Constraining Self-Interacting Dark Matter with the Milky Way’s dwarf spheroidals

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ABSTRACT
Self-Interacting Dark Matter is an attractive alternative to the Cold Dark Matter paradigm only if it is able to substantially reduce the central densities of dwarf-size haloes while keeping the densities and shapes of cluster-size haloes within current constraints. Given the seemingly stringent nature of the latter, it was thought for nearly a decade that Self-Interacting Dark Matter would be viable only if the cross section for self-scattering was strongly velocity-dependent. However, it has recently been suggested that a constant cross section per unit mass of $\sigma_T/m \sim 0.1 \text{ cm}^2 \text{g}^{-1}$ is sufficient to accomplish the desired effect. We explicitly investigate this claim using high resolution cosmological simulations of a Milky-Way size halo and find that, similarly to the Cold Dark Matter case, such cross section produces a population of massive subhaloes that is inconsistent with the kinematics of the classical dwarf spheroidals, in particular with the inferred slopes of the mass profiles of Fornax and Sculptor. This problem is resolved if $\sigma_T/m \sim 1 \text{ cm}^2 \text{g}^{-1}$ at the dwarf spheroidal scales. Since this value is likely inconsistent with the halo shapes of several clusters, our results leave only a small window open for a velocity-independent Self-Interacting Dark Matter model to work as a distinct alternative to Cold Dark Matter.

Key words: cosmology: dark matter – galaxies: halos – methods: numerical

1 INTRODUCTION
It is now clear that observations of dark matter dominated systems such as low-mass (dwarf) and low surface brightness (LSB) galaxies favour the presence of dark matter cores of $O(1 \text{ kpc})$ (e.g. Moore 1994; Kuzio de Naray et al. 2008; de Blok 2010; Walker & Penarrubia 2011; Amorisco & Evans 2012). These observations are a challenge for the Cold Dark Matter (CDM) paradigm where dark matter haloes are predicted to have density cusps, an imprint of the collisionless nature of CDM (the core-cusp problem). In a possibly related issue, it has been pointed out recently that the dark satellites of Milky-Way (MW) size halo simulations are too dense to be consistent with the kinematics of the MW dwarf spheroidals (dSphs) (the too big to fail problem; Boylan-Kolchin et al. 2011; 2012).

Although these are significant challenges to the CDM model, their solution could naturally lie in our incomplete understanding of the complex process of galaxy formation. Even though the internal dynamics of dwarfs is dominated by dark matter today, it is conceivable that earlier episodes of star formation and subsequent gas removal by supernova feedback might have been violent enough to modify the initial cuspy dark matter distribution into a cored one (e.g. Navarro et al. 1996; Pontzen & Governato 2012). Current hydrodynamical simulations have shown that such a mechanism is able to create large cores in intermediate-mass galaxies (Governato et al. 2010; 2012), and, once tidal stripping is taken into account, it might solve the too big to fail problem as well (Boylan-Kolchin 2012). It is not clear, however, if such episodes of large gas blow-outs are consistent with the star formation histories and stellar properties of LSBs and dwarfs (e.g. Kuzio de Naray & Spekkens 2011; Boylan-Kolchin et al. 2012; Penarrubia et al. 2012).

Given the large uncertainties regarding whether baryon physics can reconcile the CDM model with observations of dwarf galaxies, it is prudent to consider the alternative, which is to question the fundamental CDM hypotheses, namely, the collisionless and cold nature of CDM particles. This alternative is additionally encouraged by the null detection of several experiments that are pursuing the discovery of the favoured CDM particles, and whose sensitivity is reaching the natural values for the interaction cross sections of the particle physics models that predict them (e.g. Supersymmetry, Abazajian et al. 2012; Aprile et al. 2012).

An exciting possibility is that of self-interacting dark matter (SIDM) originally introduced over a decade ago by Spergel & Steinhardt (2000). Self-scattering between dark matter particles is...
a feature of present hidden-sector dark matter models that predict the existence of new gauge bosons. The presence of these bosons is invoked to enhance the annihilation and/or self-scattering of dark matter particles to explain a number of puzzling observations (the Sommerfeld enhancement, e.g. Arkani-Hamed et al. 2009; Buckley & Fox 2010). Collisonal dark matter is constrained by the requirements from different astrophysical observations, such as the ellipsoidal shape of haloes, the avoidance of subhalo evaporation in galaxy clusters, and the avoidance of the gravothermal catastrophe (e.g. Miralda-Escudé 2002; Gnedin & Ostriker 2001; Firmani et al. 2001). The original excitement caused by SIDM died off by the apparently strong constraints on the scattering cross section set particularly by X-ray and lensing observations of clusters in the analysis by Miralda-Escudé (2002): \( \sigma_T / m \leq 0.02 \text{ cm}^2 \text{ g}^{-1} \); such a low cross section would have no relevant impact for the dynamics of galaxies at the \( O(1 \text{ kpc}) \) scale. Peter et al. (2012) have revised this constraint and found that it was overestimated by over an order of magnitude suggesting that a current constraint is of \( O(0.1 \text{ cm}^2 \text{ g}^{-1}) \).

SIDM is clearly a viable model if the cross section depends on the relative velocity in such a way that dark matter behaves as a collisional fluid in dwarfs, and is essentially collisionless at the scale of clusters. Although this idea was phenomenologically proposed (Yoshida et al. 2000) and explored with cosmological simulations (Colín et al. 2002) a decade ago, its theoretical support has come up only recently (e.g. Ackerman et al. 2009; Feng et al. 2009; Buckley & Fox 2010; Loeb & Weiner 2011; van den Aarssen et al. 2012; Tulin et al. 2012). Moreover, earlier simulations lacked the resolution needed to reliably explore the sub-kpc region of dwarf-size haloes. It was only until recently that it was explicitly shown that theoretically motivated velocity-dependent SIDM (vd-SIDM) models produce core sizes consistent with those found in MW dSphs, and also solve the emergent too big to fail problem (Vogelsberger et al. 2012 hereafter VZL).

On the other hand, Rocha et al. (2012) have suggested that a velocity-dependent cross section is not essential since a SIDM model with a constant \( \sigma_T / m = 0.1 \text{ cm}^2 \text{ g}^{-1} \) (allowed by cluster constraints) is also consistent with the inner structure of dSphs. This seems to contradict earlier estimates made by Yoshida et al. (2000) who suggested that the average number of collisions per particle in the central core, \( n_{\text{core}} \), scales as the cube root of the halo mass. Since for this cross section, \( n_{\text{core}} \sim 2 \) for a cluster-size halo (within \( \sim 10\% \) of the virial radius, see their Fig. 2), \( n_{\text{core}} \) would thus be suppressed by a factor of \( O(100) \) in dwarf-size haloes, resulting in cores that are too small. This scaling is however imprecise since for a halo of virial mass \( M \):

\[
n_{\text{core}} \sim \langle \rho_{\text{core}} \rangle \left( \frac{\sigma_{\text{vel}}}{m} \right) t_{\text{age}} \propto \Delta_{\text{core}} \left( \frac{\sigma_{\text{vol}}}{m} \right) M^{1/3} t_{\text{age}},
\]

where \( \langle \rho_{\text{core}} \rangle \) and \( \langle \sigma_{\text{vol}} \rangle \) are the average density and velocity dispersion of dark matter particles, \( t_{\text{age}} \) is the formation time, and \( \Delta_{\text{core}} \) is proportional to the density contrast relative to the background density; all of these quantities are defined within the core of the halo. Thus, the higher concentrations and larger formation times of dwarf-size haloes reduce the \( M^{1/3} \) dependence. It is important to remark that both, Yoshida et al. (2000) and Rocha et al. (2012), extrapolated the regimes they could directly simulate to the regime of dwarfs. In this Letter, we resolve this issue by proving explicitly that a constant scattering cross-section of \( \sigma_T / m = 0.1 \text{ cm}^2 \text{ g}^{-1} \) is not able to create \( O(1 \text{ kpc}) \) cores in the dark subhaloes where the MW dSphs are expected to live; it deviates only slightly from the CDM predictions. Unless baryonic processes are invoked, the range of interesting constant \( \sigma_T / m \) values is thus very narrow: \( 0.1 \text{ cm}^2 \text{ g}^{-1} < \sigma_T / m < 1 \text{ cm}^2 \text{ g}^{-1} \).

\[ \text{Figure 1. Dependence of the momentum-transfer weighted cross-section per unit mass on the relative velocity for the different SIDM models considered here. The constant cross section cases with } \sigma_T / m \geq 1 \text{ cm}^2 \text{ g}^{-1} \text{ are likely ruled out by halo shapes based on X-ray and lensing observations of clusters (Peter et al. 2012). The models with a velocity-dependent cross-section are tuned to satisfy all current astrophysical constraints and have been shown to be consistent with the kinematics of MW dSphs (VZL).} \]

\[ 2 \text{ SIMULATIONS AND RESULTS} \]

Our analysis is based on re-simulations of the Aq-A halo (level 3 resolution) of the Aquarius project (Springel et al. 2008), which is a set of representative MW-like haloes within the CDM WMAP-1yr cosmology. This halo has a virial mass of \( M_{200} \sim 1 \times 10^{12} \, M_\odot \) within a radius of \( r_{200} \sim 246 \text{ kpc} \) (enclosing an average density of 200 times the critical density). The particle mass in the simulations is \( m_p \sim 4.9 \times 10^4 \, M_\odot \) and the Plummer equivalent gravitational softening length is \( \epsilon \sim 120 \, \text{pc} \). We use an algorithm that adds dark matter self-scattering to the \( N \)-body code GADGET-3 for gravitational interactions (last described in Springel 2005). The algorithm uses a N-body/Monte Carlo approach to represent the microphysical scattering process in the macroscopic context of the simulation. The connection between this type of approach and the Boltzmann equation is nicely described in Appendix A of Rocha et al. (2012). The details of this algorithm can be found in VZL, as well as simple controlled tests that show the agreement between the outcome of the code and analytical expectations. All dark matter models were simulated starting with the same initial conditions and their present-day self-bound subhalo population was identified using the SUBFIND algorithm (Springel et al. 2001).

In addition to CDM, we consider five SIDM cases (recently presented in Vogelsberger & Zavala 2012) to analyse the impact of self-scattering in direct detection experiments: three with a constant cross section and two with a velocity-dependent one given by a Yukawa-like interaction (e.g. Loeb & Weiner 2011). The transfer cross section scaling with the relative velocity can be seen in Fig. 2.
We note that the formula for $\sigma_T/m$ for the velocity-dependent cases is only valid in the classical regime, once quantum effects are important, the finite interaction length of the Yukawa potential cuts off the zero-velocity divergence of the cross section (see Wolf et al. 2010). Clearly, the most massive CDM subhaloes are inconsistent with the kinematics of the MW dSphs. SIDM can alleviate this problem only for a constant scattering cross-section $\sigma_T/m \gtrsim 1 \text{ cm}^2 \text{g}^{-1}$ (SIDM10 and SIDM1) or if it has a velocity dependence (vdSIDMa and vdSIDMb). Current constraints from clusters put an upper limit to the constant cross section case close to $\sigma_T/m \sim 0.1 \text{ cm}^2 \text{g}^{-1}$ (SIDM0.1). This value is too low to solve the too big to fail problem. The observational data in the bottom right can be fitted by lower mass subhaloes, not shown here since they are affected by the limited resolution of our simulations.

The circular velocity profiles at $z = 0$ encompassing the 1st and 3rd quartiles of the distribution of the 15 subhaloes with the largest values of $V_{\text{max}}(z = 0)$. The symbols with error bars are estimates of the circular velocity within the half-light radii for 9 MW dSphs (Walker et al. 2009, Wolf et al. 2010). Clearly, the most massive CDM subhaloes are inconsistent with the kinematics of the MW dSphs. SIDM can alleviate this problem only for a constant scattering cross-section $\sigma_T/m \gtrsim 1 \text{ cm}^2 \text{g}^{-1}$ (SIDM10 and SIDM1) or if it has a velocity dependence (vdSIDMa and vdSIDMb). Current constraints from clusters put an upper limit to the constant cross section case close to $\sigma_T/m \sim 0.1 \text{ cm}^2 \text{g}^{-1}$ (SIDM0.1). This value is too low to solve the too big to fail problem. We note that the extent of the too big to fail problem in CDM depends on the mass of the MW halo, if it is in the low end of current estimates, $\lesssim 10^{12} \text{ M}_\odot$, the problem may be resolved (e.g. Wang et al. 2012), although a low halo mass may generate other difficulties such as explaining the presence of the Magellanic Clouds. In the context of SIDM, the lower the mass of the MW halo, the weaker the argument against $\sigma_T/m = 0.1 \text{ cm}^2 \text{g}^{-1}$.

A simple statistical test of the agreement between the subhalo distributions of two models and the 9 dSphs is to compute the chi-square difference associated to the likelihood of having $n$ (in $n$) data points above (below) the median of the distribution of each model. Assuming that the probability distribution of finding $n$ data points is Poissonian:

$$\Delta \chi^2 = 2 \left( \ln(n^+! / n^-!) - \ln(n^+! / n^-!) \right).$$

Comparing SIDM1 and the vdSIDM models with SIDM0.1, the difference is driven solely by Draco with the former preferred over the latter with $\Delta \chi^2 \sim 4.2 (1.9)$. Using an interpolation of our three constant cross section cases, we estimate that $\sigma_T/m \sim 0.6 \text{ cm}^2 \text{g}^{-1}$ is the minimum value for which $\Delta \chi^2 = 0$ relative to SIDM0.1.

To show the typical core size and central densities that are predicted by the different SIDM models, we plot in Fig. 3 the density profile of the 15 subhaloes with the largest $V_{\text{max}}(z = 0)$ values. A value of $\sigma_T/m \sim 0.1 \text{ cm}^2 \text{g}^{-1}$ is needed for a constant cross section SIDM model to mimic the effect of the vdSIDM models and produce $\sim 1 \text{ kpc}$ cores with central densities of $\sim 10^4 \text{ M}_\odot \text{ pc}^{-3}$. If the transfer cross section is reduced to $0.1 \text{ cm}^2 \text{g}^{-1}$, then the subhaloes are only slightly less dense than in CDM, having cores (central densities) that are at least twice smaller (higher) than those in the other SIDM cases.

VZL showed that the SIDM10 and vdSIDM models have convergent density and circular velocity profiles within the central density core; we have found the same for SIDM1 and to lesser extent for SIDM0.1. Convergence is harder to achieve for CDM since, at a fixed radius, the two-body relaxation time is shorter than for SIDM (due to the reduced densities in the latter case). Power et al. (2003) showed that the density profile converges at a given radius when the two-body relaxation time is larger than the Hubble time at this radius. At the resolution level of our simulations, the convergence radius for CDM is $\sim 600 \text{ pc}$, which implies that the CDM circular velocity and density profiles shown in Figs. 2 and 4 underestimate the true dark matter content within $\sim 600 \text{ pc}$ (Springel et al. 2008), whereas for SIDM is at least half of this value. In any case, the expectation is that if the density profile of SIDM0.1 has not converged yet, higher resolution would drive it towards higher densities, not lower, bringing it even closer to CDM (this is a trend confirmed for the cases analysed in VZL, see their Fig. 9).

By using the fact that some MW dSphs have chemodynamically distinct stellar subcomponents that independently trace the same gravitational potential, Walker & Penarrubia (2011)
showed that it is possible to constrain the slopes of their inner mass profiles. They found that Fornax and Sculptor are consistent with cored density profiles while cuspy profiles with $\rho \propto r^{-1}$ are ruled out with a significance $\gtrsim 96\%$ and $\gtrsim 99\%$, respectively. We use this method to test the consistency of the SIDM models explored here. We found that all SIDM models, except for SIDM0.1, are well fitted by the following three-parameter formula:

$$\rho(r) = \frac{\rho_0 r_s^3}{(r + r_s)(r^2 + r_s^2)},$$

which is similar to the Burkert profile (Burkert 1995) but with two scale radii $r_s$ and $r_c$. The remaining case, SIDM0.1, is better fitted by:

$$\rho(r) = \frac{\rho_0 r_s^3}{(r + r_s)(r + r_c)^2}.$$  

Using these formulae, we found the best fit parameters for the massive subhaloes in each of the SIDM models. Such fits are restricted to a radial range between the softening length of our simulations $\sim 120$ pc and the radius where tidal stripping has made the outer logarithmic slope of the density profile steeper than $-3$. The latter restriction is of relevance only for the subhaloes that are affected significantly by tidal stripping within $\sim 5$ kpc. Two of these are clearly affected within $\sim 1$ kpc and should likely be removed in a more detailed analysis; they are the least consistent with the data.

To find the best fit parameters we minimise:

$$Q^2 = \frac{1}{N_{\text{bins}}} \sum_i (\ln \rho_i(r_i) - \ln \rho_{\text{fit}}(r_i))^2,$$

where the sum goes over all radial bins. Thus defined, $Q$ gives an estimate of the goodness of the fit. In Table 1 we give the best fit parameters for the median of the SIDM density profiles of the 15 subhaloes with the largest $V_{\text{max}}(z = 0)$ values. The last two have a constant cross section while the others have a velocity-dependent cross section (see Fig. 3). SIDM0.1 is likely ruled out by cluster constraints (see Rocha et al. 2012). The density profile used for the fits is given by Eq. (3) for SIDM1 and the vdSIDM models, and by Eq. (4) for SIDM0.1.

3 DISCUSSION AND CONCLUSIONS

Self-Interacting Dark Matter (SIDM) offers a promising solution to the dwarf-scale challenges faced by the otherwise remarkably successful Cold Dark Matter (CDM) model. The original idea of a velocity-independent, elastically scattering cross section died off quickly, mostly due to the apparently stringent constraint found by Miralda-Escude (2002) requiring that the cross-section per unit mass was $\sigma_T/m \lesssim 0.2$ cm$^2$ g$^{-1}$. This value is uninteresting, with earlier estimates requiring $\sigma_T/m$ to be at least of order $10^{-2}$ cm$^2$ g$^{-1}$ to create $\sim 1$ kpc cores in dwarf-size haloes (Yoshida et al. 2000, Davel et al. 2001, Peter et al. 2012). We therefore revised the constraint on collisional dark matter and found them to be overestimated by over an order of magnitude; the current constraint is $\sigma_T/m \lesssim 0.1$ cm$^2$ g$^{-1}$. Moreover, these authors have revived, in a companion paper (Rocha et al. 2012), the velocity-independent

<table>
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<th>Name</th>
<th>$\rho(r = 200$ pc)$[\text{M}_\odot \text{kpc}^{-3}]$</th>
<th>$r_s[\text{kpc}]$</th>
<th>$r_c[\text{kpc}]$</th>
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<td>0.75</td>
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<tr>
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<td>0.73</td>
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<tr>
<td>SIDM0.1</td>
<td>$2.31 \times 10^8$</td>
<td>0.97</td>
<td>0.41</td>
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Figure 3. Density profile of the 15 subhaloes with the largest $V_{\text{max}}(z = 0)$ values within CDM and different SIDM models (see Fig. 1). We show the median and 1st and 3rd quartiles of the subhalo distribution for each case. The velocity-dependent SIDM cases produce cores of approximately 600 pc. Of the constant cross section SIDM models we explored, the one that is currently allowed by cluster constraints, SIDM0.1 ($\sigma_T/m = 0.1$ cm$^2$ g$^{-1}$), only deviates slightly from CDM; the associated core sizes are less than 300 pc.
Motivated by the prospect of a viable constant cross section SIDM model, we investigate the claims from Rocha et al. (2012) using high resolution cosmological SIDM simulations of a MW-size halo. Contrary to Rocha et al. (2012), we are able to resolve the sub-kpc structure of the massive subhalo population to sufficiently small radii for comparison with the MW dSphs. We find that a velocity-independent SIDM model is consistent with the kinematics of dSphs only if $\sigma_T/m \approx 1 \text{ cm}^2 \text{ g}^{-1}$ (see Fig. 2), i.e., a value of this order is required to solve the too big to fail problem (Boylan-Kolchin et al. 2011, 2012). If the cross section is lower by an order of magnitude, the subhalo population is still too dense to be consistent with the MW dSphs. On the other hand, as shown already in VZL, velocity-dependent SIDM models with a Yukawa-like interaction (as proposed in Loeb & Weiner 2011) successfully solve the too big to fail problem.

We also use the inner slopes of the mass profiles of Fornax and Sculptor, from the analysis of Walker & Peñarrubia (2011), as examples to test the consistency of the different models we simulate here. For a velocity-independent SIDM model with $\sigma_T/m \sim 0.1 \text{ cm}^2 \text{ g}^{-1}$, we find that 13 of the 15 subhaloes with the largest $V_{\text{max}}(z=0)$ values are inconsistent with the data from Fornax (Sculptor) at $> 95(90)\%$ confidence (the other two are inconsistent at $\gtrsim 81\%$ confidence). A constant cross section ten times larger is as consistent as the velocity-dependent SIDM models explored here with only four (three) of the top 15 subhaloes excluded at $> 90(80)\%$ confidence in the case of Fornax (Sculptor); for all these cases, there are several subhaloes that are unambiguously consistent with the data.

According to the analysis of Peter et al. (2012), a constant cross section of $\sigma_T/m = 1 \text{ cm}^2 \text{ g}^{-1}$ is likely inconsistent with the observed halo shapes of several clusters. We have now shown that $\sigma_T/m = 0.1 \text{ cm}^2 \text{ g}^{-1}$ is too close to CDM to represent a distinct alternative. An interpolation of our simulations suggests that the central densities of the massive subhaloes would be consistent with the MW dSphs if $\sigma_T/m \sim 0.6 \text{ cm}^2 \text{ g}^{-1}$. We conclude that the hypothesis of a constant scattering cross section as solution to the core-cusp problem remains viable but within a very narrow range of $\sigma_T/m$ values. The challenges to make a definitive test of this hypothesis are twofold: the cluster-constraints need to be refined, and the impact of conservative baryonic processes needs to be estimated. Although adding gas physics is the next step of SIDM simulations, a challenge to make SIDM an even more attractive alternative to CDM is the prospect of explaining the observed scarcity of MW satellites and field dwarfs (e.g. Klypin et al. 1999, Zavala et al. 2009) without invoking extreme baryonic processes. As we show in Fig. 4, all allowed elastic SIDM models essentially produce the same abundance of dwarf-size haloes as in CDM. A promising possibility is that of exothermic interactions between excited and non-excited states of dark matter (e.g. Loeb & Weiner 2011). The velocity kick imparted during the collision might be large enough to cause the evaporation of low-mass haloes.

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