instructed to assess damages both to compensate for the loss in “standard of living,” (i.e., \( y^* \)) and, separately for “pain and suffering,” which would presumably be noneconomic effects (see Lewbel 2003). Another example is poverty lines. If the poverty lines for individuals have been established, then the poverty line for a couple should be defined as the expenditures required for each member of the couple to attain his or her own poverty line indifference curve.

Traditional equivalence scales do not properly answer these questions, because they attempt to relate the utility of an individual to that of a household, instead of relating the utility of the same individual in two different settings, e.g. living with a husband versus without.

**Identification and Estimation** An important result we show is that, given the in principle observable consumption demands of individual household members (which we estimate from the demands of singles given our data limitations) and the demand functions of aggregate households, a household’s sharing rule, consumption technology function, and collective based equivalence scales are all non-parametrically identified, without any assumptions regarding interpersonal comparability or utility cardinalizations. The sharing rule (which we show has a one to one relationship to relative bargaining power) and household economies of scale are similarly nonparametrically identified.

We also provide duality results that yield a convenient way to parameterize the household’s behavior for empirical analysis, analogous to the use of indirect utility functions to conveniently parameterize ordinary empirical demand functions. These duality results include specification of an income sharing function in place of the direct bargaining or social welfare function, and specification of the consumption technology in a form that is analogous to Barten (1964) scales or

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\[^8\] This is similar to the distinction Pollak and Wales (1992) make between what they call a welfare comparison versus a situation comparison.

\[^9\] The household consumption technology function is used to construct measures of the economies of scale that results from joint consumption within the household. When a couple buys the bundle \( z \) at a cost \( p'z \), it is equivalent in terms of consumption to two singles living apart who in total buy the larger bundle \( x \), which would cost \( p'x \). The economies of scale from joint consumption is then \( p'z/p'x \), which equals the cost savings, in percentage terms, that results from shared consumption.
Gorman’s (1976) general linear technology.

Finally, we provide an empirical application of our model using the Canadian FAMEX survey. The results are used to calculate estimates of equivalence scales and economies of scale of married couples versus their single counterparts.

The next section describes the basic framework and summarizes the main theoretical results of the paper, for the most part using the special case of linear technologies for clarity. Later sections then provide more formal derivations, describe our empirical implementation of the model, and discuss possible extensions and variants.

2 The Model

This section describes the proposed household model. Let superscripts refer to household members and subscripts refer to goods. Let \( U^i(x^i) \) be the direct utility function for a consumer \( i \), consuming the vector of goods \( x^i = (x^i_1, ..., x^i_n) \). We consider households consisting of two members, which we will for convenience refer to as the husband \( (i = m) \) and the wife \( (i = f) \). For many applications, it may be useful to interpret one of these utility functions as a joint utility function for all but one member of the household, e.g., \( U^f \) could be the joint utility function of a wife and her children.

2.1 Singles

When living alone, household member \( i \) maximizes a monotonically increasing, continuously twice differentiable and strictly quasi-concave utility function \( U^i(z^i) \) when facing the \( n \) vector of prices \( p \) with income level \( y^i \). Hence \( i \) solves the optimization program

\[
\max_{z^i} U^i(z^i) \quad \text{subject to} \quad p'z^i = y^i
\]

Let \( z^i = h^i(p/y^i) \) denote the solution to this individual optimization program, so the vector valued function \( h^i \) is the set of Marshallian demands of member \( i \) when living alone. Define \( V^i \) by

\[
V^i(p/y^i) = U^i[h^i(p/y^i)]
\]  

so \( V^i \) is the indirect utility function of member \( i \) when living alone. The functional form of individual demand functions \( h^i \) can be obtained from a specification of
using Roy’s identity. The Marshallian demand functions $h^m$ and $h^f$ may be estimated using ordinary demand data on observed prices, total expenditures, and quantities purchased for individuals living alone.

### 2.2 Households: decision process

Now consider a household consisting of a couple living together, and facing the budget constraint $p'z \leq y$, where $p$ is a vector of market prices. Following the standard collective approach, our key assumption regarding decision making within the household is that outcomes are Pareto efficient. This, assuming the absence of money illusion in the decision process, is equivalent to the existence of some increasing function $\bar{U}[U^f(x^f), U^m(x^m), p/y]$ that is maximized under budget and technology constraints. In what follows, we assume for convenience that $\bar{U}$ is twice differentiable. It is important to note that $\bar{U}$ may in general depend on prices. Thus, $\bar{U}$ can be interpreted as a social welfare function for the household, in which individual weights may vary with prices. Alternatively, $\bar{U}$ may stem from some specific bargaining model (Nash bargaining for instance).

All of the functions we define may depend on other variables that we have suppressed for notational simplicity. For example, the bargaining function $\bar{U}$ might depend on the relative wages of the household members, and the utility functions $U^f$ and $U^m$ might depend on demographic characteristics.

A standard result of welfare theory is that one can, without loss of generality, write the function $\bar{U}$ as $\mu(p/y)U^f(x^f) + U^m(x^m)$ for some function $\mu(p/y)$. The Pareto weight $\mu(p/y)$ can be seen as a measure of $f$’s influence in the decision process. The larger the value of $\mu$ is, the greater is the weight that member $f$’s preferences receive in the resulting household program, and the greater will be the resulting private equivalent quantities $x^f$ versus $x^m$. One difficulty with using $\mu$ as a measure of the weight given to (or the bargaining power of) member $f$ is that the magnitude of $\mu$ will depend on the arbitrary cardinalizations of the functions $U^f$ and $U^m$. Later we will propose an alternative bargaining power measure that does not depend upon any cardinalizations.

### 2.3 The Consumption Technology Function

As stated in introduction, the household purchases some bundle $z$, but individual consumptions add up to some other bundle $x$. In the technological relation $z = F(x)$, we can interpret $z$ as the inputs and $x$ as the outputs in the household’s consumption technology, described by the production function $F^{-1}$, though what
is being ‘produced’ is consumption. This framework is similar to a Becker (1965) type household production model, except that instead of using market goods to produce commodities that contribute to utility, the household essentially produces the equivalent of a greater quantity of market goods via sharing. The unobserved \( x_f, x_m \), and \( x = x_f + x_m \) are private good equivalent vectors, that is, they are respectively the quantities of transformed goods that are consumed by the female, the male, and in total.

It will often be convenient to work with a linear consumption technology, which is

\[
F(x) = Ax + a
\]

where \( A \) is a nonsingular \( n \) by \( n \) matrix and \( a \) is a \( n \) vector. This linear consumption technology has an analogous interpretation to Gorman’s (1976) linear technology (a special case of which is Barten (1964) scaling), except that we apply it in the context of a collective model.

Consider some simple examples. Let good \( j \) be food. Suppose that if an individual or a couple buy a quantity of food \( z_j \), the then total amount of food that the individual or couple can actually consume (that is, get utility from) is \( z_j - a_j \), where \( a_j \) is waste in food preparation, spoilage, etc.,. If the individuals lived apart, each would waste an amount \( a_j \), so the total amount wasted would be \( 2a_j \), while living together results in only a waste of \( a_j \). In this simple example the economies of scale to food consumption from living together is a reduction in waste from \( 2a_j \) to \( a_j \), implying that \( x_j = z_j + a_j \), so the consumption technology function for food takes the simple form \( z_j = F_j(x) = x_j - a_j \).

Economies of scale also arise directly from sharing. For example, let good \( j \) be automobile use. If \( x_f^j \) and \( x_m^j \) are the distances traveled by each household member in the car (or some consumed good that is proportional to distance, perhaps gasoline), then the total distance the car travels is \( z = (x_f^j + x_m^j)/(1 + r) \), where \( r \) is the fraction of distance that the couple ride together, which yields a consumption technology function for automobile use of \( z_j = F_j(x) = x_j/(1 + r) \). This example is similar to the usual motivation for Barten (1964) scales, but it is operationally more complicated, because Barten scales fail to distinguish the separate utility functions, and hence the separate consumptions, of the household members.

More complicated consumption technologies can arise in a variety of ways. The fraction of time \( r \) that the couple share the car could depend on the total usage, resulting in \( F_j \) being a nonlinear function of \( x_j \). There could also be economies (or diseconomies) of scope as well as scale in the consumption technology, e.g.,
the shared travel time percentage \( r \) could be related to expenditures on vacations, resulting in \( F_j(x) \) being a function of other elements of \( x \) in addition to \( x_j \).

### 2.4 The basic program

Our proposed model of this household’s consumption behavior is thus the optimization program

\[
\max_{x^f, x^m, z} \tilde{U}[U^f(x^f), U^m(x^m), p/y] \\
\]

\[
x = x^f + x^m, \quad z = F(x), \quad p'^z = y
\]

where \( z \) is the vector of quantities of the \( n \) goods the household purchases, \( p \) is market prices and \( y \) is the household’s total expenditures.

The solution of the program \( P^* \) yields the household’s demand functions, which we denote as \( z = h(p/y) \), and also private good equivalent demand functions \( x^i = x^i(p/y) \). The vector valued demand function \( z = h(p/y) \) can be estimated using ordinary demand data on observed prices, incomes, and quantities purchased by the couple.

One way to interpret the program \( P^* \) is to consider two extreme cases. If all goods were private and there were no shared or joint consumption, then \( F(x) = x \), so \( z \) would equal \( x^f + x^m \) and the program \( P^* \) would reduce to \( \max_{x^f, z} \tilde{U}[U^f(x^f), U^m(z-x^f), p/y] \) such that \( p'^z = y \), which is exactly equivalent to a standard specification of the collective model (see, e.g., Bourguignon and Chiappori 1994 or Vermeulen 2000). At the other extreme, imagine a household that has a consumption technology function \( F \), but assume that the household’s utility function for transformed goods just equaled \( U^m(x) \), as might happen if the male were a dictator that forced the other household member to consume goods in the same proportion that he does. In that case, the model would reduce to \( \max_z U^m[F^{-1}(z)] \) such that \( p'^z = y \), which for linear \( F \) is equivalent to Gorman’s (1976) general linear technology model, a special case of which is Barten (1964) scales (corresponding to \( A \) diagonal and \( a \) zero).10 Our general model, program \( P^* \), combines the consumption technology logic of the Gorman or Barten framework with the collective model of a household as either a bargaining or a social welfare maximizing group.

It should be stressed that if \( U^m \) and \( U^f \) are the utility functions of singles, the model permits changes in utility to result from living together, provided they are

10Gorman’s (1976) famous line, “If I have a wife and child, a penny bun costs threepence,” rationalizes Barten scales but implicitly assumes a dictator imposing his taste for buns on his family members.
separable from goods consumption. For instance, one could be generally happier as a result of living with a spouse, and a wife’s utility could depend on both her own attained utility level over goods (the value of $U_f$) and on $U_m$. These effects are absorbed into the function $\bar{U}$.\footnote{In addition, as will be discussed later, the consumption technology function $F$ can capture some kinds of taste changes that result from living together.}

For our identification and empirical results, we use data on single men and women to estimate the Marshallian demand functions $h^f$ and $h^m$ arising from the utility functions $U^m$ and $U^f$, and the married couple’s demand functions $z = h(p/y)$ derived from the program $P^*$. These estimates are combined to identify the consumption technology function $F$ and relevant features of the bargaining or social welfare function $\bar{U}$, which in turn yields estimates of the private good equivalents $x^f$ and $x^m$, the sharing rule, economies of scale from consumption, and the collective based equivalence scales associated with any price and income regime.

### 2.5 The Form of Household Demands

Consider the household as an open economy. From the second welfare theorem, any Pareto efficient allocation can be implemented as an equilibrium of this economy, possibly after lump sum transfers between members. Transfers can be summarized by the sharing rule $\eta(p/y)$, describing the fraction of household resources consumed by member $f$. The household’s behavior is equivalent to allocating the fraction $\eta^f = \eta(p/y)$ to member $f$, and the fraction $\eta^m = 1 - \eta(p/y)$ to member $m$. Each member $i$ then maximizes their own utility function $U^i$ given the shadow price vector $\pi$ and their own shadow income $\eta^i$ to calculate their desired private good equivalent consumption vectors $x^f$ and $x^m$.

Formally:

**Proposition 1** There exists a shadow price vector $\pi(p/y)$ and a scalar valued sharing rule $\eta(p/y)$, with $0 \leq \eta(p/y) \leq 1$, such that the solution to program $P^*$ satisfies

\[
\begin{align*}
    x^f(p/y) &= h^f \left( \frac{\pi(p/y)}{\eta(p/y)} \right) \\
    x^m(p/y) &= h^m \left( \frac{\pi(p/y)}{1 - \eta(p/y)} \right) \\
    z &= h(p/y) = F[x^f(p/y) + x^m(p/y)]
\end{align*}
\]  

(3)
This and other propositions are proved in the Appendix. Here, $\pi(p/y)$ denotes the equilibrium (shadow) prices within the household economy. In general, these shadow prices will depend in a complicated way on the consumption technology function and both members demand functions. However, $\pi(p/y)$ has a simple tractable functional form that only depends on the consumption technology when the technology is linear, as follows.

**Proposition 2** If the consumption technology function is linear, so $z = F(x) = Ax + a$ then the functions $\pi(p/y)$ and $h(p/y)$ in Proposition 1 are given by

$$
\pi(p/y) = \frac{A'p}{y - a'p}
$$

$$
z = h(p/y) = Ah^f \left( \frac{A'p}{y - a'p \eta(p/y)} \right) + Ah^m \left( \frac{A'p}{y - a'p \left( 1 - \eta(p/y) \right)} \right) + a
$$

Note that the total shadow income of the household is $\pi'(x^f + x^m) = 1$, and the sharing rule $\eta$ is given by $\eta = \pi'x^f$, which is the fraction of total shadow income consumed by member $f$. It follows that $\eta(p/y)$ is a direct measure of the weight given to member $f$ in the outcome of the household decision process.

**Proposition 3** There is a one to one, strictly monotonic relationship between the Pareto weight $\mu$ and the sharing rule $\eta$, which can be written as

$$
\mu = \left[ \frac{\partial V^f (\pi)}{\partial \eta} \right] / \left[ \frac{\partial V^m (\pi)}{\partial \eta} \right]
$$

An advantage of the sharing rule measure $\eta$ is that (unlike $\mu$) it does not depend on any chosen cardinalization of utility. Also, unlike Browning and Chiappori (1998), who find using household data alone that relative bargaining power measures can only be identified up to an arbitrary location, we show in our model that by combining data from households and from singles living alone, $\eta$ is completely identified.
3 Applications

Here we summarize some potential uses for our model of household consumption behavior. These uses are in addition to the standard applications of demand models, such as evaluation of price and income elasticities. One immediate use of the results is the estimate of the sharing rule $\eta$ as discussed in the previous section. Other applications are measures of economies of scale and adult equivalence scales.

3.1 Economies of Scale in Consumption

Previous attempts to measure economies of scale in consumption have required very restrictive assumptions regarding the preferences of household members (see, e.g., Nelson 1988). In contrast, given estimates of the private good equivalents vector $x = x^f + x^m$ from our model, we may calculate $y/px$, which is a measure of the overall economies of scale from living together, since $y$ is what the household spends to buy $z$ and $px$ would be the cost of buying the private good equivalents of $z$. This economy of scale measure takes the form

$$\frac{y}{px} = \frac{y}{p'(h[f/\eta] + h[m/(1-\eta)]).}$$

The measure relates the household’s total expenditures to the total cost of independently purchasing the same individual consumptions. It is important to note that this measure does not directly provide an estimate of adult equivalence scales. The reason is that, in general, the shadow prices used within the household are not proportional to market prices (this is exactly Barten’s intuition). It follows that individuals living alone would not buy the bundles $x^f$ and $x^m$. They could in general reach a higher level of utility by purchasing and consuming a different combination.

3.2 Adult Equivalence Scales

Given each household member $i$’s private good equivalents $x^i$, we define member $i$’s collective based equivalent income, $y^{i\ast}$, as the minimum expenditures required to buy a vector of goods that is on the same member $i$ indifference curve as $x^i$. The ratio of $y/y^{i\ast}$ is then our collective model based adult equivalence scale, since $y$ is what the household spends to get member $i$ onto the same indifference curve over goods that he or she could attain while living alone with an income level
of $y^{i\ast}$. Like the above described economies of scale and bargaining power measures, these equivalence scales do not depend on arbitrary utility cardinalizations or assumptions of interpersonally comparable utility.

Recall that $V^i(p/y^i)$ is the indirect utility function of member $i$ when living alone, and the private good equivalent vector consumed by member $i$ while in the household is $x^i(p/y) = h^i[\pi(p/y)/\eta^i(p/y)]$, where $\eta^f(p/y) = \eta(p/y)$ and $\eta^m(p/y) = 1 - \eta(p/y)$. Then $y^{i\ast}(p, y)$ is defined as the solution to the equation

$$
V^i \left( \frac{p}{y^{i\ast}(p, y)} \right) = V^i \left( \frac{\pi(p/y)}{\eta^i(p/y)} \right)
$$

(5)

This definition is ordinal, i.e., it does not depend on the chosen cardinalization for member $i$’s utility function. In applications with linear technologies, functional forms for $V^i$ and $\eta^i$ could be directly specified and $\pi$ given by Proposition 2.

Given $y^{i\ast}(p, y)$, we may calculate equivalence scales $y/y^{i\ast}(p, y)$ or the difference in income required, $y - y^{i\ast}(p, y)$. For example, at prices $p$, the quantity $y$ equals the amount of money that the household must spend so that the man in that household has the same standard of living (i.e., attains the same indifference curve over goods) that he would attain if he were to live alone with total expenditures $y_m^\ast(p, y)$. This could be applied to the calculation of poverty lines for different households. Similarly, in the case of a wrongful death or insurance calculation, a woman (or a mother and her children, if $V^f$ is defined as the joint utility function of a mother and her children) would need income $y^{f\ast}(p, y)$ to attain the same standard of living without her husband that she (or they) attained while in the household with the husband present and total expenditures $y$. This would be sufficient to compensate for the loss of economies of scale and scope from shared consumption (and for any changes in preferences over goods that result from living in the household), but would not account for grieving, loss of companionship, or other components of utility that are assumed to be separable from consumption of goods. It would also not account for any change in preferences over goods that might occur as a result of the death.

Another interesting equivalence scale to construct might be $y^{f\ast}/y^{m\ast}$, the ratio of how much income a woman needs when living alone to the income a man needs when living alone to make each as well off as they would be in a household. Interestingly, even if men and women had identical preferences, this ratio need not equal one in general, because for example if the sharing rule $\eta$ is bigger than one half, then the wife receives more than half of the household’s resources, and
hence would need more income when living alone to attain the same standard of living.

To separate bargaining effects from other considerations, other measures of collective based equivalent income could be calculated. For example, the ratio $y_f^*/y_m^*$ might better match the intuition of an equivalence scale if each $y^*_i$ equation were calculated as the solution to

$$V^i \left( \frac{p}{y^*_i(p, y)} \right) = V^i \left( \frac{\pi(p/y)}{1/2} \right)$$

which gives equivalent incomes assuming equal sharing of resources.

In other applications, one might want to consider the roles of equivalent income and sharing rule jointly. For example, given poverty lines for singles, one might define the corresponding poverty line for the couple as the minimum $y$ such that, by choosing $\eta$ optimally, each member $i$ of the couple would have an equivalent income $y^*_i$ equal to his or her poverty line as a single.

### 4 Identification

In our context, the demand functions of singles and the aggregate demands of household are observable. The identification question is thus, given the observable demand functions $h^m$, $h^f$, and $h$, can the consumption technology function $F$ and the private good equivalent demand functions $x^f(p/y)$ and $x^m(p/y)$ be estimated? We show that these functions are 'generically' nonparametrically identified, meaning that identification will only fail if the utility and technology functions are too simple (for example, a linear $F$ is not identified if demands are of the linear expenditure system form). This identification in turn implies identification of all of the above described features of the model, including the sharing rule, economies of scale, and adult equivalence scales.

A question closely related to identification is, given functional forms for the technology, the sharing rule, and the demands of singles, what is the implied functional form for household demands? This also is generically identified, and in the case of linear consumption technologies has the simple, explicit solution given in Proposition 2.
4.1 Identification of the consumption technology and sharing rule

We first investigate whether, given the observable Marshallian demand functions of singles and households, and a linear consumption technology, one can in general uniquely recover the private good equivalent functions $x^m$ and $x^f$, the consumption technology $F$, and the sharing rule $\eta$. In general, the answer is positive, provided that the number of goods is $n \geq 3$.

To see why, take some given (finite) set of price vectors $p^1, ..., p^T$ where $T \geq n + 10$. These may be interpreted as prices in different time periods. For any $p^t$ and any $y$, let $z^t$ denote the corresponding household demand; from Proposition 2, we know that

$$z^t = Ah^f \left( \frac{A'p^t}{y - a'p^t \eta(p^t/y)} \right) + Ah^m \left( \frac{A'p^t}{y - a'p^t 1 - \eta(p^t/y)} \right) + a, \quad t = 1, ..., T$$

(7)

This provides, for each $t$, $(n - 1)$ independent equations, hence a total of $(n - 1) T$ equations. The unknowns, on the other hand, are the matrix $A$, the vector $a$ and the scalars $\eta^t = \eta(p^t/y), t = 1, ..., T$; hence a total of $n^2 + n + T$ unknowns. If $n \geq 3$, then with $T \geq n + 10$, the number of equations exceeds the number of unknowns, so the system is generically identified. This allows us to recover the consumption technology. Finally, once $A$ and $a$ are known, the sharing rule $\eta(p/y)$ is identified for all $p/y$ from equation (4).

The private good equivalent functions $x^m$ and $x^f$ follow, since

$$x^f(p/y) = h^f \left( \frac{A'p}{y - a'p \eta(p/y)} \right)$$

and

$$x^m(p/y) = h^f \left( \frac{A'p}{y - a'p 1 - \eta(p/y)} \right)$$

This result is only generic since the equations (7) may fail to be linearly independent for particular functional forms. For example, it can be readily verified that identification fails when the individual demands $h^f$ and $h^m$ have the Linear Expenditure System functional form. However, such problems disappear for sufficiently complicated functional forms for members demands. This claim is illustrated in the next subsection for the Almost Ideal (AIDS) and Quadratic Almost Ideal (QUAIDS) functional forms. The latter will be used for our empirical estimation. A notable implication of these results is that our collective based
equivalence scales are completely identified from observable demand data, without any assumptions regarding cardinalizations or interpersonal comparability of utility functions.

In the appendix we show that, with some minimal assumptions, generic nonparametric identification holds not just for linear consumption technologies \( z = Ax + a \), but also for arbitrary technologies, that is, \( z = F(x) \) for general monotonic vector valued functions \( F \). This shows that our general methodology does not depend on functional form assumptions to obtain identification.

### 4.2 The case of QUAIDS individual demands

For our empirical application, we use a parametric approach. Instead of specifying the structural model as in program (P\(^*\)), the following convenient method is used to construct functional forms for estimation. First, choose ordinary indirect utility functions for members \( m \) and \( f \), and let \( h^m \) and \( h^f \) be the corresponding ordinary Marshallian demand functions. Assume a linear consumption technology \( F \). Next choose a functional form for the sharing rule \( \eta \), which could simply be a constant, or a function of measures of bargaining power such as relative wages of the household members or other distribution parameters. This sharing rule function must lie between zero and one. Proposition 2 then provides the resulting functional form for the household demand function \( h(p/y) \), and ensures that a corresponding household program exists that rationalizes the choice of functions \( h^m \), \( h^f \), and \( \eta \).

In our empirical application we assume singles have preferences given by the Integrable QUAIDS demand system of Banks, Blundell and Lewbel (1997). For \( i = f \) or \( m \), let \( w^i = \omega^i(p/y^i) \) denote the \( n \)-vector of member \( i \)'s budget shares \( w^i_k \) \((k = 1, \ldots, n)\) when living as a single, facing prices \( p \) and having total expenditure level \( y^i \). The QUAIDS demand system we estimate takes the vector form

\[
\omega^i(p/y^i) = \alpha^i + \Gamma^i \ln p + \beta^i \left[ \ln (y^i) - c^i(p) \right] + \frac{\lambda^i}{b^i(p)} \left[ \ln (y^i) - c^i(p) \right]^2 \tag{8}
\]

where \( c^i(p) \) and \( b^i(p) \) are price indices defined as

\[
c^i (p) = \delta^i + (\ln p)'\alpha^i + \frac{1}{2} (\ln p)'\Gamma^i \ln p \tag{9}
\]

\[
\ln[b^i (p)] = (\ln p)'\beta^i. \tag{10}
\]
Here $\alpha_i$, $\beta_i$ and $\lambda_i$ are $n$-vectors of parameters, $\Gamma_i$ is an $n \times n$ matrix of parameters and $\delta_i$ is a scalar parameter which we take to equal zero, based on the insensitivity reported in Banks, Blundell, and Lewbel (1997). Adding up implies that $e'\alpha_i = 1$ and $e'\beta_i = e'\lambda_i = \Gamma_i e = 0$ where $e$ is an $n$-vector of ones. Homogeneity implies that $\Gamma' e = 0$ and Slutsky symmetry is equivalent to $\Gamma_i$ being symmetric. The above restrictions yield the integrable QUAIDS demand system, which has the indirect utility function

$$V^i \left( \frac{p}{y^i} \right) = \left[ \left( \frac{\ln (y^i) - c_i (p)}{b^i (p)} \right)^{-1} + \lambda_i' \ln (p) \right]^{-1}$$

for $i = f$ and $i = m$. The singles demand functions $\omega^i (p / y^i)$ in equation (8) are obtained by applying Roy’s identity to equation (11). Deaton and Muellbauer’s (1980) Almost Ideal Demand System (AIDS) is the special case of the integrable QUAIDS in which $\lambda_i = 0$.

**Proposition 4** Assume singles have preferences given by the integrable QUAIDS, and that couples have a linear household technology. Assume $\beta^f \neq \beta^m$ and each element of $\beta^f$, $\beta^m$, and the diagonal of $A$ is nonzero. Then the household technology and the sharing rule are identified.

Proposition 4 confirms that the QUAIDS model for single’s preferences, with a linear household technology, is sufficiently nonlinear to permit identification of all the components of the couple’s model, as discussed in the previous subsection. The assumption regarding nonzero and unequal elements in Proposition 4 can be relaxed; see the proof in the Appendix for details.

## 5 Additional Results

### 5.1 Barten and Gorman Scales

Gorman’s (1976) general linear technology model assumes that household demands are given by

$$z = Ah^m \left( \frac{A'p}{y - a'p} \right) + a$$

(12)
Barten (1964) scaling (also known as demographic scaling) is the special case of Gorman’s model in which \( a = 0 \) and \( A \) is a diagonal matrix; see also Muellbauer (1977). Demographic translation is Gorman’s linear technology with \( A \) equal to the identity matrix and \( a \) non zero, and what Pollak and Wales (1992) call the Gorman and reverse Gorman forms have both \( A \) diagonal and \( a \) nonzero. These are all standard models for incorporating demographic variation (such as the difference between couples and individuals) into demand systems. The motivation for these models is identical to the motivation for our linear technology \( F \), but they fail to account for the structure of the household’s program. Even if the household members have identical preferences (\( h^f = h^m \)) and identical private equivalent incomes (\( \eta = 1/2 \)), comparison of equations (4) and (12) shows that household demand functions will still not actually be given by the Gorman or Barten model. In fact, comparison of these models shows that household demands will take the form of Gorman’s linear technology, or some special case of Gorman such as Barten, only if either one household member consumes all the goods (\( \eta \) is zero or one), or if demands are linear in prices (i.e., the linear expenditure system).

5.2 Technology, Externalities, and Preference Changes

A maintained assumption of our model is that commodities (after transformation by the consumption technology function) are privately consumed, and that preferences do not change through marriage. What if these assumptions do not hold? We will now show that our framework is compatible with more general assumptions, including externalities, public goods and preference changes. The price to pay for these generalizations is that identification will require either richer data sets or strong assumptions regarding preference changes, externalities, or public goods. We also consider here what effects violations of our maintained assumptions would have on our estimates.

**Public goods and externalities** We first show how our framework could be extended to allow for the existence of pure public goods within the household (our model already allows some degree of publicness via sharing and economies of scale). For simplicity, assume for now that there is only one public and one private good, denoted by \( X \) and \( x \) respectively, and that the household technology is separable across goods. Pareto efficiency implies that the vector \( (x^f, x^m, X) \) solves the program (with obvious notations):

\[
\max_{x^f, x^m, X} \bar{U}[U^f(x^f, X), U^m(x^m, X), p/y, P/y]
\]
\[ z = f(x^f + x^m), \quad Z = F(X), \quad pz + PZ = y \]

An equivalent formulation is

\[ \max_{x^f, x^m, z} \bar{U}[U^f(x^f, X^f), U^m(x^m, X^m), p/y, P/y] \]  

\[ z = f(x^f + x^m), \quad Z = F\left(\max(X^f, X^m)\right), \quad pz + PZ = y \]

Here, \( X^i \) can be interpreted as the quantity of the public good 'desired' by member \( i \). In principle, we allow \( X^f \) and \( X^m \) to differ, though the solution to this program always entails \( X^f = X^m \).\(^{12}\) Program (P**) is formally equivalent to program (P*) in section 2, except that \( F \) is now a function of the pair of individual consumptions, not of their sum only. In this model the demands of member \( f \) depend on the vector \( (\pi/\eta, \pi^i/\eta) \), and similarly for member \( m \). Each member \( i \)'s (Lindahl) decentralizing price \( \pi^i \) for the public good has \( \pi^i > 0 \) and, if \( f(x) = Ax + a \) and \( F(X) = BX + b \), then

\[ \pi = \frac{Ap}{y - ap - bP}, \quad \pi^f + \pi^m = \frac{BP}{y - ap - bP} \]

so the sum of individual prices for the public good plays the same role as the price of a private commodity. A similar analysis could be used to incorporate externalities within the household, whether positive or negative. Note, however, that with negative externalities individual prices \( \pi^i \) for the externality could be negative.

A more general program that encompasses both (P**) and our program (P*) as special cases is

\[ \max_{x^f, x^m, z} \bar{U}[U^f(x^f), U^m(x^m), p/y] \]

\[ z = F(x^f, x^m), \quad p'z = y \]

which allows for completely general consumption technology functions \( F(x^f, x^m) \).

Like the other programs, the general household model (\( \hat{P} \)) can be decentralized, but now the complete vector of Lindahl shadow prices will be different for members \( m \) and \( f \), being functions of the separate derivatives of \( F \) (assuming these derivatives exist) with respect to \( x^m \) and \( x^f \). In contrast, our program (P*) assumption of an additive technology \( z = F(x^f + x^m) \) makes the derivatives of \( F \)

\(^{12}\)A strict inequality - say, \( X^f < X^m \) - is ruled out because the constraints would then imply that the opportunity cost of \( (X^m - X^f) \) additional units consumed by \( f \) would be zero.
with respect to $x^m$ and $x^f$ equal, resulting in shadow prices $\pi(p/y)$ that are the same for $m$ and $f$. The problem with nonadditive technology models is that our nonparametric identification result does not hold for them. In particular, it may be the case that several different technology functions $F(x^f, x^m)$ could generate the same reduced-form demand functions. Identification would then need to rely either on detailed functional form assumptions or on the availability of additional information, such as each member’s individual consumptions $x^i_k$ for one or more goods $k$.

**Misspecifications due to public goods and externalities**  Without additional information such as richer data, if our maintained assumptions are violated then our estimates of the consumption technology function $F$ may end up incorporating features like preference changes and externalities, in addition to purely technological economies of scale and scope in consumption. This means that some calculations, in particular economies of scale and adult equivalence scales, will need to be interpreted with caution. For example, a change in taste for dancing as a result of marriage could show up as economies or diseconomies of scale in the consumption of dancing shoes. We now consider some examples.

Suppose first that our model, which assumes an additive consumption technology $Z = F(b x^f + c x^m)$, were applied to data derived from a more general technology $Z = F(x^f, x^m)$. How would this misspecification bias the results? Our additive technology implies that both individuals face the same shadow prices $\pi$. This will provide a good approximation to a more general technology if agents have similar marginal valuations for the goods, and hence similar shadow prices.

In the particular example of a public good, our additive model will provide a good approximation to actual behavior if the household members have a similar willingness to pay for the public good. As shown above, the ‘intrahousehold price’ of a public good is twice the individual price that would be estimated in our additive model. For example, in the absence of economies of scale or scope for this commodity, the shadow price should be one half the market price, leading in the Barten technology $\pi_k = A_k p_k/y$ to a coefficient of $A_k = 1/2$. If, in addition, economies of scale are present, the coefficient may become smaller than one half; conversely, congestion (or partly private use) would typically generate Barten coefficients between .5 and 1.

The analysis of positive externalities is similar to that of public goods. When externalities are negative, it points to the opposite direction. Suppose, e.g., that a commodity consumed by one member has a negative impact on the other per-
son. Since the second person’s marginal willingness to pay is negative, the first person’s individual price must exceed the household price. Now in the absence of economies of scale or scope for this commodity, the estimated shadow price would be larger than the market price, leading to a Barten coefficient greater than one. More generally, if a commodity (say, tobacco) is consumed by both members and each individual’s consumption has a negative impact on the other person’s welfare, then each individual price may exceed the household price (because at the optimum each member will have a positive marginal willingness to pay to get rid of the other person’s consumption). Overall, in the absence of preference changes, an estimated Barten coefficient larger than one would be suggestive of the presence of negative consumption externalities for the commodity under consideration.

**Changes in preferences** A similar situation arises when marriage induces preference changes. Externalities versus changes in preferences would be hard if not impossible to distinguish empirically. For example, if we found that households consume proportionally more restaurant meals than either single males or single females, this could be due either to externalities (each member’s marginal utility of restaurant dining might be enhanced by the presence of their spouse) or by a simple change in preferences (members liking restaurant food better when married). The two explanations have different implications, e.g., under the externality story only dinners taken with one’s spouse are increased in value, in contrast to the preference change explanation. In practice, available data would hardly allow the estimation of such subtle distinctions.

One way to address the issue of preference change is to explicitly incorporate parameters to measure taste changes resulting from marriage in the model, in addition to the household technology parameters. For example the mapping from market prices to shadow prices could be given by:

\[ \pi_k = L_k A_k p_k / y \]  

where \( A_k \) is a technological economies of scale parameter with value one for a purely private good, and the parameter \( L_k \) captures common taste changes over the ‘private good equivalent’ \( x_k = (z_k / A_k) \). If \( L_k = 1 \) then tastes for good \( k \) do not change with marriage. If \( L_k < 1 \) then married individuals like good \( k \) more than when they are single.

In this example, the household consumption technology is of the Barten type \( z = Ax \) where \( A \) is diagonal, and the utility function of an individual \( i \) living alone is, as before, \( U^i(z^i) \), but now the individual’s tastes also change in a purely Barten
way, from $U^i(z^i)$ before marriage to $U^i(Lz^i)$ after marriage, where $L$ is diagonal. More formally, we should say that member $i$’s indifference curves are defined by $U^i(z^i)$ before marriage and by $U^i(Lz^i)$ after. The member’s actual level of utility presumably also changed after marriage, so $U^i(Lz^i)$ also undergoes some unobservable monotonic transformation after marriage. This transformation is absorbed into $\tilde{U}$.

The household’s optimization function is then

$$\max_{x^f,x^m,z} \tilde{U}[U^f(Lx^f), U^m(Lx^m), p/y]$$

$$z = A(x^f + x^m)$$

$$p'z = y$$

and the resulting household demands are exactly as before, except that everywhere we had $A_k$ we will now have $L_k A_k$.

If we could separately identify $L$ and $A$ (this identification issue is discussed below), then we may define equivalence scales $y^{i*}/y$ as in equation (5) using either the preferences of member $i$ as a single or the preferences of member $i$ as a member of a couple, that is, we could define $y^{i*}$ either as the solution to

$$V^i \left( \frac{p}{y} \right) y^{i*} = V^i \left( \frac{A'p}{\eta y} \right)$$  \hspace{1cm} (14)$$

or as the solution to

$$V^i \left( \frac{L'p}{y} \right) y^{i*} = V^i \left( \frac{A'L'p}{\eta y} \right)$$  \hspace{1cm} (15)$$

In general these two definitions will differ. An equivalence scale based on equation (14) makes member $i$ as well off alone as she was in the couple, based in both situations on the preferences (and hence indifference curves) she has a single, while the equation (15) definition makes her as well off based on her personal indifference curves as a household member. A third possible calculation, which would not require separately identifying $L$ and $A$, would be the solution to

$$V^i \left( \frac{p}{y} \right) y^{i*} = V^i \left( \frac{A'L'p}{\eta y} \right)$$  \hspace{1cm} (16)$$

which attempts to compensate member $i$ for both taste change and economies of scale, however, this definition entails the same untestable interpersonal comparability or choice of cardinalization assumptions that afflict traditional equivalence scale calculations, since in this case actually equating utilities would require replacing $V^i$ on the right of equation (16) with the unobservable monotonic transformation discussed above.
Identification Our general identification results do not hold when preferences may change. This fact is easy to see from the equations above. The model incorporating preferences changes is observationally equivalent to our original model with technology \( z = LA(x^f + x^m) \). In estimation, without additional information we cannot separately identify the \( A_k \)'s and the \( L_k \)'s but only their product.

In our model it would be reasonable to interpret a Barten technology coefficient for good \( k \) of less than 0.5 or greater than 1 as partially reflecting externalities or taste changes, e.g., as an estimate of \( L_k A_k \) for some taste change \( L_k \). If we are prepared to assert \textit{a priori} that the good \( k \) (e.g., clothing) is purely private and does not entail externalities, then we would be assuming \( A_k = 1 \) and the estimate would serve to identify the change in taste \( L_k \) for that good. Identifying information could instead come from direct measurement of sharing within the household, e.g., \( A_k \) for auto use might be measured from direct observation of the percentage of time that couples use their car jointly.

Rather than attempt to separately identify taste changes, it may be reasonable to assume that, at least for some goods, the dollar effect of a change in tastes is small. In equivalence scales this could mean using equation (16) as an approximation to equation (15), because only the product \( L_k A_k \) is identified and \( L_k \) is assumed to be relatively close to one. For example, joint consumption of heating is likely to have a much larger effect on measured cost savings of living together than on any change in tastes for heat. The portion of \( F \) that corresponds to taste changes versus consumption technology is also largely irrelevant for calculations such as demand elasticities, or the interpretation of the sharing rule \( \eta \) as a measure of bargaining power.

Many applications of equivalence scales deal with appropriate compensation for a change in status, for example, calculation of the appropriate level of life insurance on a spouse, or compensation in legal cases of wrongful death. In these circumstances, we would like to compensate for both changes in tastes and the loss of purely technological economies of scale in consumption, and there would be no need to separately identify each. This would still exclude noneconomic effects such as grief and loss of companionship. Note, however, that this means using equation (16), and any attempt like this to calculate compensation for changes in taste rather than technology raises the traditional equivalence scale identification issues.

Future work may lead to additional data or alternative assumptions that allow for more complete separation of taste changes, externalities, and purely technological economies from sharing. While our particular identifying assumptions are open to debate and (we hope) improvement, we believe that the model we provide
here is an appropriate framework for analyzing these issues.

6 Empirical Application

An implication of our propositions is that one may directly specify a functional form for \( \eta \) instead of for \( U \), which greatly simplifies empirical implementation of the model. In our empirical implementation the sharing rule \( \eta \) is directly parameterized and estimated as part of the household’s demand functions. The simplest specification simply lets \( \eta \) equal a constant to be estimated. More generally, \( \eta \) is specified as a function of variables that affect bargaining power, such as the relative incomes and other demographic characteristics of the household members.

Given estimates of ordinary demand functions \( h^f \) and \( h^m \) from singles data, equation (4) may then be estimated from household demand data, where the parameters to be estimated are the technology parameters \( A \) and \( a \), and any parameters in the sharing rule \( \eta(p/y) \). To maximize efficiency, we estimate these technology and sharing rule parameters for couples simultaneously with the ordinary demand function parameters in \( h^f \) and \( h^m \) for singles.

6.1 Data

We use Canadian FAMEX data from 1974, 1978, 1982, 1984, 1986, 1990 and 1992. The Canadian FAMEX is a multi-staged stratified clustered survey that collects information on annual expenditures, incomes, labour supply and demographics for individual (‘economic’) households. The survey is not nationally representative. In particular, in most years only information from respondents living in large cities is collected (hence the high proportion of city dwellers in our samples below). The survey is run in the Spring after the survey year (that is, the information for 1978 was collected in Spring 1979). All of the information is collected by interview so that the expenditure and income data are subject to recall bias. Although this may give rise to problems, the FAMEX surveying method has the great advantage that information on annual expenditures is collected. Thus the FAMEX has much less problem with infrequency bias than do surveys based on short diaries. It is also the case that since the survey year coincides with the tax year (January to December) the income information is thought to be unusually reliable since it is collected at about the time that Canadians are filing their (individual) tax returns. These are often explicitly referenced by the enumerators.
Prices are taken from Statistics Canada. When composite commodities are created, the new composite commodity price is the weighted geometric mean of the component prices (a Stone price index) with budget shares averaged across the strata (couples, single males and single females) for weights. Thus, the weights are not the individual household budget shares.

We have three strata: single females, single males and couples with no one else present in the household. We sample only younger agents: the single females and wives in couples are aged less than or equal to 42; single males and husbands are all aged less than or equal to 45. All agents are in full year full time employment and we have excluded any household with non–negative net income or non-negative individual gross incomes. About 5% of couples in the original sample drawn did not own cars; car ownership has a major impact on demand patterns and modelling this 5% turned out to present problems. Consequently we also select on couples having a car. The (non-durable) goods we model are: food at home, restaurant expenditures, clothing, vices (alcohol and tobacco), transport, services and recreation.

Table 1 presents summary statistics for our three strata. As can be seen, income and total expenditure is lowest for single females and highest for couples. Total expenditure on nondurables is between 41% and 45% of net income. Home ownership is significantly higher for couples than for singles. Single males have significantly higher budget shares for restaurant, vices and transport than single females. Single females have higher budget shares for food at home, clothing and services. Of course, this does not necessarily mean that single men and women have different preferences since these are conditional on demographics and real total expenditure which differ across the two strata. Below we shall present tests for whether single men and women have the same preferences. Finally the budget shares of couples are much closer to those of single females than to those of single men for all goods except for transport and services.

6.2 Singles Budget Shares

Our model starts with a utility derived functional form for the budget shares of singles. For this we use the QUAIDS model described in equations (8), (9), (10), and (11), with $i = m$ for single men and $f$ for single women. We also estimate a QUAIDS for couples to provide an initial comparison with singles and with our collective household model.

We allow the $\alpha$ and $\beta$ parameters in the QUAIDS to depend on demographics.
<table>
<thead>
<tr>
<th></th>
<th>Single females</th>
<th>Single males</th>
<th>Couples Husband</th>
<th>Couples Wife</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sample size</strong></td>
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<td>1574</td>
<td>1610</td>
<td></td>
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<tr>
<td><strong>Net income</strong>*</td>
<td>26,137</td>
<td>30,890</td>
<td>56,578</td>
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<tr>
<td><strong>Total expenditure</strong>*</td>
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<td>13,645</td>
<td>23,175</td>
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<td>-</td>
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<td></td>
</tr>
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<td>bs(food at home)</td>
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<td>0.16</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>bs(restaurant)</td>
<td>0.11</td>
<td>0.15</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>bs(clothing)</td>
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<td>0.09</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>bs(vices)</td>
<td>0.07</td>
<td>0.12</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>bs(transport)</td>
<td>0.21</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>bs(services)</td>
<td>0.17</td>
<td>0.10</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>bs(recreation)</td>
<td>0.11</td>
<td>0.13</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>car</td>
<td>0.64</td>
<td>0.78</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>home owner</td>
<td>0.13</td>
<td>0.23</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>city dweller</td>
<td>0.85</td>
<td>0.81</td>
<td>0.83</td>
<td></td>
</tr>
<tr>
<td>age</td>
<td>29.9</td>
<td>31.1</td>
<td>30.2</td>
<td>28.2</td>
</tr>
<tr>
<td>higher education</td>
<td>0.20</td>
<td>0.25</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>Francophone</td>
<td>0.19</td>
<td>0.17</td>
<td>0.21</td>
<td>0.18</td>
</tr>
<tr>
<td>Allophone</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>White collar</td>
<td>0.43</td>
<td>0.40</td>
<td>0.39</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Notes. * mean for 1992 (when prices are unity).

Table 1: Descriptive statistics
Specifically we take:

\[ a_k^i = a_{k0}^i + \sum_{m=1}^{M_\alpha} a_{km}^i d_m \]  

(17)

\[ \beta_k^i = \beta_{k0}^i + \sum_{m=1}^{M_\beta} \beta_{km}^i d_m \]  

(18)

where \( M_\beta = 2 \) for singles (dummies for owning a car and being a home owner) and \( M_\beta = 1 \) for couples (a home owner dummy).\(^{13}\) For the intercept demographics we have \( M_\alpha = 18 \) for couples and \( M_\alpha = 13 \) for singles. These demographics include some variables that are common to everyone in the household: four regional dummies; a house ownership dummy and a dummy for living in a city. For singles we also have a car ownership dummy. As well we include individual specific demographics: age and age squared, a dummy for having more than high school education, dummies for being French speaking or neither English nor French speaking and an occupation dummy for being in a white collar job. Although these demographics are highly correlated within couples we need to include both sets so that we can allow for the dependence of couples’ budget shares on both sets of individual characteristics. We have 24 and 28 parameters per good/equation for singles and couples respectively.

Before presenting estimates for our model, we graph the QUAIDS fits for the different goods for the three strata (with homogeneity and symmetry imposed). We estimate by GMM and take account of the endogeneity of total expenditure by using as instruments for singles: all of the demographics and log relative prices (the included variables) plus log real net income\(^{14}\) and its square, the product of real net income with the car and home ownership dummies and absolute log prices. For instruments for couples we use the instruments used for singles (without, of course, the car dummy variables), with individual specific values for husbands and wives, where appropriate. We also include logs of the individual gross incomes and the ratio of the wife’s gross income to total gross income (we discuss these variables below). In all we have 25 instruments per equation for singles and 32 instruments per equation for couples. Without symmetry there is one degree of over-identification for each equation for singles and four per equation for couples. For singles, the \( \chi^2 \) (6) statistics for the over-identifying restrictions are 5.8

\(^{13}\)Recall that we have selected only couples who own a car.

\(^{14}\)This is the log of nominal net income divided by a Stone price index computed for our seven non-durables goods.
(probability = 44%) and 12.4 (5.4%) for single females and single males respectively. For couples, the over-identifying $\chi^2$ (24) statistic is 40.6 (1.8%). We then impose symmetry on both sets of estimates; the respective $\chi^2$ (15) statistics for single females, single males and couples are 25.3 (4.6%), 28.0 (2.1%) and 28.6 (1.8%).\(^{15}\)

Much of our analysis is based on single men and women having different preferences, so we present tests for the sets of parameters being the same. For the six equations modelled we have the following $\chi^2$ (24) statistics for equality: 52 (food), 118 (restaurant), 686 (clothing), 95 (recreation), 117 (transport) and 622 (services). Thus we decisively reject that single men and single women have the same preferences.

To show the differences between predicted demands for singles and couples, we calculate predictions for agents who live in Ontario in 1992, the year and region for which prices are set to unity. For the other demographics we take the means for Ontario in 1992. For total expenditure we take a range from the first decile to the ninth decile. Figures 1 to 7 present the plots for our seven goods. In the figures we shift the plots for couples to the left to make them more ‘readable’; the shift chosen is purely for display purposes.\(^{16}\) The relative positions for single men and single women are similar to the unconditional budget shares given above for all goods except for food at home for which single men have a higher predicted budget share for a given level of total expenditure.\(^{17}\) We find that food at home is a necessity for all strata and that transport and services are necessities for singles. Restaurants and recreation are luxuries for all strata and vices are a luxury for singles. Clothing is a necessity for low income single women but a luxury for high income single women (the QUAIDS quadratic term is significant). The other important feature of these graphs is that vices are a necessity and transport is a luxury for low income couples. One objective of this paper is to see how well we can reconcile these predictions with a model in which we have common preferences for each sex (whether they be married or single) and a parsimonious set of sharing rule and technology parameters.

\(^{15}\)Note that this QUAIDS model for couples, which we only estimate for comparison, is a unitary model, while our proposed model is a collective model, which in general need not satisfy Slutsky symmetry.

\(^{16}\)Since the leftward shift we have used to graph the budget shares for couples is arbitrary, we cannot interpret the relative positions of the couples.

\(^{17}\)The lower mean value for single men in Table 1 is because they have a higher mean value for total expenditure and the food budget share is decreasing in the latter.
6.3 The Joint Model

For our empirical application, we assume singles demands have the integrable QUAIDS functional form described in equations (8), (9), (10), (11), (17) and (18). The joint model therefore includes one set of $\alpha^i, \beta^i, \Gamma^i$ and $\lambda^i$ parameters (including demographic components) for men, $i = m$, and another complete set of these parameters for women, $i = f$. Let $w^f_k = \omega^f_k (p/y)^f$ and $w^m_k = \omega^m_k (p/y)^m$ denote the QUAIDS budget share of consumption of good $k$ for single women and single men, respectively.

For couples we assume a Barten type technology function defined as

$$z_k = A_k x_k$$

for each good $k$, and so is equivalent to the general linear technology $z = Ax + a$ when the matrix $A$ is diagonal and overheads $a$ are zero. The shadow prices for this technology are

$$\pi_k = \frac{A_k p_k}{y}$$

where the couple faces prices $p$ and has total expenditure level $y$. This technology function is particularly convenient for budget share models. Let $w_k = \omega_k (p/y)$ be the budget share of good $k$ for the household, so $\omega_k (p/y) = p_k h_k (p/y)/y$. With the technology function (19) and corresponding shadow prices (20), equation (4) yields the following simple expression for the couple’s budget shares

$$\omega_k (p/y) = \eta \omega^f_k \left( \frac{\pi}{\eta} \right) + (1 - \eta) \omega^m_k \left( \frac{\pi}{1 - \eta} \right)$$

Equation (21) shows that, with a Barten type technology, the budget shares of the couple equal a weighted average of the budget shares of its members, with weights given by the income sharing rule $\eta$ and $1 - \eta$. Equation (21) clearly illustrates the point discussed earlier that $\eta$ represents both the fraction of equivalent resources controlled by the wife (by appearing in her budget share demands) and the extent to which the household’s demands resemble her demands (by appearing as the weight on her shares).

The parameters of the couples model consist of all the QUAIDS parameters of both the single’s models $\omega^f$ and $\omega^m$, the Barten scales $A_k$ which we model as constants, and the parameters of the sharing rule $\eta$. Our model for $\eta$ is

$$\eta = \frac{\exp (s' \delta)}{1 + \exp (s' \delta)}$$
where \( s \) is a household specific vector of distribution factors and \( \delta \) is a vector of parameters. This logistic form bounds the sharing rule between zero and one. Based in part on the bargaining models of Browning, Bourguignon, Chiappori and Lechene (1994) and Browning and Chiappori (1998), the distribution factors that comprise \( s \) are a constant, the wife’s share in total gross income, the difference in age between husband and wife, a home-ownership dummy and log deflated (by a Stone price index) household total expenditure.

The joint system consists of the vectors of budget shares for singles \( \omega^f(p/y^f) \) and \( \omega^m(p/y^m) \) and the vector of budget shares for couples \( \omega(p/y) \). For efficiency these are all estimated simultaneously, since all the parameters in the singles models also appear in the couples model.

We estimate the joint system by GMM. We assume that the error terms are uncorrelated across households but are correlated across goods within households. Let \( \omega_h \) be the \((n - 1)\) vector of budget shares for the first \( n - 1 \) goods consumed by household \( h \). Denote the vector of all parameter values by \( \theta \) and let \( \hat{\omega}_h(\theta) \) be the predicted budget shares for household \( h \). The error vector for household \( h \) is thus given by \( u_h(\theta) = \omega_h - \hat{\omega}_h(\theta) \). Let the numbers of couples, single females and single males be given by \( H_c, H_f \) and \( H_m \) respectively. Denote the \((1 \times g_c)\) vector of instruments for couples by \( z^c_h \) and similarly for singles (where \( g_c = 32 \) and \( g_f = g_m = 25 \)). The vector of moment conditions for couples is given by the \(((n - 1)g_c \times 1)\) vector:

\[
\mu^c(\theta) = \sum_{h=1}^{H_c} u'_h (I_{n-1} \otimes z^c_h)
\]

and similarly for singles. The overall weighting matrix is given by:

\[
W = \begin{bmatrix}
W_c & 0 & 0 \\
0 & W_f & 0 \\
0 & 0 & W_m
\end{bmatrix}
\]

where:

\[
W_c = \left( \sum_{h=1}^{H_c} (I_{n-1} \otimes z^c_h)' \tilde{u}^c \tilde{u}^c' (I_{n-1} \otimes z^c_h) \right)^{-1}
\]

and similarly for the weighting matrices for singles. The residuals \( \tilde{u}^c \) are taken from a first stage GMM with an identity weighting matrix. For the singles these first stage residuals are from the QUAIDS estimates reported earlier. For couples, these first stage residuals are from estimates of the couples budget share system.
(21), estimated using just couples data.\textsuperscript{18} The GMM criterion is the sum of the GMM criteria for the three strata with these weighting matrices, that is,
\[
\min_{\theta} \left\{ \mu^c (\theta)' W^c_c \mu^c (\theta) + \mu^f (\theta)' W^f f (\theta) + \mu^m (\theta)' W^m m (\theta) \right\}
\]
where $\theta$ is the full parameter vector. In our system we have at least 258 preference parameters ($6 \times 24 - 15 = 129$ symmetry constrained QUAIDS parameters for each of men and women, plus the sharing rule and Barten scale parameters). We have 492 instruments (for each good there are 25 instruments for each single strata and 32 for couples), giving a maximum degrees of freedom of 234 for the simplest model. This is a very large system that takes a long time to converge for any given specification. Consequently we are restricted in the inferences and diagnostics we present. For such a non-linear system conventional measures of standard errors can be quite misleading (see Gregory and Veall (1985)\textsuperscript{19}) but extensive quasi-likelihood testing of the ‘significance’ of particular parameters are prohibitively time consuming.

6.4 Estimation

We start with an extremely parsimonious specification and add extra parameters until we find a ‘satisfactory’ fit. We postpone detailed discussion of the implications of the models until we have a preferred specification.\textsuperscript{20} As a benchmark, we first estimate with no technology and equal sharing ($\eta = 0.5$ and $A_k = 1$ for all $k$). This corresponds to a model in which there are no economies of scale for any

\textsuperscript{18}Without singles data, the parameters from the couples model are not all identified, but all that is required for the first stage are unrestricted residual estimates. One could, alternatively, use the residuals from the QUAIDS estimates for couples, but doing so would lead to asymptotically inefficient parameter estimates in the second stage, because the joint model for couples is not a special case of a QUAIDS model. We found that use of residuals from the couples QUAIDS model estimates gave similar qualitative results but significantly higher values for the test statistics for over-identification reported below.

\textsuperscript{19}This is a problem that is endemic to large, highly nonlinear models, often resulting in substantial differences in the $p$-values of asymptotically equivalent Wald and likelihood ratio tests of variable exclusion hypothesis.

\textsuperscript{20}Estimation is made extremely difficult by the time it takes to estimate any variant (several days) and the fact that there are multiple local minima for each variant. For example, for our preferred specification we found over ten local minima, some of which give quite different implications. As always in this case, we report results for the estimates that gave the lowest value for the over-identifying restriction test statistic, but the possibility remains that these are not at a global minimum.
Table 2: Results

<table>
<thead>
<tr>
<th>Model</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharing rule</td>
<td>0.5</td>
<td>0.68</td>
<td>0.70*</td>
</tr>
<tr>
<td>Barten scale, food</td>
<td>1</td>
<td>0.93</td>
<td>0.82</td>
</tr>
<tr>
<td>Barten scale, rest</td>
<td>1</td>
<td>0.78</td>
<td>0.79</td>
</tr>
<tr>
<td>Barten scale, cloth</td>
<td>1</td>
<td>1.70</td>
<td>1.88</td>
</tr>
<tr>
<td>Barten scale, vices</td>
<td>1</td>
<td>3.19</td>
<td>3.00</td>
</tr>
<tr>
<td>Barten scale, trans</td>
<td>1</td>
<td>0.55</td>
<td>0.51</td>
</tr>
<tr>
<td>Barten scale, serv</td>
<td>1</td>
<td>0.59</td>
<td>0.59</td>
</tr>
<tr>
<td>Barten scale, recr</td>
<td>1</td>
<td>1.31</td>
<td>1.38</td>
</tr>
<tr>
<td>df</td>
<td>234</td>
<td>226</td>
<td>222</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>432.0</td>
<td>236.1</td>
<td>224.1</td>
</tr>
<tr>
<td>P-value (%)</td>
<td>0</td>
<td>30.8</td>
<td>44.8</td>
</tr>
</tbody>
</table>

* at mean of sharing of factors

goods and a fixed equal sharing rule. The $\chi^2$ (234) statistic for the restrictions is 432 so that this model is, as we would expect, decisively rejected. The next model we estimate (model 2) allows for a variable share parameter and seven variable Barten terms, so we have extra parameters ($\eta, A_1, ..., A_7$). The results are given in the second column of Table 2. As can be seen, allowing for unequal sharing and Barten scaling improves the fit dramatically: the $\chi^2$ (8) statistic for these parameters all having the same value as the benchmark model is 195.9. Moreover this model gives a satisfactory value for the test of the over-identifying restrictions. The next model, 3, replaces a constant sharing rule with the model of equation (22) which includes the four household specific distribution factors described earlier. In the third column of Table 2 we present the estimated values of $\eta$ (at the mean of the data) and the $A_k$’s. The $\chi^2$ (4) statistic for the inclusion of the demographics is 12.0 so that the sharing factors are jointly significant. We shall take this variant as our preferred specification.21

At our preferred estimates transport and services are estimated to be almost wholly public (the Barten scale is close to one half). We also find economies of

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21We also tried adding fixed costs $a$ to the consumption technology. Although these improved the fit somewhat they yielded extreme values for some of the equivalence scales considered later. Given the time taken to estimate any variant of the model we decided not to explore further models with fixed costs.
scale for food at home and food in restaurants. According to our previous discussion, these findings may also reflect the public nature of these commodities and/or the existence of positive externalities associated with their consumption. Since these four goods account for a high share of total expenditure, we shall expect to find fairly significant economies of scale. Recreation, clothing and particularly vices have Barten scales above unity. If assume these good are purely private, then these scales could be interpreted as taste changes, specifically, that married people ‘like’ these goods less than singles do. Alternative explanations are possible, for instance, negative externalities could explain the large coefficient obtained for vices. We return to this issue below.

6.5 Implications of parameter estimates

6.5.1 The fit of the model

We first discuss the fit of the preferred model 3. Our model imposes strong restrictions on the differences between budget shares of singles and couples, so we might expect to see large differences between the QUAIDS fits and our model fits. On the other hand, the over-identifying restrictions for our model are not rejected which suggests that the restrictions may not be significant. In figure 2 we present the Engel curve fits for our seven goods for the reference group used for figure 1. Comparing the fits for singles with those of figure 1 (the unconstrained QUAIDS fits) we see that the two sets of estimates are very similar. This is to be expected given that both use a QUAIDS specification. For couples, the Engel curves for ‘eating out’, ‘clothing’, ‘services’ and ‘recreation’ are also close to the QUAIDS fits. The ‘food at home’ Engel curve is somewhat different, but food is still found to be a necessity, though the Engel curve for couples is now between that of single females and single males. The Engel curves for ‘vices’ appears somewhat different (‘vices’ are now estimated to be a luxury for couples and the budget shares are very close to those for single women) but given the imprecision of the fit for this good, the differences are probably not statistically significant. ‘Transport’, however, is very different; we now find that it is unequivocally a necessity for couples and that the budget share is about half way between those of singles.

The QUAIDS for all strata and our model are not strictly nested, so we compute an informal quasi-likelihood test statistic for comparing the two.22 To do this we compare the over-identifying test statistic for the two sets of estimates, using

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22Formal tests for comparing nonnested models are impractical, and not likely to be very reliable, given the size and complexity of the models being compared.
### 6.5.2 The sharing rule

We now present some of the substantive implications of our estimates. We begin with the sharing rule. When considering the values presented here it is most important to keep in mind that, as discussed earlier, the sharing rule $\eta$ encompasses both the wife’s share of private consumption and the influence or power the wife has on the household’s demand for public or joint consumption.

Table 3 gives the values of the sharing rule for different sets of characteristics. The benchmark household has a home-owning husband and wife with the same gross income and age and with median total expenditure for Ontario in 1992.23 Table 3 presents the estimates of the level and variations in the sharing rule. At our benchmark value the wife’s share is 0.65. As we discussed, if this was purely

<table>
<thead>
<tr>
<th>Household characteristics</th>
<th>Wife’s share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>0.65</td>
</tr>
<tr>
<td>First quartile total expenditure</td>
<td>0.60</td>
</tr>
<tr>
<td>Third quartile total expenditure</td>
<td>0.68</td>
</tr>
<tr>
<td>Renters</td>
<td>0.77</td>
</tr>
<tr>
<td>Wife’s income share = 0.25</td>
<td>0.65</td>
</tr>
<tr>
<td>Wife’s income share = 0.75</td>
<td>0.66</td>
</tr>
<tr>
<td>Husband ten years older</td>
<td>0.65</td>
</tr>
<tr>
<td>Wife ten years older</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Table 3: Sharing rule implications

the same weighting matrix. The QUAIDS system (with symmetry imposed) has 411 parameters and our system has 270, so that we have 141 ‘restrictions’. If we use the weighting matrix from the our system then the over-identifying statistic for the QUAIDS system is 61.0 and for our system it is 224.1. The difference of 163.1, which has a probability value of 9.4%, suggesting that our system, although very restrictive, would not be rejected against the QUAIDS system. On the other hand, if we use the weighting matrix from the QUAIDS system then the over-identifying statistics for the QUAIDS system is 91.0 and for our system it is 475.2. This gives a difference of 414.2 which suggests that the QUAIDS system, with its 141 additional parameters, fits a good deal better.

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23In the estimation results presented above the values of the sharing parameter given (0.70) was at the mean of data for the whole sample.
for private goods then this would be implausibly high but it also reflects the effects of joint or public consumption. More generally, this number says that household demand functions look more like women’s demand functions than men’s demand functions.

From rows 2 and 3 we see that the wife’s share is strongly increasing in total expenditure so that in poorer households the wife’s share is much lower. This impact of total expenditures on the sharing rule means that household’s demands are not unitary, that is, they are not equal to demands that would arise from the maximization of a single, ordinary utility function.

Other rows of this table show that the home-ownership status variable has a very strong impact with renting households apparently having a much higher share for the wife. As we saw above, the pattern of home ownership between singles and couples is very different so this result is probably due at least in part to a selection effect rather than a genuine difference in intra-household allocations. We leave the important (but difficult) task of accounting adequately for selection effects to future work. Neither the wife’s share of income nor their relative ages has much impact on the sharing rule. This is very different from the results reported in Browning, Bourguignon, Chiappori and Lechene (1994), which examines conventional sharing rules for private goods using similar data on couples. Note that the latter paper did not include the home ownership dummy in the sharing rule and included real household income instead of total expenditures. Nonetheless the qualitative differences are striking and suggest that allowing for household technology may have a major impact on our conclusions regarding intra-household allocations.

6.5.3 Economies of scale

We now consider estimates of the private good equivalent consumption for married men and married women and the resulting economies of scale. Given estimates of the budget share systems for singles, $w_k^i = \omega_k^i \left( \frac{p}{y_i} \right)$, and for households (equation 21), the private good equivalent quantities for each household member are given by

$$x_k^f = \frac{\eta}{\pi_k} \omega_k^f \left( \frac{\pi}{\eta} \right)$$

and

$$x_k^m = \frac{1 - \eta}{\pi_k} \omega_k^m \left( \frac{\pi}{1 - \eta} \right),$$

37
the economies of scale to consumption are
\[ \frac{p'z}{p'x} = \frac{y}{p'(x_k^f + x_k^m)}, \tag{25} \]

In Table 4 we present results for four different cases. First we present results with the sharing rule in the household fixed at one half (but with the same parameter estimates for the Barten scales as for the estimated value of the sharing rule parameters) and then for the mean estimated benchmark value of 0.65 (models A and B and models C and D respectively). The other variation is in the value of the Barten scales. If these reflected only publicness of goods then they would be bounded between one half (purely public) and unity (purely private). In practice, as discussed in section 5 these values may also capture consumption externalities within the household and preference changes consequent on being married, and so may lie outside these bounds. For example, if ‘clothing’ is purely private but the members of a couple ‘like’ clothing less than when they were single, then we would estimate a Barten scale of greater than unity. This would lead to smaller apparent economies of scale. To partly purge the estimates of these effects, we also present results with the Barten scales capped at unity (models B and D). Referring to Table 2 above, we see that the goods which are capped are clothing, vices and recreation; our setting of the Barten scales to unity for these goods is equivalent to assuming that the three goods are purely private and that the estimates of greater than unity reflect preference changes.

In Table 4 the first set of estimates (labelled “Ebs, good”) give the equivalent budget shares for different goods for each member. At unit market prices these are defined as:

\[ \tilde{\omega}^f_{i} = \frac{\pi_i h^f_i (\pi / \eta)}{\eta} = \frac{A_i h^f_i (A_1 / \eta, ..., A_n / \eta)}{\eta} \]
\[ \tilde{\omega}^m_{i} = \frac{\pi_i h^m_i (\pi / (1 - \eta))}{(1 - \eta)} = \frac{A_i h^m_i (A_1 / (1 - \eta), ..., A_n / (1 - \eta))}{(1 - \eta)} \tag{26} \]

In practice, we use the QUAIDS estimates for men and women directly to calculate the individual budget shares, with Barten scales and shares replacing market prices and total expenditure. These estimates allow us to look at the ‘tastes’ of married men and women, allowing for economies of scale, sharing and preference changes. The second set of estimates (labelled “Ratio, good”) give the ratios of equivalent consumptions:

\[ \text{ratio for good } i = h^f_i (A_1 / \eta, ..., A_n / \eta) / h^f_i (A_1 / (1 - \eta), ..., A_n / (1 - \eta)) \tag{27} \]
These estimates present a picture of the relative contributions to the ‘purchase’ of equivalent consumption by each person.

The first column of Table 4 presents results with equal sharing and unmodified Barten scales. Thus men and women face the same shadow prices and have the same total expenditure. Women have higher budget shares for food at home, clothing and household services and lower budget shares for eating out and vices. Since we impose equal sharing these differences in budget shares translate directly into the ratios for equivalent consumption given in the lower half of the table. For example, wives purchase more than twice as much of the ‘services equivalent’ as do their husbands but less than one half of the ‘vices equivalent’.

One of the most important implications of the estimates is the scale economy parameter (for the benchmark values), see equation (25). This is estimated to be 0.79. That is, two singles would need $27\% \left(= 100 \times \left(1 - 0.79 \right) \right)$ more total expenditures than a couple if they are to consume the same equivalent private consumptions. Once again we warn that this value likely reflects both pure household technology economies of scale and externalities or taste changes, and so should be treated with caution.

The estimates for model A assume that all changes in demand patterns are due to economies of scale in consumption. As discussed above, with this rationalization values of the Barten scale should be between one half and unity. In model B we cap the values that are greater than unity to unity (but continue to maintain equal sharing). This lowers the shadow price of clothing, vices and recreation with respect to model A. Thus these changes induce both (compensated) price effects and an income effect from agents facing significantly lower absolute prices for three goods which have an aggregate budget share of about $30\%$. For women, the budget shares for clothing and recreation rise and for men they fall. The budget shares for vices does not change much. These own price responses reflect that the demands for clothing and recreation are (uncompensated) price inelastic for men but price elastic for women.\footnote{The budget shares increases with an own price increase if and only if the own price elasticity is greater than minus unity.} There are also important (uncompensated) cross-price effects. In particular, the budget shares for household services change quite radically and the ratio of equivalent consumptions for services is much smaller. As we would expect, the extent of scale economies is now greater (since couples face lower shadow prices for some goods) and two singles would now require $41\%$ more total expenditure to buy the same equivalent consumption levels.

In models A and B we imposed equal sharing. The next two models assume...
that the wife receives the benchmark value of 65% of household total expenditure to buy goods at the shadow prices given by the Barten scales. Comparing the predictions for models A and C we see that, as we would expect the women’s budget shares rise for necessities and fall for luxuries (and conversely for men) but the changes are relatively small. The ratios of expenditures are now all above unity which simply reflects the fact that the wife now has an equivalent total expenditure which is almost twice that of her husband’s. More importantly, the effects largely cancel out in the estimate of the scale economy parameter and we have virtually the same estimated value in both models. We conclude that the extent of sharing within the household does not significantly affect the (technological) economies of scale. The final model (D) in which we allow for unequal sharing and we cap the Barten scales gives some differences in details but the same broad conclusions.

As well as providing estimates of the extent of economies of scale, these es-
timates also provide an additional check on our identifying assumptions. In our modelling we have not made a distinction between men and women’s clothing for couples. If we assume that clothing is a private good and that wives only consume women’s clothing and husbands only consume men’s clothing then we can compare our estimate of the ratio of equivalent consumption of clothing with the actual levels of men and women’s clothing. For the 1992 data we have a mean ratio of 1.40.\textsuperscript{25} This is to be compared to the model C estimate of 2.15. The latter value is largely due to the very high sharing parameter we have estimated. As can be seen, if we have equal sharing then the ratio is 1.08. If we specify that the ratio should be the same as for men and women’s clothing then we obtain a sharing parameter of 0.55. This suggests that we could use the information on men and women’s clothing and our auxiliary assumptions to achieve more precise estimates. We shall return to this point later.

6.5.4 Welfare comparisons

The results presented in the last subsection relate to the technology and sharing within the household. As we emphasised earlier, the finding (for models B and D) that two singles require about 40% more total expenditure than a couple to buy the same equivalent consumption levels represents an upper bound on the amount needed to be as well off as they would be if living together. In practice, singles buy different goods when living alone and can achieve the same level of utility as when married at lower cost. In this subsection we present estimates of these adult equivalence scales.

Let $V^i \left( \frac{p}{y^i} \right)$ be the indirect utility function (up to any arbitrary monotonic transformation) that gives rise to member $i$’s budget share demand functions $\omega_k = \omega_k \left( \frac{p}{y^i} \right)$. Then the minimum expenditure required by member $i$, while living alone, to attain the same indifference curve over goods that he or she attains as a member of the household is the number $y^{i*}$ that solves the equation

$$V^f \left( \frac{p}{y^{f*}} \right) = V^f \left( \frac{\pi}{\eta} \right)$$

\textsuperscript{25}This estimate is based on estimated quadratic log equations for budget shares for men and women’s clothing. We use the 1992 data and control for the same demographics as in our specification. Given estimates of the budget shares at the mean of the data for Ontario in 1992, we take ratios of the predictions.
for $i = f$, and

$$V^m \left( \frac{P}{y_{m*}} \right) = V^m \left( \frac{\pi}{1 - \eta} \right)$$

(29)

for $i = m$. We may then calculate the male adult equivalence scale for the household as $y/y_{m*}$ and for a female as $y_{f*}/y_{m*}$. Alternatively, to separate issues of bargaining power from other considerations, we may evaluate these equations and the resulting equivalence scales substituting $\eta = .5$ into equations (28) and (29).

In Table 5 we present the adult equivalence scales and also the (technological) scale economy parameter, for comparison purposes. If we impose equal sharing and do not cap Barten scales then we estimate that single men and women need about 57% of the total expenditure of a couple to be as well off. This is somewhat below the scale economy estimate of 63.5% (half of 127% from above) for model A. Thus the ability to change expenditure patterns when single leads to a significant decrease in the needs of singles. If we exclude the (presumed preference) changes due to Barten scales being above unity, then the individual scales rise to about 68%, still assuming equal sharing.

If instead of equal sharing we take the benchmark share of 0.65, then we find that men are better off when single even with less than half of household income. This is the result of two effects: we estimate that married women receive more private goods and they have more influence on the demand for shared goods.

If we take a more moderate share of 0.55 for the wife, as implied by the discussion at the end of the last subsection, then the estimates still differ radically from the case of equal sharing. For the unmodified Barten scales single men now need only half of the total expenditure of a couple to be as well off. These estimates would suggest that there are no material gains to marriage for men (in terms of gains from economies of scale in consumption of nondurables). Women then need 62% of a couple's total expenditure to be as well off single as married. The mean for men and women in models $A$ and $C$ are very similar, suggesting that the 'aggregate' scale is largely invariant to the sharing parameter. Finally, if we cap the Barten scales then women and men need 74% and 63% respectively. As we have emphasised above, there is no single 'correct' adult equivalence scale and the value one would use depends on the context. Our estimates suggest the range of possible equivalence scale values are quite wide, with values of between 58% and 74% for women and between 50% and 68% for men.
<table>
<thead>
<tr>
<th>Model</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wife’s share</td>
<td>0.5</td>
<td>0.5</td>
<td>0.55</td>
<td>0.55</td>
</tr>
<tr>
<td>$A_k \leq 1, \forall k$</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Scale economy</td>
<td>0.79</td>
<td>0.71</td>
<td>0.79</td>
<td>0.72</td>
</tr>
<tr>
<td>AE scale, women</td>
<td>57.9</td>
<td>67.7</td>
<td>62.5</td>
<td>73.9</td>
</tr>
<tr>
<td>AE scale, men</td>
<td>55.8</td>
<td>69.5</td>
<td>50.2</td>
<td>62.6</td>
</tr>
</tbody>
</table>

Table 5: Adult equivalence scales

7 Conclusions

We model households in terms of the utility functions of its members, a bargaining or social welfare function, and a consumption technology function. By employing a collective model and a separate consumption technology, we combine data from singles and couples to identify equivalence scales, consumption economies of scale, household members’ bargaining power and other related concepts, without assumptions regarding cardinalizations or interpersonal comparability of preferences. We also provide duality results that facilitate the empirical application of the model. These include the use of an income sharing rule to model the outcome of household bargaining or social welfare calculations, and shadow prices that embody economies of scale and scope arising from joint consumption. Our empirical application of the model uses data from the Canadian FAMEX. We demonstrate generic nonparametric identification of our model.

Given our framework, one useful area for further work would be the development of alternative identifying assumptions. Without preference changes or externalities within the household, our model is very much over identified, in fact, in our QUAILS specification most of the parameters of the entire consumption technology can be identified from the couple’s demands for each good. This over-identification might be exploited by relaxing the model in ways that incorporate and identify possible externalities and changes in preferences between individuals in households and those living alone. Direct observation of some elements of the equivalent consumption vectors $x, x^f, x^m$ would greatly aid identification, by distinguishing components of $F$ that are due to changes in preferences versus pure economies of scale or scope in consumption. Such data might include direct measures of the quantity of food wasted (discarded) by singles and couples, or measures of the fraction of time couples drive together versus alone, or simply treating mens and womens clothing as separate, private goods.
Alternative identification conditions that minimize the data requirements on singles would be valuable not only to relax assumptions regarding changes in preferences, but also to extend the model to directly include the utility functions of children, which for now must be assumed to be joint with the utility of an adult household member.

Other useful directions for future research would be the development of empirically tractable duality and identification results (akin to our Propositions 1 to 4) for more general, nonlinear consumption technology functions as discussed in section 5.

While our particular identifying assumptions should be open to debate and improvement, we believe that the model we have provided is an appropriate framework for estimating and analyzing adult equivalence scales, consumption economies of scale, household members’ bargaining power, and other concepts relating to household preferences, consumption, and demand behavior.

Appendix

Here we provide proofs and some additional related results.

Proof of Proposition 1

Consider the household as a two person economy facing the technological constraint \( p F(x^f + x^m) = y \). From the welfare theorems, any equilibrium is Pareto efficient, and any Pareto efficient allocation can be decentralized as an equilibrium for some lump-sum transfers. In particular, consider the Pareto efficient allocation characterized by program (P*) and let \((x^m, x^f)\) be the corresponding individual demands. The decentralization is that the household can satisfy its objective function, Program P*, by telling each member \(i\) to choose a utility maximizing private consumption vector \(x^i\) (program \(P^i\)), but subject to a budget constraint \(\pi^i x^i = \eta^i\). For some shadow price vector \(\pi = \pi (p/y)\) and shadow incomes \(\eta^i = \eta^f (p/y)\) (having \(F\) depend on \(x^f\) and \(x^m\) only through their sum results in shadow prices \(\pi\) being the same for both members). The second welfare theorem implies that agent \(i\), facing prices \(\pi\) and income \(\eta^f\), will choose the bundle \(x^i = h^i (\pi/\eta^i)\), and the household then actually purchases the vector \(z = F(x)\) where \(x = x^f + x^m\).

By homogeneity, we may without loss of generality normalize the price vector \(\pi\) such that the total shadow income \(\pi' x = 1\), and define the sharing rule \(\eta(p/y)\) by \(\eta = \eta^f = \pi' x^f\), and \(\eta^m = 1 - \eta\). The sharing rule \(\eta\) is the fraction of
the household’s shadow income that is allocated to member $f$, and therefore is a measure of her weight or bargaining power in the household’s program. She then uses this income to ‘buy’ her vector of private good equivalents $x^f$, paying shadow prices $\pi$. Similarly, member $m$ allocates income $1 - \eta = \eta^m$ to obtain private good equivalents $x^m$, paying shadow prices $\pi$.

**Proof of Proposition 2**

If the technology is linear, so $z = F(x) = Ax + a$, then Program $P^*$ has the constraint, $(Ax + a)' p = y$, which is equivalent to $x' \pi = 1$ with $\pi(p/y) = A' p / (y - a' p)$. One may then immediately verify that Program $P^*$ is equivalent to the decentralization described in the proof of Proposition 1, using this expression for $\pi(p/y)$. Note that, in general, shadow prices $\pi(p/y)$ will depend on both the technology $F$ and preferences and bargaining behavior of the household. However, in this household economy, when the technology is linear the ‘production’ side alone determines the equilibrium shadow prices.

**Proof of Proposition 3**

A first order condition of the household’s program is

$$\frac{\partial \widetilde{U}}{\partial x^i} \frac{\partial U^i}{\partial x^i} = \lambda_0 \pi_k$$

for each member $i$ and good $k$, where $\lambda_0$ is a Lagrange multiplier. It follows that

$$\frac{\partial \widetilde{U}}{\partial \eta^i} \frac{\partial V^i}{\partial \eta^i} = \frac{\partial \widetilde{U}}{\partial x^i} \frac{\partial U^i}{\partial x^i} \frac{\partial U^i}{\partial \eta^i} = \lambda_0$$

so

$$\frac{\partial \widetilde{U}}{\partial \eta^f} \frac{\partial V^f}{\partial \eta^f} = \frac{\partial \widetilde{U}}{\partial \eta^m} \frac{\partial V^m}{\partial \eta^m}$$

It can then be directly verified that all of the first order conditions from the program $P^*$ are unchanged when $\widetilde{U}$ is replaced with $\mu(p/y)U^f(x^f) + U^m(x^m)$, where

$$\mu = \frac{\partial \widetilde{U}}{\partial \eta^f} \frac{\partial V^f}{\partial \eta^f} = -\frac{\partial \widetilde{U}}{\partial \eta^m} \frac{\partial V^m}{\partial \eta^m}$$

This establishes the relationship between $\mu(p/y)$ and $\eta(p/y)$. Properties of this relationship follows from standard properties of indirect utility functions with respect to income.
Proof of Proposition 4

The AIDS quantity demand for good \( j \) consumed by single \( i \) is

\[
h^i_j (p/y^i) = \frac{y^i_j}{p^i_j} \left( \alpha^i_j + \Gamma^i_j \ln p + \beta^i_j \left[ \ln (y^i_j) - c^i_j (p) \right] \right)
\]

where

\[
c^i_j (p) = (\ln p)' \alpha^i_j + \frac{1}{2} (\ln p)' \Gamma^i_j \ln p
\]

\[
\ln \left[ b^i_j (p) \right] = (\ln p)' \beta^i_j
\]

\( \alpha^i_j \) and \( \beta^i_j \) are \( n \)-vectors of parameters, \( \Gamma^i_j \) is an \( n \times n \) matrix of parameters, \( e' \alpha^i_j = 1 \), \( e' \beta^i_j = \Gamma^i_j e = 0 \) where \( e \) is an \( n \)-vector of ones, and \( \Gamma^i_j \) is symmetric.

Linear technology implies that the shadow price for good \( j \) is

\[
\pi^i_j = \frac{A^i_j}{y - a' \ p} = \frac{\sum_k A_{kj} p_k}{y - a' \ p}
\]

where \( A^i_j \) is the transpose of the \( j \)’th column of \( A \) and \( A_{kj} \) is the \( k \)’th element of \( A^i_j \).

With AIDS singles and linear technology, the couple’s quantity demand for good \( k \) is

\[
h^k_k (p/y) = \sum_j A_{kj} \left[ \frac{\eta}{\pi^i_j} \left( \alpha^f_j + \Gamma^f_j \ln \pi + \beta^f_j \left[ \ln (\eta) - c^f_j (\pi) \right] \right) + \frac{1 - \eta}{\pi^i_j} \left( \alpha^m_j + \Gamma^m_j \ln \pi + \beta^m_j \left[ \ln (1 - \eta) - c^m_j (\pi) \right] \right) \right] + a_k
\]

\[
h^k_k (p/y) = \left( y - a' \ p \right) \sum_j A_{kj} \left[ \eta \left( \frac{1}{A^i_j p} \right) \left( \alpha^f_j + \Gamma^f_j \ln \left( \frac{A^i_j p}{y - a' \ p} \right) + \beta^f_j \left[ \ln (\eta) - c^f_j \left( \frac{A^i_j p}{y - a' \ p} \right) \right] \right) + (1 - \eta) \left( \frac{1}{A^i_j p} \right) \left( \alpha^m_j + \Gamma^m_j \ln \left( \frac{A^i_j p}{y - a' \ p} \right) + \beta^m_j \left[ \ln (1 - \eta) - c^m_j \left( \frac{A^i_j p}{y - a' \ p} \right) \right] \right) \right] + a_k
\]

Given \( h^k_k (p/y) \) for each good \( k \), the constants \( a_k \) are identified as the intercept terms, and we may identify the summation by \( [h^k_k (p/y) - a_k] / (y - a' \ p) \). Now use

\[
c^i_j (\pi) = c^i_j \left( \frac{A^i_j p}{y - a' \ p} \right) = (\ln A^i_j p)' \alpha^i_j - (\ln y - a' \ p) + \frac{1}{2} (\ln (A^i_j p))' \Gamma^i_j \ln (A^i_j p)
\]

to write this summation term as
For each good $k$, the coefficient of $\ln y$ in this expression is

$$\sum_j \frac{A_{jk}}{A_{jk}p} \left[ \frac{\eta}{(\ln\eta) - ((\ln A_p)\alpha_j + \ln(A_p))} \left( \beta_j^f \left[ (\ln (\eta) - ((\ln A_p)\alpha_j + \ln(A_p)) + \frac{1}{2}(\ln (A_p))\Gamma_j^f \ln (A_p)) \right) \right) + (1 - \eta) \left( \beta_j^m \left[ (\ln (1 - \eta) - ((\ln A_p)\alpha_j + \ln(A_p)) + \frac{1}{2}(\ln (A_p))\Gamma_j^m \ln (A_p)) \right) \right) \right]$$

where the summation is now over all goods $j$ for which $A_{kj}$ is not equal to zero. Variation in $y$ therefore provides identification of the above expression for each good $k$. Variation of $p$ in the above expression for each $k$ identifies $A_{kj}$.

For triplets of goods $j, k, \ell$ having $A_{kj} \neq 0$ and $A_{kj} \neq 0$. The above expression with $\ell = k$ identifies $\eta$, since $\beta_j^m$ and $\alpha_j^f$ are identified from singles, and therefore $A_{kj}$ is also identified. Define $\bar{A}_{kj} = A_{kj}/A_{jj}$ and $d_j = A_{jj}$. Identification of $A_{kj}$ implies identification of $\bar{A}_{kj}$ for all $j, \ell$. What remains is to identify each $d_j$.

The term $A_{kj}/A_{j}p = 1/\sum_\ell (\bar{A}_{kj}/A_{jj})p_\ell$ is identified for each $j$, which implies based on the large summation term above that

$$\sum_j \frac{\eta\beta_j^f + (1 - \eta)\beta_j^m}{\bar{A}_{kj}/A_{jj}}$$

is identified for each good $j$. The component of this expression that is quadratic in $\ln (A_p)$ is

$$\ln (A_p)\left( -\eta\beta_j^f \frac{1}{2}\Gamma_j^f - (1 - \eta)\beta_j^m \frac{1}{2}\Gamma_j^m \right) \ln (A_p)$$

$$= \sum_k \sum_\ell \ln \left( \sum_j d_j \bar{A}_{kj}p_j \right) \ln \left( \sum_j d_j \bar{A}_{kj}p_j \right) \phi_{k\ell}$$
where \( \phi_{k\ell} \) and \( A_{kj} \) are already identified for all \( j, k, \ell \). Variation in \( p \) in this expression then identifies \( d_{j} \), which completes the identification.

Note that the technology and sharing rule are substantially overidentified. Given the demands of singles, the couple’s demands for each good \( k \) separately identify \( \eta, a \) (from the \( y - a'p \) term) and \( A_{\ell j}/A_{kj} \) for all \( j, k, \ell \) having \( A_{kj} \neq 0 \). Therefore, observing couple’s demands for all goods, not just one, greatly overidentifies these terms. In particular, if the \( k'th \) row of \( A \) has all nonzero elements, then (given the demands of singles) the technology and the sharing rule can be completely identified just by observing couple’s demands for the \( k'th \) good. In addition to the overidentification resulting from observing the couple’s demands for multiple goods, only the terms involving the intercept, the coefficient of \( \ln y \), and the quadratic in \( \ln (Ap) \) were used for identification, so all the other terms in the couple’s demands, such as the linear in \( \ln (Ap) \) term, provide additional overidentifying restrictions. These various overidentifying assumptions may be used to relax the assumptions regarding unequal and nonzero parameters in the statement of Theorem 3.

Similarly, the above proof applies when single’s have QUAIDS demands, since in that case the only difference in the model is the addition of more terms (in particular, a quadratic in \( \ln y \)) which provide more overidentifying restrictions.

**Nonparametric Identification**

We now show that, given only the observable Marshallian demand functions of singles and households, we can in general recover the private good equivalent functions \( x^m \) and \( x^f \), the consumption technology \( F \), and the sharing rule \( \eta \), without parameterizing any of these functions. As a result, applications that are defined in terms of these functions, such as economies of scale to consumption and adult equivalence scales, are also nonparametrically identified.

Given observed demand functions, if the technology function \( F \) and sharing rule \( \eta \) are identified, then it immediately follows that all of the features of the model are identified. The goal is therefore to establish nonparametric identification of \( F \) and \( \eta \). Let \( \rho = p/y \) and let \( \Omega_F \) be the space of increasing \( C^2 \) functions with range and domain equal to the positive orthant, and let \( \Omega_\eta \) be the space of \( C^2 \) functions with domain equal to the positive orthant and range \((0, 1)\). Define a mapping \( T \) by the following procedure. Take some element \((F, \eta) \in \Omega_F \times \Omega_\eta\), and treat \( F, \eta \) as if they were the true technology function and sharing rule by first defining \( x(\rho) = F^{-1}[h(\rho)] \) and \( \tilde{\pi}(\rho) = DF(x') \cdot \rho \) evaluated at \( x = x(\rho) \). Next let \( \pi(\rho) = \tilde{\pi}(\rho)/[\tilde{\pi}(\rho)x(\rho)] \) and \( x^m(\rho) = h^m[\pi(\rho)/(1 - \tilde{\eta}(\rho))] \) (as an aside, note these equations demonstrate that if the true \( F, \eta \) is known then the other fea-
tures of the model such as shadow prices and private good equivalents are also identified) Now define the function $(\tilde{F}, \tilde{\eta}) = T (\bar{F}, \bar{\eta})$ by

$$\tilde{F} [x (\rho)] = h (\rho)$$

$$\tilde{\eta} (\rho) = C_f [U_f [x (\rho) - x^m (\rho)], \pi (\rho)]$$

where $C_f (U_f, \pi)$ is the cost function that corresponds to the utility function $U_f$. This construction of $\tilde{\eta} (\rho)$ does not depend on the choice of cardinalization for $U_f$, as can be seen by equivalently expressing $\tilde{\eta} (\rho)$ in terms of Marshallian demand functions as the solution to $x (\rho) - x^m (\rho) = h [\pi (\rho) / \tilde{\eta} (\rho)]$. The true technology function and sharing rule pair $F, \eta$ are then a fixed point of the mapping $(\tilde{F}, \tilde{\eta}) = T (\bar{F}, \bar{\eta})$. In general, $T$ might not be a contraction mapping, and so may have several fixed points. However, for suitably regular demand functions (those for which the tangent application to $T$ is Fredholm) each fixed point will be locally unique as a consequence of Smale’s generalized transversality theorem. This provides local generic identification. Regularity conditions such as monotonicity and domain constraints further restrict the set of feasible functions solutions $F, \eta$. Global identification results if only one fixed point is present or if, among all fixed points, only one satisfies the required regularity conditions.

The above paragraph describes formally what we mean by ’generic’ identification. A similar case of generic identification appears in Chiappori and Ekeland (1999). Informally, the existence of the mapping $T$ shows that a sufficient number of demand functions are identified to generally permit recovery of $F$ and $\eta$, so these functions will be identified as long as the demand functions are not too simple. An illustration of this point are the results provided earlier, which show that a linear technology is not identified if the member’s demand equations are also linear (specifically, the linear expenditure system), but are identified for nonlinear member demands like the Almost Ideal or QUAIDS.

References


Figure 1: QAIDS fits for singles and couples.
Figure 2: Model fits for singles and couples.