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Published in:
International Journal of Modern Physics D

DOI:
[10.1142/S0218271806008279](https://doi.org/10.1142/S0218271806008279)

Publication date:
2006

Citation for published version (APA):
A. de Diego, J., Nunez, D., & Zavala Franco, J. (2006). Pioneer anomaly? *International Journal of Modern Physics D*, 15(4), [533]. <https://doi.org/10.1142/S0218271806008279>

Pioneer anomaly? Gravitational pull due to the Kuiper belt

José A. de Diego^a, Darío Núñez^b, Jesús Zavala^b

^a*Instituto de Astronomía,*

Universidad Nacional Autónoma de México

A. P. 70-262, 04510 México, D. F., MEXICO

^b*Instituto de Ciencias Nucleares,*

Universidad Nacional Autónoma de México

*A. P. 70-543, 04510 México, D. F., MEXICO**

(Dated: February 2, 2008)

In this work we study the gravitational influence of the material extending from Uranus orbit to the Kuiper belt and beyond on objects moving within these regions. We conclude that a density distribution given by $\rho(r) = \frac{1}{r}$ (for $r \geq 20UA$) generates a constant acceleration towards the Sun on those objects, which, with the proper amount of mass, accounts for the blue shift detected on the Pioneers space crafts. We also discuss the effect of this gravitational pull on Neptune, and comment on the possible origin of such a matter distribution.

PACS numbers: PACS numbers: 04.80.-y, 95.10.Eg, 95.55.Pe

Mankind is now in direct contact with regions beyond the Solar System. The space probes launched in the 70's already passed the orbit of the last Solar system planet, Pluto, and are still working! It is really a homage for their designers. In particular, the Pioneer probes 10 and 11 were designed in such a cunning way that their position, via the Doppler effect, can be determined with great accuracy (the tracking system have the sensitivity to measure frequency changes at the level of mHz/s , [1]).

When these probes were still within the Solar system, at 20 UA, that is, between Uranus, which has a mean distance from the Sun of 19.13 UA, and Neptune, at 30 UA, the frequency received at Earth start showing an unaccounted effect, a blue shift that is usually interpreted as a constant acceleration with a magnitude of

$$a = 8.74 \pm 1.33 \times 10^{-8} \frac{\text{cm}}{\text{s}^2}, \quad (1)$$

and directed towards the Sun [1]. The cause for such a blue shift was unknown but more remarkable was the fact that such phenomenon kept being present. In figure 1, we show a diagram of the Solar system showing the Pioneers' trajectories and positions. This figure is a reproduction taken from the original at [2].

There have been a number of possible explanations for the cause of the blue shift, and there is now an important number of works that discard any internal effect of the probes as the cause, such as heat reflected off the probes and possible gas leaks, (see [1], [2] and [3]), implying that it is due to an external cause. Now the Pioneers are farther than 70 UA, well beyond Pluto's orbit, 39.3 UA, and the effect is still there. Following Occam's razor, the simplest explanation is that a **constant** force, independent of the distance, is producing the measured blue shift.

In this way, the facts are that our first encounter with the external regions of the Solar system and beyond are

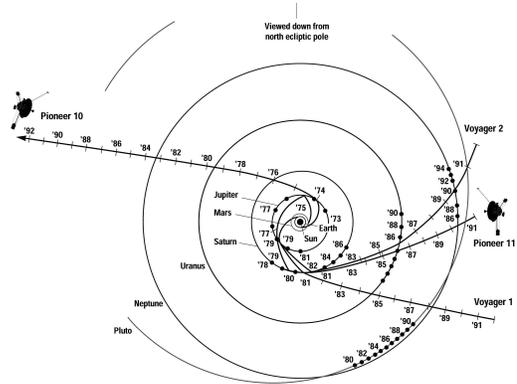


FIG. 1: The Pioneers trajectories in the Solar System [2].

showing the presence of something out there which affects the motion of bodies. It is interesting to mention that this effect started clearly as the probes passed Uranus orbit, still within the Solar system, and it has not being observed, so far, in the trajectories of the planets in that region, Neptune and Pluto.

Any number of explanations have being put forward to account for this effect, including dark energy (see for example [4]), quantum oscillations of the spacetime, branes ([5]) and you name it. (For other interesting alternatives see [6], [7] and [8]). Our point of view is try first to explain the phenomenon with local, everyday physics, and if this is not enough, then use other alternatives. Also, it is clear that dark energy contribution to the motion of bodies at local scales is very much smaller than the one detected at the Pioneers trajectories. Indeed, the effects of dark energy start to being noticeable only at galaxy cluster scales! (see, for instance, [9]). However, it is remarkable that, taking the accepted value for the cosmological constant, considering it as the source of the dark energy, $\Lambda = 3\Omega_{DE} \left(\frac{H_0}{c}\right)^2$, with $\Omega_{DE} = 0.7$, the ratio of dark energy density to the critical density of the Universe,

*Electronic address: jdo@astroscu.unam.mx, nunez@nucleares.unam.mx, zavala@ciqa.unam.mx

H_0 the Hubble constant today (we take $h = 0.7$ [10]), and c the speed of light, we get $\Lambda = 1.2h \times 10^{-56} \text{ cm}^{-2}$, and if we want to construct an acceleration associated with it, we obtain that $a_\Lambda = c^2 \sqrt{\Lambda} = 9.79 \times 10^{-8} \frac{\text{cm}}{\text{s}^2}$, a value close to the one observed on the Pioneers [11]. Nevertheless, as mentioned, the cosmological effects are negligible at Solar systems scales, so this is just a remarkable coincidence.

In the present work we develop a more local and common idea. The Solar system started from the proto-planetary cloud which, by gravitational collapse, formed the Sun and planets but some material remained in the form of small structures, tiny rocks, and dust, revolving around the Sun, forming belts. Those tiny rocks within the orbits of the planets, were ultimately swept by them, and the ones beyond form belts, two mayor regions beyond Neptune's orbit: The Kuiper belt, going from 30 UA to 70 UA, where it joints with the Öort cloud, which extends up to 4000 UA [12, 13], and internal belts within the orbits of the four major external planets. We study the gravitational pull generated by those belts on the objects moving within them and conclude that, if their density distribution goes as $\rho(r) \propto \frac{1}{r}$, a constant acceleration pull is produced, directed towards the center, which could account for the observed blue shift on the Pioneer probes.

As an illustration, let's take the simplest case of a solid spherical distribution of matter, an object moving within a media of a given density distribution $\rho(\vec{r})$, will be accelerated according to the following Newtonian law:

$$\vec{\nabla} \cdot \vec{a} = -4\pi G \rho(\vec{r}), \quad (2)$$

with G the gravitational constant, $G = 6.67 \times 10^{-8} \frac{\text{cm}^3}{\text{gr s}^2}$, and \vec{a} the acceleration of the object. Thus,

$$\oint_S \vec{a} \cdot \hat{n} dA = -4\pi G \int_V \rho(\vec{r}) d^3r, \quad (3)$$

with \hat{n} the normal vector to the surrounding surface to the volume V . Considering that the acceleration is only radial: $\vec{a} = a_P \hat{r}$, and taking a spherical shell of radius r , as the surface of integration, we get

$$a_P = -4\pi G \frac{1}{r^2} \int_0^r \rho(r') r'^2 dr', \quad (4)$$

from which is clear that, in order to have a constant acceleration, the density on the media must go as:

$$\rho(r) = \frac{\alpha}{r} \quad (5)$$

Moreover, we can compute the proportionality constant α , from the observed value of the Pioneers' acceleration; obtaining that:

$$\alpha = \frac{a_P}{2\pi G} = 0.21 \frac{\text{gr}}{\text{cm}^2} \quad (6)$$

And then, the density of the sphere needed to produce such an acceleration is, for example at 20 UA

where it's claimed that the anomaly starts: $\rho(20UA) = 7 \times 10^{-16} \frac{\text{gr}}{\text{cm}^3}$. However, this solid sphere is not the answer to the problem because it starts form $r = 0$, i.e. from the Sun, and the anomaly doesn't appear until 20 UA. The next possibility is a spherical shell starting at 20 UA with the same density distribution as in eq. (5). In this case the acceleration within the shell is not constant: for the region inside 20 UA the acceleration is zero, for the region inside the shell, we can calculate the acceleration by taking the difference between the influence of two spheres centered in the Sun; the smaller one with a radius $r_1 = 20$ UA and a mass M_1 produces a radial acceleration in a test particle located at $r > 20UA$ given by:

$$a_1 = -\frac{GM_1}{r^2} = -\frac{G2\pi\alpha r_1^2}{r^2} \quad (7)$$

The bigger sphere with radius r produces an acceleration whose magnitude is given by eq. (4): $a_2 = -2\pi G\alpha$; therefore, the acceleration inside the shell for a test particle is:

$$a_P = -2\pi G\alpha \left(1 - \frac{r_1^2}{r^2}\right). \quad (8)$$

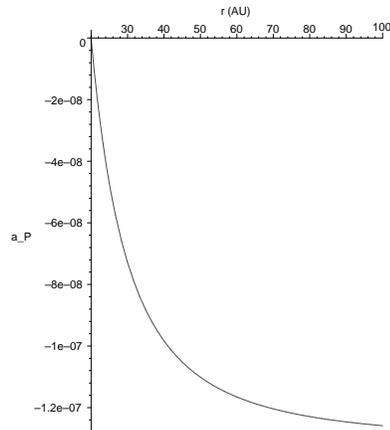


FIG. 2: Deacceleration caused in a test particle located at r due to a spherical shell starting from $r_1 = 20$ UA to $r_2 = 100$ UA, the acceleration is in units of $1 \times 10^{-8} \frac{\text{cm}}{\text{s}^2}$.

For this case the resulting acceleration is not independent of the radius but as r gets larger compared to r_1 the acceleration approaches a constant value. If we fix this value to eq. (1), then the value of α and $\rho(20UA)$ are the same as in the case of the solid sphere. Even when the acceleration is not constant within the sphere, this case can't be discarded because of the uncertainty in the value of the anomaly (around 15%, see eq. (1); see also fig. 2 of [1]); from figure 2 we can see that for an important region inside the sphere the calculated acceleration is in accordance with the anomaly taking into account the uncertainty in the observational data.

The third case is a cylindrical ring, with height h and borders at $r_1 = 20$ UA and $r_2 = 100$ UA and a surface density $\sigma(r) = \alpha_s/r$ (in this case, r is the radius in

cylindrical coordinates), and the solution is not as trivial as the two previous. In the appendix we describe the method used to calculate the acceleration of an infinitely thin disk ($h \rightarrow 0$), the result appears in figure (3). The following important features can be seen in the figure: the acceleration is almost constant between 40 UA and 80 UA with a change of less than 3%; near the edges of the ring a significant change in the acceleration takes place, this is due to the fact that \vec{a} is singular at the edges.

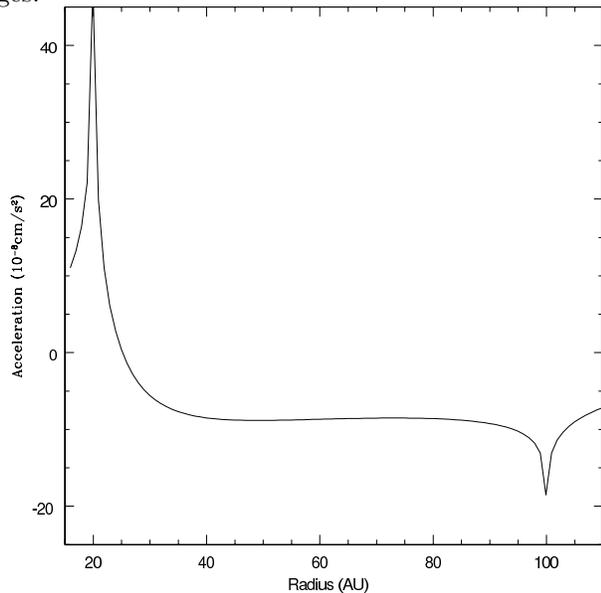


FIG. 3: Acceleration caused in a test particle located at r due to an infinitely thin disk going from $r_1 = 20$ UA to $r_2 = 100$ UA, the acceleration is in units of $1 \times 10^{-8} \frac{\text{cm}}{\text{s}^2}$.

The first feature of the results show us that a constant acceleration can be produced in a large region inside the ring by such a simple model. The region of this constancy can be extended backwards (from 40 UA to 20 UA as the Pioneer anomaly requires) by extending backwards the ring, it has to start then at $r_1 < 20$ UA. However it's important to mention that in order to give a better estimate of where this ring could start in reality, it's necessary to have more precise data than the one reported for the Pioneer anomaly, specially in the zone when the anomaly starts to be significant, because it's in this region where the larger uncertainty in the data is present (see figure 2 of [1]).

Regarding the second feature, we can say that the singularities can disappear taking a smooth but fast change in the surface density in the edges and not a cut-off as the one we used in the calculations, this is of course a more realistic case. It's also important to say at this point that when the 2D case we have chosen here is taken to 3D for a cylindrical ring of height h the results doesn't change significantly, the calculation has been made somewhere else [14] (see figures 5 and 8 of the paper), for this case, in the region of constant acceleration, the change in magnitude between the 2D and 3D models is almost null, the

main difference is near the edges where the spikes become less extreme for the 3D case.

In order to obtain the magnitude we need for the acceleration, the surface density at $r = 20$ UA of the thin ring must be, according to eq. (4) in the appendix:

$$\sigma(20UA) = 0.81 \frac{\text{gr}}{\text{cm}^2} \quad (9)$$

In the realistic case of 3D this ring becomes a cylindrical ring of height h that has a volumetric density given by $\rho = \sigma/h$, then for $r = 20$ UA, and assuming $h = 1$ UA we have: $\rho(20UA) = 5.4 \times 10^{-14} \frac{\text{gr}}{\text{cm}^3}$.

The total mass of a belt with such a density, considering a disk with a thickness of 1 UA, and ranging from 20 to 100 UA, gives $M_{\text{disk}} \approx 306M_{\oplus}$, two order of magnitudes larger than the current estimates on the Kuiper's belt mass [15].

A first possible explanation for this discrepancy, which is obtained considering that the belt is formed out of pure dust, could be the presence of tiny ice rocks and gas which have not been accounted for by those studies. In the same work, [15], the authors did consider, as in our present work, the gravitational influence of the Kuiper belt as a possible explanation to the acceleration seen on the Pioneers, but discarded that influence based on two reasons: First that the acceleration profile is not constant across the data range, (this is because they did not consider a density distribution going as $\frac{1}{r}$), and second that the total mass exceeded the current estimations, $M_{\text{disk}} = 0.3M_{\text{Earth}}$, for the dust in the Kuiper belt region from 30 to 100 UA.

Nevertheless, even though such mass distribution does explain the deacceleration observed in the Pioneers, it is unlikely that baryonic matter could be responsible for this effect, there would have to be a very high amount of small objects implying a large collision rate and optical and infrared signatures that have not been detected. Thus, in order to keep the model and maintain that the observed deacceleration is due to gravitational pull, the other possible candidate is the dark matter within the Solar system. The fact that the galaxies, including ours of course, are surrounded by dark matter halos is already a well established fact. The numerical simulations, namely the NFW, predicts the dark matter halo and give a well known density distribution [16]. Evaluating that density at the position of our Solar system, and considering that the gravitational influence of the Sun extends to the Öort cloud (around 7×10^4 UA), we see that there are about $500M_{\text{Earth}}$ of dark matter whose dynamics is dictated by the Sun. It is still an open question on how this Solar dark matter halo reacts to that influence. When the proto Solar system was being formed, the Solar dark matter halo was pulled inwards and it could be that the final configuration is an spherical shell or a belt in a region close to the Sun with properties similar to the ones described in the last paragraphs, giving in this way the mass needed to explain the observed deacceleration on both spacecrafts.

Given the properties required for the dark matter particles, the natural candidates are WIMPs (Weakly Interacting Massive Particles). Indeed, one most favoured WIMP to be the dark matter particle is the neutralino, predicted by one of the extensions of the standard model of particles, the Minimal Supersymmetric Standard Model (MSSM), which complies with the characteristics needed for a dark matter particle, namely mass of the order of 100GeV, making it a member of the Cold Dark Matter paradigm, very small cross section, $\equiv 10^{-9} \text{ GeV}^{-2}$, electrically neutral and stable ([17, 18]). Detection of the neutralino is nowadays an exciting field of research, neutralinos may be detected either directly through their interactions with ordinary matter or indirectly through their annihilation decay products. In the first case, several experiments on Earth are looking for signals associated with dark matter scattering with baryonic matter (nucleus recoils for example) ([19, 20]) with no conclusive result yet. The interpretation of this data depends on the spatial distribution of density and velocity for dark matter in the Earth neighborhood which is poorly known. The model presented here could serve as an attempt to give a prediction for the density distribution of dark matter using the data of the Pioneer's anomaly, but only for regions beyond 20UA, therefore it has no effect for direct detection experiments on Earth. Indirect detection is based on looking for signals of the remnants resulting from dark matter annihilation, the neutralino being a Majorana particle can annihilate with itself. The production of these remnants, positrons, photons, neutrinos, etc., can be enhanced in regions of high dark matter density such as the center of galaxies (for an excellent review on this topic see [17]).

The properties of the neutralino fit very well with the model presented in this work. Their presence influence the dynamics of the objects mostly through gravitational interaction and has a negligible dispersion with ordinary matter. And it is for this last reason that there could not be a measurable contribution to the Pioneer anomaly due to dispersion effects.

Also, it is important to point out that the belt would also affect Neptune's orbit, modifying its period in the following way (using the approximation of circular orbits):

$$T^2 = 4\pi^2 \frac{d_N^3}{GM_\odot} \left(\frac{1}{1 + \frac{a_P}{d_N^2} GM_\odot} \right), \quad (10)$$

that is, Neptune also feels the acceleration due to this matter distribution. Such acceleration changes Neptune's period in 0.29 seconds *per* period, implying a shift in Neptune's center of mass of 1.62 kilometers after each

revolution. It is a very tiny quantity but future probes could be designed in such a way as to be able to measure these effects.

The radial density profile (going as $1/r$) needed to explain a constant acceleration towards the Sun, can be explained by models of Solar System formation (in the case of baryonic matter only). In [21], the authors made an analysis of the dust population in the outer solar system region; making a computational approach (see fig. 4) considering source objects from the Jupiter-family comets and the Kuiper Belt. They obtained a radial distribution of dust for the region between 20 to 70 UA, that has a sharp peak near 20 UA and from there falls like $\frac{1}{r}$, just as our model predicts. After that, it has a smaller peak near 50 UA (see fig. 5 of this paper that is a reproduction of fig. 5 of [21]). This result is encouraging for our model but given the large amount of mass needed to reproduce the actual deacceleration measured by the Pioneers, it would have to be remade with the inclusion of dark matter to see if the density profile remains the same with the required amount of mass.

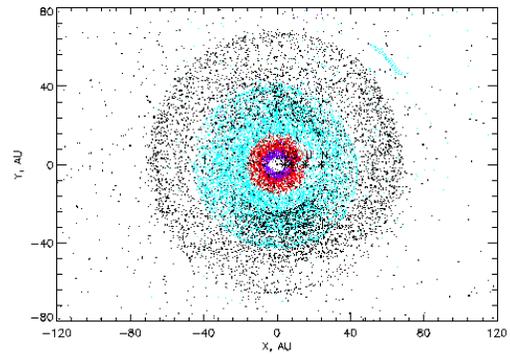


FIG. 4: Numerical simulation of the density distribution of matter at the Solar system in the ecliptic plane [21].

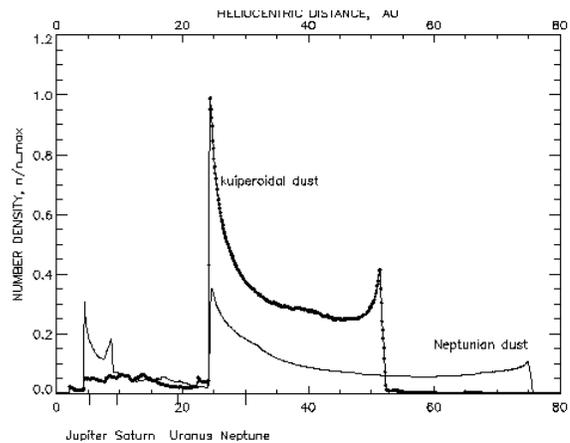


FIG. 5: Numerical simulation of the radial density distribution of matter at the Solar system in the ecliptic plane [21].

Following this scheme, the mass density distribution

that appears after the orbit of Uranus, is a result of an inward transport of material from the Kuiper Belt into this orbit due mainly to the gravitational influence of Neptune and Uranus. If the radial distribution goes in fact like $\frac{1}{r}$ (at least between 20 to 100 UA), as the kind of models presented by Gor'kavyi *et al.* [21], and with a total mass of $306M_{\oplus}$, then the so called Pioneer anomaly can be explained by the gravitational force that this distribution produce on every test particle inside it, as was shown previously, making the observed blue shift not an anomaly, but a natural consequence of the gravitational pull due to the material present in these regions.

Indeed, the actual dark matter distribution within the Solar System is still an open question. A numerical simulation has to be performed to give an appropriate description on this subject, taking into account the effects of the formation and evolution of the Solar system on the dark matter Solar halo. The outcome of these analysis would determine whether or not the matter distribution proposed in the present model is plausible. In particular, it is important to determine the influence due to the outer planets. This would answer the question if a gap of 20 AU starting from the Sun and then a region with density falling like $1/r$ that has not been trashed away by Neptune is possible. The gravitational influence of the outer planets on the baryonic matter has formed Solar belts in these regions, which probably has a matter distribution similar to the one required for dark matter.

Also, we want to comment that while this work was being finished, we learned that Nieto *et. al* [2] were also considering the effect of the material in the Kuiper belt on the space probes. However, in their approach they studied the drag force of this material on the probes, obtaining a different density distribution, mainly that the density should be constant in order to explain the constant acceleration pull observed on the Pioneers.

Finally, we remark that it is very important to perform new and more detailed studies on the density distribution and total mass of the matter belts in the regions beyond Uranus' orbit to verify, or discard, the explanation for the Pioneer acceleration presented in this work. **In order to do so, we consider it is necessary to send several probes to the outer regions of the Solar System, like the TAU Probe (Thousand Astronomical Unit), in several directions, including those perpendicular to the Solar system plane, and equipped with the needed instruments in order to determine, via the deacceleration effects, the nature and characteristics of the phenomenon which causes these effects, and then see if it is compatible with a dark matter distribution.**

We acknowledge that the required amount of mass needed to reproduce the magnitude of the Pioneers deacceleration is indeed very large considering the actual estimates of mass in such a region, however the idea proposed

in this work is appealing due to it's simplicity and has already generated some attention in the scientific community (see for example the works that have appeared while this work was being considered for publication, [14, 22]). We think that the proposal described above considering dark matter as a possible explanation for the unaccounted mass is worth to follow and, as we mentioned above, it would be interesting to perform numerical simulations for the Solar System formation which include dark matter in order to study its final distribution.

This work was partially supported by the grants DGAPA-UNAM IN113002, IN122002. JZ also acknowledges the scholarship support by CONACyT and DGEP. We want to thank our colleagues in the Gravitational department of the Nuclear Science Institute for much encouragement and helpful discussions on the present idea. We also thank the anonymous referee of the present work for several remarks which contributed to better sketch the problems and advantages of the model proposed.

I. APPENDIX

In the present appendix we will describe the procedure to obtain the exact result for the acceleration caused by a distribution of matter in a infinitely thin ring going from $r_1 = 20$ UA to $r_2 = 100$ UA with a surface density distribution: $\sigma(\vec{r}) = \alpha_s/r$.

The method is straightforward, using Newton's law of gravitation, we calculate the acceleration caused by the distribution of matter on a test particle; the case of interest for us, is only when the position of the test particle (the Pioneers) is on the plane of the disk, therefore we need only two dimensions to do the calculations.

An infinitesimal element of mass dm_i and area ds_i , produces an acceleration $d\vec{a}_i$ in the test particle placed in the position \vec{r} given by:

$$d\vec{a}_i(\vec{r}) = -\frac{Gdm_i(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} = -\frac{G\sigma(\vec{r}_i)ds_i(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} \quad (11)$$

where $\sigma(\vec{r}_i)$ is the surface density of the mass element. Then all we have to do is to sum the contribution of all the mass elements in the ring. To do so, we use the following simplifications: if we assume that $\sigma(\vec{r}_i) = \sigma(r_i)$, then the symmetry of the problem allow us to put, choosing a cartesian coordinate system, $\vec{r} = x\hat{e}_x$ and $\vec{r}_i = r_i(\cos\theta_i\hat{e}_x + \sin\theta_i\hat{e}_y)$, where r_i and θ_i are the coordinates of the mass element dm_i in polar coordinates. The total contribution to the acceleration in the direction of \hat{e}_y is zero given the symmetry of the ring; for the component in the direction \hat{e}_x the integral over the whole distribution (θ going from 0 to 2π and r_i going from r_1 to r_2) gives the radial acceleration caused on the test particle as a function of it's radial position. To perform these calculations we made a numerical program in fortran 77. The resulting acceleration as a function of the position r

of the test particle, appears in fig. (4) of the paper, the value of α_s was adjusted to obtain the magnitude of the acceleration in the Pioneer anomaly, the value is:

$$\alpha_s = 2.43 \times 10^{14} \frac{\text{gF}}{\text{cm}} \quad (12)$$

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