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Information Structure and the Tragedy of the Commons in Resource Extraction

Rabah Amir and Niels Nannerup*

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Abstract

This paper considers the well-known Levhari-Mirman model of resource extraction, and investigates the effects of the information structure of the dynamic game – open-loop, Markovian or history-dependent – on the equilibrium consumption path and the overall utility of the agents. The open-loop regime yields a Pareto-optimal outcome. The Markovian regime leads to the most pronounced version of the tragedy of the commons. History-dependent behavior yields an outcome set that is intermediate between the other two cases.

The analysis suggests that in environments characterized by a dynamic (and no market) externality, forcing agents to commit to open-loop behavior would constitute welfare-improving regulation.

Key words and Phrases: Dynamic resource games; Open-loop, closed-loop and trigger strategies; Pareto optimality; Regulation.

JEL codes: Q20, C73.

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1. Introduction

The tragedy of the commons is one of the most readily accepted conclusions in economic analysis. In the abscence of clear-cut property rights assignment or in the presence of public goods, the emergence of the tragedy of the commons is almost always a foregone conclusion. In other words, a first-best outcome typically cannot prevail in such environments when agents' behavior is assumed noncooperative. Among the many diverse settings characterized by this outcome, common-property resource extraction is one of the most natural.

Of the several different competing models of noncooperative resource extraction, the pioneering work of Levhari and Mirman (1980) remains the leading discrete-time model, and one of the most influential overall. Their model postulates two agents as joint owners of a renewable resource, with each of them maximizing the infinite-horizon discounted sum of utilities depending on own consumption only. An important feature of their model is that it reflects no market externality. They refer to the only interdependence in their model, the common-property feature, as a "dynamic externality". Adopting the specific framework of log utility and isoelastic growth function for the resource, they provided a simple analysis of the Markovian (also known as feedback or closed-loop no-memory) equilibrium of the dynamic game based on a closed-form solution with linear consumption strategies. Their results confirm that, relative to the first-best or cooperative solution, the Markovian equilibrium leads to overconsumption (at every stock level, and hence at every period) and to a lower overall utility level for each agent. Their model has recently been extended by Fisher and Mirman (1996) to cover the more general case where both externalities are simultaneously present. Also see Datta and Mirman (1999).

In a novel attempt to model history-dependent behavior in such an environment, Cave (1987) subsequently considered the symmetric version of this model and analysed "cooperative" equilibria secured by trigger strategies. In his setting, the two agents agree on extraction paths that mutually improve on the Markovian outcome, with reversion to the latter for the indefinite future constituting the punishment mode in case a deviation from the "cooperative path" is detected[1] . At any point in the course of the game, the players recall all the previous history of play: states as well as actions. Some recall of history is obviously necessary for the players to be able to monitor compliance with the cooperative extraction path. The equilibria thus derived have the desirable property of subgame-perfection. Ingeniously exploiting the rich structure of this framework, Cave (1987) provides a complete characterization of the associated large equilibrium set. In particular, whether or not this set includes a Pareto-optimal extraction path depends on a derived simple condition on the parameters on the problem.

The present paper is an attempt to understand the effects of the information structure of the dynamic game of Levhari and Mirman (1980) on the characteristics of the resulting equilibria. The main underlying question is whether a monotonic relationship exists between the level of information available to the agents and the efficiency properties of the resulting equilibria. We consider three different information structures, listed in order of increasing information for the players as open-loop, closed-loop no-memory, and history-dependent strategies. For the latter two cases, we rely completely on the results of Levhari-Mirman and Cave, as described above, and in detail in the body of the paper. Our first task then is to investigate the structure of equilibrium under open-loop behavior by the agents: Consumption at any period depends only on the initial stock level and on the date. The agents

[1] While the idea that strategic interaction over time with recall of past play can induce more cooperative behavior from the players was well-established in the theory of repeated games, the economic applications of the theory of dynamic games (with a state variable) have generally restricted attention to non-history-dependent behavior. In this sense, Cave (1987) has pioneered this type of analysis in economic applications of dynamic games. The complexities involved in such a task explain the specific nature of the model that Cave adopted.

are thus committed at the very beginning of the game to a fully specified course of play that cannot be altered along the way, as no dynamic information becomes available in the course of the game. The game may thus be viewed as a one-shot game with infinite sequences (of consumption levels) as strategies.

Our first main result is that the (easily computed) open-loop equilibrium coincides with the symmetric Pareto-optimal solution. While at first very surprising, we argue that this is less so when one considers that the only externality present in the game is the dynamic externality. In other words, this result would survive an extension to more general functional forms for the utility and growth functions, but would not extend to a more general setting that includes (say) a market externality[2] .

This finding, together with the results of the two previous studies, leads to the following assessment: Equilibrium efficiency does not depend in a monotonic way on the level of information available to the agents. Rather, efficiency as a function of information is U-shaped, with a maximum at the lowest information level (i.e. with open-loop strategies) and possibly at the maximal information level (i.e. with full recall of history), with a local maximum being guaranteed there, and a minimum at the intermediate level of information (i.e. with Markovian strategies). An intuitive account of this interesting conclusion is provided in the body of the paper, together with a detailed description of the general properties of each of the information structures at hand.

Besides shedding some light on the relationship between information structure and equilibrium efficiency in a dynamic game, our results offer one policy implication worth exploring:

[2] In certain continous time formulations of resource extraction models, the Pareto optimality of open-loop equilibria has been known for some time, see Chiarella et. al. (1984), Kaitala et. al. (1985) and Dockner and Kaitala (1989). On the other hand, to the best or our knowledge, there is no discrete-time counterpart in the literature. Furthermore, it is well-known that fundamental differences often exist between the continuous-time and corresponding discrete-time formulations. In particular, existence of Markovian equilibrium fails in the contiuous-time version of Levhari and Mirman's model, our basic framework here!

If regulatory action could induce the agents to behave according to open-loop strategies in environments characterized by the sole presence of the dynamic externality, the resulting equilibrium would be a first-best outcome. A possible way of accomplishing this is to force the agents to submit a vector of specified consumption levels, over some fixed horizon, to which they would remain committed[3] . If the players are then forced to stick to their announcements the outcome would be Pareto-optimal over the given horizon. An interesting property of such a scheme is that it is likely to require much less monitoring to ensure compliance than current regulation via quota assignment. The reason for this important feature follows from an interesting property of open-loop equilibria: If all other players are using open-loop strategies, a given player cannot (unilaterally) improve on his payoff by using more complex strategies[4] . In other words, a unilateral violation of this scheme would not be worthwhile to the perpetrating agent.

By contrast, the standard quota regulation is subject to unilateral violations since, given the quotas assigned to the other players, the best response of a player is often to consume more than his own quota. Indeed, in practice, this widespread regulation scheme has enjoyed rather limited success. Likewise, OPEC has frequently had major difficulties in ensuring compliance of the member states with their assigned extraction quotas, the explanation being again the unilateral incentive for over-extraction, given the partners' compliance with their quotas[5] .

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[3] See Reinganum and Stokey (1985) for an analysis of the role of the period of commitment to extraction paths in a dynamic game.

[4] In other words, an open-loop Nash equilibrium (that is derived by allowing only open-loop deviations for the players) remains an equilibrium if the players are allowed to use any more general classes of strategies.

[5] It is worthwhile to note here that, in view of the abscence of a supranational governing body to ensure direct compliance with assigned quotas, OPEC has to rely on threat-secured compliance instead. Indeed, sustained deviations in the form of over-production in the past have sometimes triggered reactions (e.g. by Saudi Arabia) that resulted in a glut in the oil market. Thus, OPEC would fit quite nicely a Cave-style analysis.

On the other hand, a model of OPEC would generally not satisfy the result that the open-loop equilibrium is Pareto-optimal, since OPEC reflects a market externality and no dynamic externality (as oil wells

The rest of the paper is organized as follows. Section 2 presents the basic model. In Section 3-6, the solutions under the three different information structures and the Pareto-optimal case are exposited. The main comparative results and their derivation, an associated discussion, with policy implications, form Section 7. Section 8 provides an observation on the sustainability of the natural resource. Finally, a short conclusion forms Section 9.

2. The model

There are two identical agents sharing the rights to exploit a renewable resource. The resource stock develops over time according to a biological growth rule given (upon a normalization of units) by

$$x_{t+1} = x_t^{\alpha}, \qquad 0 < \alpha \leq 1, \tag{1}$$

and this rule is common knowledge to both agents. Following Levhari and Mirman (1980) and Cave (1987), the utility of agent $i, i = 1, 2$ is assumed to be :

$$u^i(c_t^i) = \log c_t^i, \tag{2}$$

where[6] c_t^i is his resource consumption at time t. Let $0 < \delta < 1$ be the common discount factor of the agents. The objective for agent i is then to maximize the sum of the discounted utility of consumption, i.e.:

$$\max \sum_{t=0}^{\infty} \delta \log c_t^i$$

subject to

$$x_{t+1} = (x_t - c_t^1 - c_t^2)^{\alpha}.$$

Upon specification of the strategies of the two players, the formulation of an infinite horizon dynamic game between the agents will be complete. Over the next four sections

are the private property of the states involved), in complete contrast to the Levhari-Mirman world.

[6] Throughout, we denote agents by superscipts and calendar time by subscripts.

of the paper we compare the equilibria that arise under different information structures or, equivalently, under different sets of strategies used by the agents, in terms of their efficiency properties and their consequences on the resource stock.

It is worth noting already that this model is specific on two important counts. First, the utility and growth functions have special (though commonly used) functional forms. Second, the agents face no market externality here, the only externality being what Levhari and Mirman referred to as a dynamic (common-property) externality.

In what follows, we can consider each of the four solutions separately. For each case, we provide a definition and summary of the salient features of the strategies at hand, and then derive the associated equilibrium. Further details may be found in Basar and Olsder (1999), Fudenberg and Tirole (1986) or Amir (2000).

3. The Open-loop Equilibrium

3.1 Definition and General facts

This subsection provides a definition of open-loop strategies in *deterministic* Markovian dynamic games[7] and a summary of their general properties and limitations. In the present setting, an open loop (henceforth OL) strategy σ for an agent is an infinite sequence $\sigma(x_0) = (c_0, c_1, ...) \in R^\infty$ specifying the resource consumption level at every time period t over an infinite horizon as a function of the initial stock x_0 and calendar time t only.

Open-loop behavior thus rests on the premise that the players simultaneously commit at the beginning of the game to a completely specified list of actions, without any possibility of update or revision during the entire course of the game. The players receive no new information, not even about the value of the current state. Hence, no contingency planning

[7] These are defined by the facts that the one-period reward and the next state are time-invariant and depend only on the current state and actions.

of any sort is possible.

Several important observations concerning open-loop equilibria should be noted. Interestingly, while some of these properties may appear complex, they actually all follow directly from the well-known properties of Markovian dynamic programming. To begin with, in *deterministic* Markov dynamic optimization, there always exists an optimal open-loop strategy (under minor regularity conditions). In other words, in one-person problems, restricting oneself to open-loop policies results in no loss of value compared to using more sophisticated behavior such as Markov or history-dependent policies. This fact is certainly intuitive, as is its failure in the presence of chance moves or stochastic transitions[8] .

The game-theoretic partial analog[9] of the above fact is perhaps less intuitive, though no less important. In deterministic dynamic games, an open-loop equilibrium remains an equilibrium when the strategy spaces are expanded to include Markovian or history-dependent strategies. The reason is that if the rival is using open-loop strategies, a player cannot achieve a higher payoff by using more sophisticated strategies than open-loop. This follows directly by invoking the above fact for the player's best-response problem which, given the open-loop strategies of the rivals, is a *deterministic* (Markovian) dynamic optimization problem.

Open-loop equilibria are generally not subgame-perfect; in other words, they are not time-consistent. If the game is restarted at some state along the equilibrium trajectory, the new open-loop equilibria for the game starting at that state will not necessarily coincide with the restriction of the original equilibrium strategies to the subgame starting at that same state.

Open-loop equilibria are typically much simpler to analyze than Markovian equilibria.

[8] The model of Levhari and Mirman (1980) has been investigated in a general framework with a stochastic growth law for the resource in Amir (1996).

[9] No general results are known about the comparison of equilibrium payoffs under open-loop vs. Markovian (or closed-loop no-memory) behavior. Examples can be given for either type of strategy (when followed by all players) to be better than the other type, in terms of resulting equilibrium payoffs.

This relative simplicity is at the heart of the widespread use of open-loop strategies in the early stages of the adoption of the tools of strategic dynamics in economics[10] . In recent years, economists have generally agreed that the commitment to a completely specified course of action over the indefinite future is not a realistic behavioral postulate in most cases of interest in economic dynamics. Furthermore, subgame-perfection (or time-consistency) is broadly viewed as a desirable property of dynamic equilibria. Consequently, focus has markedly shifted to Markovian behavior.

3.2 The Open-Loop Solution

In order to characterize the OL equilibrium, we begin with an analysis of the best response problem of agent 1 (say). Fix an open loop strategy $\tau^2 = (c_0^2, c_1^2, c_2^2, ...) \in R^\infty$ for agent 2 that is feasible, i.e., that is such that $c_0^2 \leq x_0$ and $x_{t+1} = (x_t - c_t^2)^\alpha \geq 0, t \geq 1$. Given the OL strategy τ^2, agent 1 clearly faces a Markov-stationary dynamic optimization problem, and his best-response may be solved for via dynamic programming as follows[11] . With $V_{\tau^2}(x)$ denoting the optimal total discounted utility agent 1 can obtain when the initial resource stock is x and agent 2 follows the OL strategy τ_2, we have the standard optimality equation:

$$V_{\tau^2}(x) = \max_{0 \leq c^1 \leq x - c^2} \{\log c^1 + \delta V_{\tau^2}[(x - c^1 - c^2)^\alpha]\}. \tag{3}$$

The first order condition for the RHS maximization is

$$\frac{1}{g(x)} = \delta\alpha V'_{\tau^2}[(x - g\,(x) - c^2)^\alpha](x - g(x) - c^2)^{\alpha-1}, \tag{4}$$

[10] In the economics of natural resource exploitation and sustainability, studies that rely on the open-loop information structure tend to be older. They include, among many others, Salant (1976), Lewis and Schmalensee (1980), and Dasgupta and Heal (1979).

[11] We caution the reader that this is a highly unusual step. Indeed, open-loop equilibria are typically solved for via the maximum principle, as dynamic programming produces feedback equilibria, by its very construction. Nonetheless, a minor alteration in this method will work here, the benefit being that this facilitates direct step-by-step comparisons with the other cases, all of which are typically solved via dynamic programming methods.

where $g(x)$ is the optimal consumption when the stock is x. Along an optimal consumption path, we have

$$V_{\tau^2}(x) = \log\left[g\left(x\right)\right] + \delta V_{\tau^2}[(x - g\left(x\right) - c^2)^{\alpha}], \qquad \text{for all x.}$$

Assuming (for now) that V_{τ^2} and g are differentiable, we have

$$V'_{\tau^2} = \frac{g'\left(x\right)}{g\left(x\right)} + \delta\alpha V'_{\tau^2}[(x - g\left(x\right) - c^2)^{\alpha}](x - g\left(x\right) - c^2)^{\alpha-1}\left(1 - g'\left(x\right)\right). \tag{5}$$

Substituting (4) into (5) yields the envelope result

$$V'_{\tau^2} = \frac{1}{g\left(x\right)}. \tag{6}$$

Using (6), (4) can be rewritten as the Euler (functional) equation:

$$\frac{1}{g\left(x\right)} = \frac{\delta\alpha\left(x - g\left(x\right) - c^2\right)^{\alpha-1}}{g\left[(x - g\left(x\right) - c^2)^{\alpha}\right]}. \tag{7}$$

An analogous equation can be derived for agent 2 given a feasible consumption path by agent 1. However, invoking symmetry, we can postulate that $g\left(x_t\right) = c_t^2$, for all t, at an open-loop equilibrium. Furthermore, we can postulate (and later confirm) that (7) has a solution g that is linear in x. Then pluging $g\left(x\right) = \lambda x$ in (7) leads to

$$\frac{1}{\lambda x} = \frac{\delta\alpha\left[(1-2\lambda)x\right]^{\alpha-1}}{\lambda\left[\left(1-2\lambda\right)x\right]^{\alpha}}.$$

After simplification,

$$\lambda = \frac{1-\delta\alpha}{2} \qquad \text{or} \qquad g_{ol}(x) = \left(\frac{1-\delta\alpha}{2}\right)x. \tag{8}$$

Viewed as $g^*\left(x_t\right) = \left(\frac{1-\delta\alpha}{2}\right)x_t$, $t = 0, 1, ...$, this is the closed-loop (or feedback) representation of the symmetric open-loop equilibrium consumption strategies[12] .

[12]Stating these strategies in this form is very convenient for the purposes of this paper, as this allows for direct comparisons with the other cases. The alternative, i.e. giving an infinite vector of consumption levels depending only on x_0, would make this step needlessly complicated.

A simple way to calculate the corresponding equilibrium value function for each agent is to substitute (8) in the functional equation (3) yielding

$$V_{ol}(x) = \log\left(\frac{1-\delta\alpha}{2}x\right) + \delta V_{ol}\left[(\delta\alpha x)^{\alpha}\right], \tag{9}$$

and then postulating the 'guess' $V_{ol}(x) = A\log x + B$ into (9), we get

$$A\log x + B = \log\left(\frac{1-\delta\alpha}{2}x\right) + \delta\left\{A\log\left[(\delta\alpha x)^{\alpha}\right] + B\right\}.$$

Identifying terms leads to

$$V_{ol}(x) = \frac{\log x}{1-\delta\alpha} + \frac{\log(1-\delta\alpha) + \delta\alpha\log(\frac{\delta\alpha}{2})}{1-\delta}. \tag{10}$$

Under the open-loop equilibrium, the resource stock evolves according to the difference equation

$$x_{t+1} = \left[x_t - 2\left(\frac{1-\delta\alpha}{2}x_t\right)\right]^{\alpha} = (\delta\alpha x_t)^{\alpha}.$$

At a steady-state equilibrium of the resource stock, $\overline{x}$, we have $\overline{x} = (\delta\alpha\overline{x})^{\alpha}$ or

$$\overline{x} = (\delta\alpha)^{\frac{\alpha}{1-\alpha}}$$

4. The Closed-loop (no memory) Equilibrium

4.1 Definition and General facts

A feedback or closed-loop memoryless strategy for an agent is a function γ from the set of all possible stock levels to the set of possible consumption levels[13] . Since with such strategies, players are allowed to condition their extraction only on the value of the current stock, they necessarily consume the same amount of resource every time the same stock is observed, regardless of calendar time.

[13]With the above terminology originating in the control theory literature, the standard game-theoretic way of referring to such strategies is as Markov-stationary, see e.g. Amir (1996). The latter terminology is more consistent with the way we defined the general class of games at hand, but is less prevalent in the resource economics literature.

It is well-known that a feedback or Markov-stationary equilibrium of a Markov-stationary infinite-horizon dynamic game remains an equilibrium even when history-dependent strategies are allowed. This also follows from the fact that with the rival playing a Markov-stationary strategy, a player's best-response problem is a Markov-stationary dynamic programming problem, for which it is known that there exists a Markov-stationary optimal policy. Note in this case that this argument is equally valid in the presence of chance moves (i.e. stochastic transitions)[14] .

Furthermore, an equilibrium in Markovian (or feedback) strategies is always subgame-perfect in a strong sense: Uniformly in the initial state.

4.2 The Feedback Solution

The results of this subsection are due to Levhari and Mirman (1980). Their solution proceeds via backward induction to derive the closed-loop equilibrium as the length of the horizon is increased, and then obtaining the infinite-horizon equilibrium as a limit. Here, we analyse instead the infinite-horizon problem directly to maintain comparability with the other cases.

A feedback strategy γ is feasible if $\gamma(x) \leq x$ for all $x \geq 0$. We consider only the set of strategy pairs (γ^1, γ^2) that are jointly feasible here, in the sense that $\gamma_1(x) + \gamma_2(x) \leq x$ for all $x \geq 0$. In other words, we are really considering a generalized game, in Debreu's (1952) terminology.

Fixing a feasible strategy $\gamma^2(\cdot)$ by agent 2 (say), the best-response problem of agent 1 can be analysed as follows. Let $V_{\gamma^2}(x)$ denote the optimal value agent 1 can obtain when agent 2 follows the consumption function $\gamma^2(\cdot)$ and the initial state is x. Then agent 1's

[14] It is of interest to observe that these important justifying arguments, as well as the so-called one-shot deviation principle, follow directly from the general properties of dynamic programming.

best response strategy is the solution to the standard optimality equation:

$$V_{\gamma^2}(x) = \max_{0 \leq c^1 \leq x - c^2} \{\log c^1 + \delta V_{\gamma^2}[(x - c^1 - \gamma^2(x))^\alpha]\}. \tag{11}$$

The first order condition is (with $g(x)$ denoting the optimal consumption when the stock level is x):

$$\frac{1}{g(x)} = \delta \alpha V'_{\gamma^2}[(x - g(x) - \gamma^2(x))^\alpha](x - g(x) - \gamma^2(x))^{\alpha - 1}. \tag{12}$$

We can clearly write

$$V_{\gamma^2}(x) = \log[g(x)] + \delta V_{\gamma^2}[(x - g(x) - \gamma^2(x))^\alpha], \quad \text{for all x.}$$

Differentiating with respect to x gives

$$V'_{\gamma^2} = \frac{g'(x)}{g(x)} + \delta \alpha V'_{\gamma^2}[(x - g(x) - \gamma^2(x))^\alpha](x - g(x) - \gamma^2(x))^{\alpha - 1}\left(1 - g'(x) - \gamma^{2\prime}(x)\right). \tag{13}$$

Substituting (12) into (13) yields the envelope relation

$$V'_{\gamma^2} = \frac{1 - \gamma^{2\prime}(x)}{g(x)}. \tag{14}$$

Substituting (14) into (12), the Euler equation follows

$$\frac{1}{g(x)} = \frac{\delta \alpha (x - g(x) - \gamma^2(x))^{\alpha - 1}}{g[(x - g(x) - \gamma^2(x))^\alpha]}\left\{1 - \gamma^{2\prime}\left[(x - g(x) - \gamma^2(x))^\alpha\right]\right\}. \tag{15}$$

Postulating a symmetric equilibrium with strategies linear in the stock, i.e. $\gamma^2(x) = g(x) = \lambda x$, (15) becomes

$$\frac{1}{\lambda x} = \frac{\delta \alpha [(1 - 2\lambda) x]^{\alpha - 1}}{\lambda [(1 - 2\lambda) x]^\alpha}(1 - \lambda).$$

This simplifies to

$$\lambda = \frac{1 - \delta \alpha}{2 - \delta \alpha}.$$

Hence, the symmetric closed-loop (no-memory) equilibrium consumption strategy is

$$g_{cl}(x) = \frac{1 - \delta \alpha}{2 - \delta \alpha} x. \tag{16}$$

This is clearly jointly feasible, i.e. $2g^*(x) < x$, for all x. The corresponding value function may by computed, as before, via the optimality equation (11) ,upon substituting (16) for both agents:

$$V_{cl}(x) = \log\left(\frac{1-\delta\alpha}{2-\delta\alpha}x\right) + \delta v_{cl}\left[\left(\frac{\delta\alpha x}{2-\delta\alpha}\right)^{\alpha}\right]. \tag{17}$$

Substituting the guess $V_{cl}(x) = A\log x + B$ into (17) and identifying corresponding terms yields

$$V_{cl}(x) = \frac{\log x}{1-\delta\alpha} + \frac{\log\frac{1-\delta\alpha}{2-\delta\alpha} + \frac{\delta\alpha}{1-\delta\alpha}\log\frac{\delta\alpha}{2-\delta\alpha}}{1-\delta}. \tag{18}$$

The steady-state equilibrium level of resource stock satisfies

$$\overline{x} = (\overline{x} - 2\frac{1-\delta\alpha}{2-\delta\alpha}\overline{x})^{\alpha},$$

so that

$$\overline{x}_{cl} = \left(\frac{\delta\alpha}{2-\delta\alpha}\right)^{\frac{\alpha}{1-\alpha}}.$$

5. The symmetric Pareto-optimal solution

In view of the symmetry across agents, we focus only on the (unique) symmetric solution out of the set of all Pareto-optimal outcomes. This is obviously equivalent to the single-agent or monopoly problem:

$$\max \sum_{t=0}^{\infty} \delta^t \log c_t^i$$

subject to

$$x_{t+1} = (x_t - c_t)^{\alpha}.$$

The dynamic programming solution may be obtained from the previous section upon setting $\gamma^2(x) = 0$. The resulting optimality equation, envelope relation and Euler equation are, respectively

$$V(x) = \max_{0\le c\le x}\{\log c + \delta V[(x-c)^{\alpha}]\},$$

$$V'(x) = \frac{1}{g(x)},$$

and

$$\frac{1}{g(x)} = \frac{\delta\alpha (x - g(x))^{\alpha-1}}{g[(x - g(x))^{\alpha}]}.$$

The optimal consumption policy is then

$$g_{po}(x) = (1 - \delta\alpha) x. \tag{19}$$

Following the same method as before, the corresponding optimal total discounted utility per agent is

$$V_{po}(x) = \frac{\log x}{1 - \delta\alpha} + \frac{\log(1 - \delta\alpha) + \frac{\delta\alpha}{1-\delta\alpha}\log(\delta\alpha) - \log 2}{1 - \delta}. \tag{20}$$

The steady-state equilibrium level of resource stock is easily seen to be

$$\overline{x}_{po} = (\delta\alpha)^{\frac{\alpha}{1-\alpha}}.$$

6. The trigger-strategy equilibria

Cave (1987) extends the analysis of Levhari and Mirman (1980) by incorporating history-dependent behavior, in the specific form of threat or trigger strategies. We here review the main results of Cave's analysis. Unlike closed-loop (no memory) strategies, agents now have access to the entire past history of play and condition their actions on it at each time period. A trigger-strategy equilibrium is characterized by two phases. The first is a cooperative phase where players specify a mutually beneficial extraction path (e.g. a Pareto-optimal path) for the entire length of the game. The second is a punishment phase where a player would "pull the trigger" and revert to the punishment part of his strategy as soon as he detects a deviation by the rival from the cooperative path. Following Cave, we consider the equilibria that result from the threat of reversion to the (unique) feedback equilibrium of Levhari-Mirman, so that the resulting equilibria are clearly overall subgame-perfect.

One may appropriately interpret this set-up as a self-enforcing cooperative agreement in a noncooperative framework. By its very construction, such a trigger-strategy equilibrium has the property that no agent wishes to defect from either the agreement or the threats associated with out-of-equilibrium behaviour, whence Cave's "Cold Fish War" reference.

Cave exploits the special structure of this model by specifying consumption by agents in fractions h^i of the current resource stock. The agents select a pair $h = [h^1, h^2]$ of extraction rates in each period from the set of feasible extraction rate vectors given by $Y = \left\{h \in R_+^2 : \sum h^i \leq 1\right\}$, so that at date t, $c_t^i = h_t^i x_t$. By solving recursively for the resource stock at time t as a function of a given initial stock, x_0, we find $x_t(h, x_0) = (1-H)^{\lambda(t)} x_0^{\alpha^t}$, where $H = h^1 + h^2$ and $\lambda(t) = \alpha(1-\alpha^t)/(1-\alpha)$. The present value to agent 1 for a given extraction rate vector h is then

$$V(h, x_0) = \sum_{t=0}^{\infty} \delta \log\left[h^1 x_t(h, x_0)\right] = \frac{\Psi^1(h) + (1-\delta)\log(x_0)}{(1-\delta)(1-\alpha\delta)}, \tag{21}$$

where $\Psi^1(h) \triangleq (1-\alpha\delta)\log(h^1) + \alpha\delta\log[1-H]$. A strategy χ for an agent is an infinite sequence of extraction rates $\chi = (f_0, f_1, ...)$, such that each f_t is a map from the history of states and actions up to time t to the set of consumption levels.

Specifically, let $b = [b_1, b_2, ...]$ be a sequence of pairs of extractions representing cooperative behaviour and let z_t be the history of play, that is, the pairs of extraction rates during the first t periods. A closed-loop supported equilibrium strategy supporting cooperative behaviour for player 1 is then

$$f_t^1(z_{t-1}) = \begin{cases} b_t^1 \text{ if and only if } z_{t-1} = [b_1, b_2...], \\ \frac{g_{cl}(x_t)}{x_t} \text{ otherwise,} \end{cases}$$

where g_{cl} is agent 1´s equilibrium closed loop (no memory) strategy.as given by (16). Given this, cooperative behaviour is closed-loop supported for player 1 if and only if :

$$V(h,x) \geq Max_{c^1}\left\{\log c_1 + \delta V_{\gamma^2}\left[x - c_1 - b^2 x\right]\right\}. \tag{22}$$

If a pair of cooperative extraction rates for agent 1 and 2, $d = [b^1, b^2]$, is closed-loop supported for both players, d belongs to the set of closed-loop supported extractions, CL. From (18) and (21) we have that $d \in CL$ if and only if (i, j =1, 2; $j \neq i$)

$$F^i(h) = \Psi^i(h) - (1-\delta)\log\left(1 - h^j\right) \geq F^i\left(\frac{\gamma^i}{x_t}, \frac{\gamma^j}{x_t}\right) = K, \tag{23}$$

where $K = (1-\alpha\delta)\log(1-\alpha\delta) + \alpha\delta\log\alpha\delta - \delta\log(2-\alpha\delta)$.

The set CL of extractions can be determined by varying H in (23). This leads to a convex, compact, symmetric, and non-empty set. It follows from the very definition of CL, i.e. (23) that the closed loop equilibrium extractions$\left(\frac{\gamma^i}{x_t}, \frac{\gamma^j}{x_t}\right)$ always constitute the largest total extraction rate in CL. (See Cave (1987) for details.)

By use of the symmetry assumption, it is now clear from (19) and (23) that there exists a Pareto optimal solution in the set of closed loop supported extraction rates if and only if

$$F^1\left(\frac{1-\alpha\delta}{2}, \frac{1-\alpha\delta}{2}\right) = F^2\left(\frac{1-\alpha\delta}{2}, \frac{1-\alpha\delta}{2}\right) \geq K,$$

or

$$\delta\log(2-\alpha\delta) - \delta(1-\alpha)\log 2 - (1-\delta)\log(1-\alpha\delta) \geq 0. \tag{24}$$

7. Comparative results

We state our main results in the form of

Proposition 1 *(i) The open-loop equilibrium coincides with the symmetric Pareto-optimal solution, in that both lead to the same consumption path and the same total discounted utility for each agent.*

(ii) The closed-loop (no memory) equilibrium leads to overconsumption (at every possible stock level) and to a lower total discounted utility level for each agent relative to the symmetric Pareto-optimal solution.

(iii) The trigger-strategy equilibrium set is such that any of its selections leads to a consumption level and a total discounted utility level that are respectively lower and higher than the corresponding levels under a closed-loop equilibrium.

Furthermore, the trigger-strategy equilibrium set includes a Pareto-optimal point if and only if (24) holds.

Proof. We provide only an outline of proof, as the actual analysis was conducted in the previous section. (i) appears from comparing (8) and (19). Total consumption is clearly the same in the OL equilibrium and the Pareto optimal solution. In consequence, the OL equilibrium and the Pareto optimal solution have the same total discounted utility per player. (ii) follows by comparing (16) with (19) and (18) with (20), respectively. Clearly, total consumption in the CL equilibrium is higher than the Pareto optimal level while total discounted utility is highest in the Pareto optimal solution. (iii) follows from the very definition of the trigger strategies.□

Some important direct consequences of this Proposition are given next. The obvious proofs are omitted. The first compares the steady-state levels of resource under the different regimes.

Corollary 2 *The steady-state equilibrium levels of the resource stock satisfy:*
$\overline{x}_{cl} \leq \overline{x}_{ts} \leq \overline{x}_{po} = \overline{x}_{ol}$

While open-loop equilibria generally fail the desirable property of subgame-perfection, the present case forms an interesting exception. Indeed, due to the coincidence of the open-loop and the Pareto-optimal solutions, we obviously have:

Corollary 3 *The open-loop equilibrium of the dynamic game at hand is subgame-perfect, or equivalently, gives rise to a time-consistent play of the game.*

We now provide an extensive discussion of our conclusions. The main result sheds light on the important issue of how the *extent* of the tragedy of the commons depends on the information structure of the agents in common-property resource extraction. For the well-known model of Levhari and Mirman, noncooperative open-loop behavior is outcome-equivalent to perfect cooperation, and thus leads to a first-best or Pareto-optimal outcome. Feedback or Markovian behavior leads to the most pronounced version of the tragedy of the commons, of all cases considered. Finally, history-dependent behavior in the form of trigger-strategies leads as usual (due to Folk-Theorem-type arguments) to a myriad of solutions, all reflecting an intermediate outcome, ranging from the feedback solution all the way to *possibly* the first-best situation (depending on the parameters of the problem).

We now attempt to provide an intuitive understanding of these results. In the open-loop case, agents are deprived of the possibility of observing the current stock and reacting in a dynamic sense to the catches of the rival. This lack of awareness of the period-by-period effects of the rival's catches on the resource stock leads the agents to avoid the over-consumption that is otherwise inherent to noncooperative resource extraction. That this leads all the way to Pareto optimality is rather surprising, and is due to the special structure of the model (the sole presence of the dynamic externality). Under feedback behavior, the agents are fully aware of the consequences of the rival's catch on the resource level, and react accordingly, causing the standard tragedy of the commons to be fulfilled. Feedback information, on the other hand, is not sufficient to allow agents to make threat-secured agreements, as these require history-dependent information for the agents to be able to detect deviations from agreed-upon extraction paths. Increasing the agents' information levels as Cave did allows them to make such cooperative agreements and reduce the extent of the tragedy of the commons, possibly all the way to first-best.

Our main result can thus be succinctly paraphrased as follows: *The efficiency of noncooperative equilibria in common-property resource extraction is not monotonically related to the level of information on which the players' decisions are based.* Rather, it is U-shaped, with a global maximum at the lowest level of information (open-loop strategies), and maybe another at the maximum level of information (i.e. history-dependent strategies).

While the coincidence of the open-loop and the Pareto-optimal solutions is easily shown to extend to general utility and growth functions and asymmetric agents, the trigger-strategy equilibrium is much harder to characterize at such a level of generality, as suggested by Cave's specific analysis on which we relied here.

The two corollaries follow directly from the main result, and are self-explanatory. The subgame-perfect property of the open-loop equilibrium turns out to be of possible practical relevance to a possible regulatory application of our analysis, as discussed below.

8. A Policy Implication

A natural question at this point is whether our conclusions offer any policy-relevant insights. Consider a government regulator with a mandate to prevent or limit over-extraction of a natural resource (such as fish or game) over a specified time horizon. One possible way to proceed would be to require the consuming agents to (independently and simultaneously) submit individual consumption plans over the entire time horizon with the understanding that they will remain committed to these announcements, and penalized for any future deviations. According to our analysis, *if the dynamic externality is the only one present* and if uncertainty plays no role in the context at hand[15] , this noncooperative mechanism would lead to a Pareto-optimal solution over the specified time horizon. This offers an interest-

[15]Recall that, as can be intuitively expected, the open-loop information structure loses many of its equilibrium properties in the presence of uncertainty in the state transition law (see Section 3).

ing possible alternative to the usual centrally-negotiated cooperative solution consisting of allocating quotas to the agents, that is worth exploring.

If the specified time horizon is long enough, and in view of the coincidence of the open-loop and the Pareto-optimal solutions, we know from Easley and Spulber (1981) that this rolling-horizon solution should remain fairly close to the Pareto-optimal solution of the infinite-horizon problem.

More broadly, while we agree with the now well-known assessment that open-loop behavior in a long-term game is generally inappropriate as an approximation of real-life volontary behavior in most economic settings, we argue here that open-loop behavior may quite naturally arise from Pareto-improving *regulation* in some specific settings. Indeed, forcing a commitment from all agents and policing their compliance with the announcements may be welfare-improving for all, in some situations characterized in particular by the independence of the one-period rewards on the rival's actions.

A key property of such a mechanism is that it would essentially be cheat-proof , at least in a unilateral sense, since if all the other agents employ open-loop strategies, a given agent cannot improve his payoff by following some more complex strategy, so he might as well do the same (this fact is an important property of open-loop equilibrium and was discussed in Section 3.1). Thus the monitoring regulator need only worry about deviations from the announced paths that involve more than one agent. This potentially makes the monitoring problem less demanding than in the case of regulation via centrally-allocated extraction quotas. Another desirable feature of this mechanism is that it is likely to be less demanding to administer, as it allows policy-makers to avoid the typically lengthy and strenuous negotiations that are often associated with quota negotiations[16] .

[16]For resource conservation, the main approach taken by regulation authorities is to stipulate a total allowable catch for the whole fishery based on biological studies on the actual stock population. Individual quotas are subsequently awarded, usually, to agents with a history of participation in the fishery (and through

In the language of industrial regulation (see e.g. Perry, 1984), our proposal can be regarded more as an instance of structural policy than of behavioral policy. In other words, the aim of the regulator is to influence the strategic environment of the regulated agent by constraining his allowable strategy set, and not to interfere directly with the agents' postulated behavior of individual utility maximization (taking the rival's action as given). In contrast, central quota assignment is an instance of the latter mode of regulation.

Naturally, the notion that commitment is important in many classes of noncooperative games has been around for quite some time, and this is only a new instance (at least to us) of this general fact.[17] Yet, as simple as this idea is in the context of noncooperative dynamic resource exploitation, it does not seem to have been proposed before.

9. Information Structure and Resource Depletion

In this section, we make the observation that the differences in noncooperative resource extraction between open-loop and feedback behavior can in some cases be so pronounced as to lead to the long-run resource sustainability in the former case and depletion in the latter case. To this end, it is necessary, following Levhari and Miman (1980) again, to postulate another form of natural growth function, as (1) always leads to a strictly positive resource stock at steady-state, regardless of the information sructure.

Consider the log utility function (2) and a linear growth relationship for the resource

negotiations at the international level.). To implement a cost-effective fishery individuable transferable quotas are increasingly being used. Under this system total quotas being isssued are still determined by the regulation authorities, while trade in quota units is allowed so that they go to agents who value them the most. Such management systems of centralized control are used for most regulated fish populations. Clark (2000) provides a broad discussion of traditional and alternative fishery management techniques.

[17]For instance, within the theory of multiperiod mechanism design it proves essential for the principal to be able to commit to a long term contract at the beginning of a relationship. In the opposite case serious ratchet effects may arise leading to substantial difficulties for contract theory, see for example Laffont and Tirole (1993), chapter 9.

evolution

$$x_{t+1} = ax_t \qquad \text{with } a > 1.$$

Under the open-loop information structure, the same steps as before lead to the unique equilibrium (per-agent consumption) strategies[18] :

$$g_{ol}(x) = \frac{1-\delta}{2}x, \ \forall x \geq 0.$$

Likewise, the feedback equilibrium strategies are given by

$$g_{cl}(x) = \frac{1-\delta}{2-\delta}x, \ \forall x \geq 0.$$

Hence, the evolution of the resource stock under the OL equilibria is given by

$$x_{t+1} = \frac{a\delta}{2-\delta}x_t,$$

while under the feedback equilibrium, it follows

$$x_{t+1} = a\delta x_t.$$

It is easy to verify then that under a moderate natural growth rate of the resource, i.e. if $\frac{2-\delta}{\delta} < a < \frac{1}{\delta}$, we have $\lim_{t\to\infty} x_t = \infty$ under the open-loop solution, and $\lim_{t\to\infty} x_t = 0$ under the feedback solution. In other words, the resource grows without bound under noncooperative open-loop extraction and is depleted over time under feedback extraction, a rather drastic difference.

On the other hand, in case of a high growth rate, i.e. if $a > \frac{1}{\delta}$, then $\lim_{t\to\infty} x_t = \infty$ in both cases, while in case of a low growth rate, i.e. if $a < \frac{2-\delta}{\delta}$, then $\lim_{t\to\infty} x_t = 0$ in both cases.

[18]Since $\log ax_t = \log a + \log x_t$, it is easy to see that the equilibrium policies are independent of a. Then setting $a = 1$, one can obtain these policies from the corresponding solutions in the previous sections, simply by setting $\alpha = 1$.

10. Conclusion

Adopting the simple framework proposed by Levhari and Mirman (1980), this paper has provided a comparative analysis of the effects of the information structure on the equilibrium extraction path for a common-property natural resource. To allow a more intuitive understanding of the relationship between information and extraction paths, we derive all solutions using a dynamic programming approach. The main result is that equilibrium efficiency is U-shaped in the level of information available to the agents, with in particular the open-loop equilibrium being Pareto-optimal. We argue that the main result extends to more general functional forms, provided the special structure of the Levhari-Mirman model is preserved, with in particular the abscence of a market-type externality.

This result suggests a mechanism design type of approach to the problem of regulation of common-property resource extraction, in the sense that if agents can be induced to behave according to open-loop strategies, the equilibrium would be self-enforcing, and thus require relatively minor monitoring. This proposal would also save on the usual tedious bargaining that always accompanies the process of quota-setting. It must of course be stressed that this proposition is valid only in rather special environments as reflected in the special structure of the model used here.

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