New Problems and Solutions in Basic University Teaching

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Abstract

In this paper we will examine some of the problems and difficulties in modern university teaching and how these difficulties were overcome and the problems were solved.

Because the syllabus in Danish (and other European) high schools has been substantially weakened over the last decade and especially since 2002, the university students have experienced new serious problems in their first year learning. This has had the consequence that many students dropped out of their studies and that many others failed at their first year exams. This was not acceptable and therefore something had to be done.

In this paper we will only deal with the first year courses in mathematics for economists at The University of Copenhagen, and it is told how the teaching in mathematics was changed during the last two years such that the rate of failure dropped considerably. Many resources were spent to reach this aim, and it was very important to engage and activate the students and to give them more personal excitement, such that they also obtained higher ability for studies on their own hand.

The result of the new way of teaching was remarkable and the aim of a much lower rate of failure was reached. Furthermore the students saw mathematics not only as a scientific tool useful in modern economic theory but also as a cultural and academic discipline with a long and interesting historical development. The students got much more knowledge and they obtained a personal attitude to their mathematical education.

Both for the students and for the Department of Economics this was a success.

Introduction

During the last 10 years Danish university teaching has been confronted with several new challenges.

The classic well known university study tradition established by Immanuel Kant and Wilhelm von Humboldt and introduced in Denmark by the famous philologist Johan Nicolai Madvig (1804-1886) has now been replaced with other ideals (Flexner 1930, Gibbons 1994, Humboldt 1792, Huxley 1876, Jaspers 1923 and 1946, Kant 1798, Korsgaard 2004, Petersen 1993). In the classic tradition "the free academic study" with no narrow limitations and with an individually high degree of intellectual absorption the students could form their own academic personality. In this tradition famous Danish scientists such as Niels Bohr, Harald Bohr, August Krogh, Johan Ludvig William Valdemar Jensen, and Børge Jessen were taught during their studies at the University of Copenhagen.

In the first half of the 20th century only a few per cent of the Danish population were educated at high schools and at the universities. The high school teaching and the university studies were for the few privileged, but in the 1960ies this changed considerably. Many more young people got a high school exam on a relatively high academic level and many more were matriculated at the Danish universities and got an academic degree (Damberg 2006). Still the university teaching was based on the tradition from Kant, von Humboldt, and Madvig.

After 1990 the high school teaching was changed in several steps. First of all many new subjects, that were not academic and were not taught at the universities, were introduced in high school teaching and at the same time the professional level in many subjects such as mathematics, physics

and linguistics was reduced in a disastrous way (Damberg 2006). The students at high school were no longer trained sufficiently in basic academic and scientific methods, and when they started their university studies they got so many problems in their learning, that many dropped out very fast and many others were not able to pass their first year university exams. These problems increased substantially after the year 2000.

Also after 2000, and especially after 2003 when a new legislation for the Danish universities was decided by the Danish parliament, the classic ideals for university studies have been changed. The free study has been replaced with short well determined courses and many exams, such that the students should be able to obtain a university degree faster than before. But this also implies that the university education must be very efficient with very definite courses such that it has become difficult for the students to obtain a deeper and abstract understanding of their syllabus. At the same time the Danish universities were transformed from classic universities into mass universities with many students. The classic ideals were replaced with other ideals based on management principles. This is a development that is also known from other western countries (Damberg 2006, Käufer 2000, and Olesen 2007).

The very increasing gap between the aim of the university education on a high academic level and the poor knowledge and the bad study ability the students now have from high school has become a more and more substantial problem for basic university teaching. In 2007 approximately 30 percent of the first year students at The Department of Economics were not able to pass their written exam in mathematics. This percentage has been only around 15 – 20 before 2003 and therefore something to prevent this catastrophic development had to be done (Olesen 2007). In this paper it is told how we managed this big problem in 2008, and it is also told how we are building up a new way of substantial academic teaching with future perspective for first year students.

Mathematics At The Department Of Economics

Mathematics is taught at several institutes at The University of Copenhagen. The Institute of Mathematics at The Faculty of Natural Sciences has teaching and research on pure mathematics as its main purpose. But there are also other university institutes in Copenhagen where both research and teaching on mathematics – especially applied mathematics and how to use mathematics in other fields – take place. One of these institutes is The Department of Economics at The Faculty of Social Sciences, another is The Institute of Analytic Chemistry at The Faculty of Pharmaceutical Sciences.

Since 1970 all students of economics are forced to pass a first year written exam on mathematics. This course has changed to some extent over this period of more than 35 years, but the main topics of the course are still fundamental linear algebra and classic mathematical analysis. The course at The Department of Economics is being taught over two terms and has since 1990 been consisting of a lecture of two hours and two or three hours of class teaching, such that the students end up with a mathematical standard that is equivalent to the standard the first year students obtain at The Institute of Mathematics (Olesen 2007).

Since high school teaching on both mathematics and also many other subjects was weakened considerably in Denmark at the beginning of the 21st century the university teaching on mathematics at The Department of Economics had to be changed.

In 2002 the so called "Standard attempt" (Da. "Standardforsøg") was introduced in Danish high school teaching (Damberg 2006, Olesen 2007). In mathematics approximately 30 per cent of the syllabus was taken away and replaced by some interdisciplinary projects. This implied that the first year students didn't have the mathematical capability they usually had had before, and therefore it was decided that the first year teaching on mathematics at The Department of Economics should be split up into two separate courses, "Mathematics 1" and "Mathematics 2" – both lasting one term. The mathematical requirements as a whole should be changed and reduced a little. Furthermore, for the best students, a new more advanced and optional course in mathematical analysis was established (Olesen 2007).

Since high school teaching in Denmark is lasting through three years the new educational system at The Department of Economics was introduced at the beginning of the autumn term in 2005, and in June 2006 (after one academic year of teaching) the first students passed their exams on mathematics according to this system.

In the course "Mathematics 1", which is an introductory course, the following items are taught (Olesen 2007):

1. Mathematical notation and terminology, elementary mathematical logic and the theory of sets.
2. Theory of functions, calculus concerning functions of one real variable, elasticity of functions of one real variable, elementary calculus and integration theory.
3. Implicit functions and implicit differentiation.
4. Real sequences and difference equations of the first order.
5. Infinite series and convergence criteria, annuities and geometric series.
6. Introduction to the theory on functions of several real variables and partial derivatives.
7. Applications of mathematics within other subjects.

After having attended this course in their first term, the students must pass an oral exam. The oral examination lasts approximately 30 minutes.

In the course "Mathematics 2" the students are taught the following items (Olesen 2007):

1. Basic linear algebra: Vectors, vector spaces, inner products, and subspaces.
2. Matrices in general, echelon matrices, non-singular matrices, and operational matrices.
3. Solving systems of linear equations, inverting non-singular matrices.
4. Determinants of square matrices.
5. The matrix equation of a linear mapping and the eigenvalue problem for square matrices.
6. Spectral theory for symmetric matrices and quadratic forms.
7. Solving ordinary differential equations of the first order.
8. Finding optimal points for functions of several real variables, both with and without constraints.
9. Multiple integrals.

After having attended the course "Mathematics 2", the students must pass a written exam that lasts 4 hours. The examination requirements consist of all the items of both courses, "Mathematics 1" and "Mathematics 2". There are 5 problems to be solved: 2 in linear algebra and 3 in mathematical analysis. Assessing the problems 50 per cent are given to the problems in linear algebra and 50 per cent to the problems in mathematical analysis. To pass the written exam the students must have a score of at least 50 per cent (Olesen 2007).

Evaluation Creteria

During the period 1971 – 2007 all university exams in Denmark were assessed using the following grades: 13, 11, 10, 9, 8, 7, 6 (which were all passing grades), and 5, 03, and 00 (which were all failure grades). The top grades were 10, 11, and 13, where 13 was an extremely outstanding top grade that was given only rarely. The middle grades were 7, 8, and 9, and the grade 6 was given for just passing the exam. The failure grades were 00, 03 and 5, where 00 was only given if almost nothing was correct (Olesen 2007).

Between the percentage of scores and the grades the following scale of equivalence was used:

Percentage	**0 – 39**	**40 – 49**	**50 – 59**	**60 – 67**	**68 – 75**	**76 – 83**	**84 – 91**	**92 – 100**
Grade	**00 and 03**	**5**	**6**	**7**	**8**	**9**	**10**	**11 and 13**

In August 2006 the Ministry of Education and the Ministry of Research decided to introduce a new scale of grading that had only 7 different grades and that was comparable to the Anglo-Saxon scale and the international ECTS-scale (Olesen 2008).

From September 2007 all Danish university exams should be assessed using this new scale consisting of the grades: 12, 10, 7, 4, 02 (which are passing grades), and 00, and – 3 (which are failure grades). The top grades are 12 and 10, the middle grades are 4 and 7, and the grade 02 is given for just passing the exam. The failure grades are 00 and – 3, where – 3 is only given if almost nothing is correct. Compared to the ECTS-scale we have that 12 = A, 10 = B, 7 = C, 4 = D, 02 = E, 00 = Fx, and – 3 = F.

Between the percentage of scores and the new (international) grades the following scale of equivalence is used:

Percentage	**0 – 10**	**11 – 49**	**50 – 59**	**60 – 67**	**68 – 83**	**84 – 91**	**92 – 100**
Grade	**- 3**	**00**	**02**	**4**	**7**	**10**	**12**

The Exam Results In Summer 2006 and In Summer 2007

After having assessed all the written exams in "Mathematics 2" in summer 2006 and in summer 2007 the following grades were given (Olesen 2007):

Grade	**13**	**11**	**10**	**9**	**8**	**7**	**6**	**5**	**03**	**00**
Numbers in 2006	**3**	**11**	**24**	**21**	**29**	**18**	**25**	**6**	**31**	**3**
Numbers in 2007	**8**	**20**	**18**	**15**	**17**	**18**	**36**	**23**	**44**	**3**

In summer 2006 171 students attended the written exam in "Mathematics 2", and 131 of them passed the exam. So 40 failed. The rate of failure was 23.4 per cent. The average grade for all students was 7.1, and the average grade for the students who passed the exam was 8.4.

This was not in particular notable. The exam result was satisfactory and it was almost the same as we had seen during the forgoing years. But in June 2007 things turned out to be different. Now 202 students were examined in "Mathematics 2". So there were many more students than the year before. But the number of students who passed the exam was almost the same: Only 132. Hence 70 students failed. The rate of failure increased to 34.7 per cent and that many students failing this exam had never been experienced before. The average grade for all students was 6.8 and the average grade for the student who passed the exam was 8.5. This development was disastrous and something radical had to be done. But first of all it was necessary to analyze this terrible exam result a little closer.

In stead of looking at the absolute numbers of students having the different grades it might (as a beginning) be useful to look at the percentages and compare the different rates from 2006 and 2007. This is shown in the following table:

Grade	**13**	**11**	**10**	**9**	**8**	**7**	**6**	**5**	**03**	**00**
Percentage in 2006	**1.8**	**6.4**	**14.0**	**12.2**	**17.0**	**10.5**	**14.6**	**3.5**	**18.1**	**1.8**
Percentage in 2007	**4.0**	**9.9**	**8.9**	**7.4**	**8.4**	**8.9**	**17.8**	**11.4**	**21.8**	**1.5**

This table shows that the rate of top grades were nearly constant (22.2 percent in 2006 and 22.8 percent in 2007). For the group in the middle there was a dramatic decrease from 39.7 per cent in 2006 to 24.7 per cent in 2007. For the poorest grades of failure (00 og 03) there was an increase from 19.9 per cent in 2006 to 23.0 per cent in 2007, and the very big increase for students failing the exam can be seen under the grade 5 with 3.5 per cent in 2006 and 11.4 per cent in 2007 (Olesen 2007).

Analyzing the Exam Result From 2007

In September and October 2007 all these written exams were analyzed very carefully. The question was: What happened and what was the deeper cause when a student failed? Could a simple pattern be found?

To find a possible answer to this interesting question the following issues in the written exams were set up: Text and symbols, understanding of numbers, elementary concepts, elementary calculus (all these issues coming from high school syllabus), linear algebra, elementary university analysis, and advanced university analysis (all these issues have only been taught at the university). And how the students had managed to handle these different issues during their exams were divided into three categories: "Good", "Middle" and "Bad".

The result from this analysis is seen in the following table. In the three columns we have the absolute number of students in the different categories and in parenthesis just behind we see this number as a percentage of the total number of students:

Total number of students: 202	Good	Middle	Bad
Text and symbols	**27 (13)**	**127 (63)**	**48 (24)**
Understanding of numbers	**43 (21)**	**115 (57)**	**44 (22)**
Elementary concepts	**32 (16)**	**85 (42)**	**85 (42)**
Elementary calculus	**59 (29)**	**41 (20)**	**102 (50)**
Linear algebra	**57 (28)**	**78 (39)**	**67 (33)**
Elementary university analysis	**62 (31)**	**96 (48)**	**44 (22)**
Advanced university analysis	**56 (28)**	**34 (17)**	**112 (55)**

>From this table we see that between one fifth and one fourth of all the students had severe problems writing a mathematical and logical text and understanding simple calculating with real numbers and fractions. Furthermore 42 per cent didn't know very much about elementary high school mathematics and 50 per cent were not able to use elementary calculus. This was a big surprise and we had never seen things like this before. Many students had a very poor mathematical knowledge from high school and no doubt the cause of this terrible development was the introduction of the "Standard attempt" where the high school syllabus was reduced with approximately 30 per cent and many central topics were marginalized. When the students had such a poor mathematical basis they were handicapped when they attended their university exams. The table also shows that many students had difficulties solving advanced university mathematical problems, which is not very surprising, but on the contrary they managed the other fields in university mathematics much better (Olesen 2007).

From this we see that a very important reason for failure is poor knowledge from high school and the main reason for passing the exam in "Mathematics 2" is good knowledge in linear algebra and elementary university analysis. If the rate of failure should decrease considerably (and that was necessary) we had to teach in high school mathematics at the beginning of the university courses and also we had to exploit the students' new special competences in group working that was now coming in front in Danish high school teaching (Damberg 2006).

New Solutions in Basic University Teaching

Soon after the exam in "Mathematics 2" had taken place in June 2007 and already before the analysis was done it was decided to change the teaching on mathematics at The Department of Economics. First of all we wanted the students to be more active during their classes (Olesen 2008). To reach this aim we had to exploit the students' new competences in group working immediately. This implied that the class teaching was reorganized. Until then the teaching assistants had a great amount of freedom in their teaching. The only request was that all the exercises mentioned in the students' curriculum should be solved and discussed, but now we introduced a specific schedule for all classes. The students were forced to work in groups consisting of 3 - 4 students. All exercises from the curriculum must be solved (or maybe rather must be tried to be solved) and discussed in the groups before classes are given. Here the easiest exercises are then presented during one lesson by the students, proofs are discussed and trained during the next lesson under guidance of the teaching assistant, and at last during the third lesson the more difficult exercises are presented on the blackboard by the teaching assistant.

Was this a success? Did this reorganizing of the classes have any impact? The answer is absolutely "yes". When the students attended the oral exam in "Mathematics 1" in January 2008 only few students failed and therefore we wanted to go further and give the students more teaching during their next term in spring 2008.

From high school they had even worse mathematical knowledge than the students had the year before. This was an obvious fact and during the lessons on mathematics in autumn 2007 it had been quite clear, that the first year students didn't understand how to prove a theorem, and they didn't understand even simple definitions and concepts. What we a few years ago could explain during a 2 hour lecture we now need 3 hours to explain such that the students could understand most of the definitions, concepts and mathematical proofs, and that they afterwards were able to solve mathematical problems on their own.

What had to be done was quite clear: In the spring term of 2008 we introduced 3 hour lectures on mathematics and still we forced the students to work together in small groups during their classes. Furthermore we established special study forums in which they could obtain professional guidance to solve mathematical problems and exercises, and they were trained in using the methods of mathematical research, such that they obtained some basic knowledge from the theory of science which is very important for their future studies (Olesen 2007). A great part of the ideological basis of the teaching was still taken from the classic university ideals.

During the spring term 2008 we had already noticed that this change of the first year studies at The Department of Economics had a considerable impact on the students' study activity and on their understanding of university mathematics. But this change required many more resources than before and we hoped that this investment would be paid back, since we supposed that many more students now would be able to pass their exams (both in "Mathematics 2" and in other subjects).

The Result of The Exam In Summer 2008

On Tuesday 10 June 2008 the students had their written exam in "Mathematics 2". It was rather exciting to assess the 208 exams, and the result was surprisingly positive which is seen in the following table, now using the new international scale of grades (Olesen 2008):

Grade	**- 3**	**00**	**02**	**4**	**7**	**10**	**12**
Number of students in 2008	**6**	**46**	**36**	**29**	**33**	**22**	**36**

Of the 208 students attending the written exam in "Mathematics 2" 156 passed and 52 failed. The rate of failure was only 25 per cent, so there was a considerable decrease of 9.7 per cent in this rate compared to 2007. Let us now compare the grades given in 2007 and 2008 which is possible since we group all grades in the categories "Top", "Middle", "Just passed", and "Failed" (mentioned earlier in this article) and since the number of students is nearly unchanged from 202 in 2007 to 208 in 2008. This comparison is seen in the following table:

Grade	Top	Middle	Just passed	Failed
Number of students in 2007	**46**	**50**	**36**	**70**
Number of students in 2008	**58**	**62**	**36**	**52**

Of course it would be better to compare the percentages in different categories in stead of looking at the absolute numbers, and if we do this we find the following table:

Grade	Top	Middle	Just passed	Failed
Percentage of students in 2007	**22.8**	**24.8**	**17.8**	**34.7**
Percentage of students in 2008	**27.9**	**29.8**	**17.3**	**25.0**

We notice that apart from the rate of failure has decreased the rates of top and middle grades have increased and the rate of "just passed" exams is almost constant.

Now it might be interesting to analyze the result in 2008 in almost the same way as we did with the result from 2007. So once again we consider the three categories "Good", "Middle", and "Bad" and the following issues: Text and symbols, Elementary concepts, Elementary calculus (all included in high school syllabus), Linear algebra, Elementary university analysis, Multiple integrals, and Differential equations (all these issues from university teaching).

The result from this analysis is seen in the following table. As we did earlier: In the three columns we have the absolute number of students in the different categories and in parenthesis just behind we see this number in per cent of the total number of students (Olesen 2008):

Total number of students: 208	Good	Middle	Bad
Text and symbols	**17 (8)**	**172 (83)**	**19 (9)**
Elementary concepts	**50 (24)**	**70 (34)**	**88 (42)**
Elementary calculus	**49 (24)**	**77 (37)**	**82 (39)**
Linear algebra	**70 (34)**	**115 (55)**	**23 (11)**
Elementary university analysis	**62 (30)**	**119 (57)**	**27 (13)**
Multiple integrals	**69 (33)**	**10 (5)**	**129 (62)**
Differential equations	**102 (49)**	**30 (14)**	**76 (37)**

>From this table we see that 42 per cent of the students had very severe problems with their elementary mathematical concepts which should have been well known knowledge from high school. And 39 per cent are not acquainted with simple high school calculus. On the other hand the students were able to learn a lot of university mathematics. This tells us that Danish high school teaching is really bad and that the students don't learn enough elementary mathematics which is very important for their further studies. The poor high school teaching builds up a terrible barrier

that has to be penetrated if the students should be prepared for university studies. We also see that at the Department of Economics we have been able to break down this barrier – at least to some extent – but to reach this aim we had to spend 50 per cent more resources on the lectures. To repair on what high school education should have done is quite expensive but the investment is good because more students pass their first year university exam and even with higher grades.

Conclusion—New Elements in Basic University Teaching

In this paper we have seen that the Danish high school teaching doesn't prepare the students sufficiently for university studies, but this is, however, not only a Danish phenomenon. In Norway (Described by professor Knut Sydsæter, Oslo) and in Sweden (Described by professor Anders Borglin, Lund, Scania) this disastrous development is even worse than it is in Denmark and also in other western countries this problem is seen very clearly.

We have now empirically demonstrated that it is possible to change the basic university teaching such that more students pass their first year exam without decreasing the academic standard, but we have to invest many more resources in teaching and we have to reorganize both the lectures and the classes to reach our aims.

Since we cannot expect an improvement in high school teaching in the near future we have to develop our basic university teaching even more and this development must follow the same fundamental principles we have used during spring term 2008. Therefore we will do the following:

(A) The lectures will in both terms last three hours each week. Then it is possible to teach in elementary mathematical concepts, in mathematical terminology, in basic set theory and in proof techniques. Also it is possible to teach in theory of science and in mathematics as a cultural subject.
(B) The classes (still lasting three hours a week) will continuously be organized in the same way as they were last academic year. The students are forced to work together in groups such that they feel responsible for each other.
(C) In both terms there will be 4 evening sessions lasting 3 hours where the students under professional guidance can work concentrated with more complicated mathematical problems and with more or less advanced applications of mathematical modeling.

To teach continuously in the cultural and historical aspects of mathematics is very important – also at our department – actually at every department where mathematics to some extent is included in the studies.

At Danish high schools it is now common to consider mathematics just as a useful tool for solving numerical problems using a strong electronic calculator. That mathematics first of all is a cultural subject and a philosophical subject – and in fact a highly developed science - is not pointed out sufficiently to the students at high school. Therefore at the Department of Economics we want to teach mathematical method and mathematics both as a cultural and as a philosophical subject throughout the two first year terms (using the textbook 12, Olesen 2007). Also we will teach on the history of mathematics such that the students will obtain a better understanding of the scientific development. It is our hope that these initiatives will enrich our university teaching and will increase the students' engagement in their learning process and improve their excitement for university studies at our department in general.
Long time ago, in 1798, the famous German philosopher and natural scientist Immanuel Kant wrote his interesting work "Der Streit der Fakultäten" (The Conflict between the Faculties, Kant 1798). In

his book Kant considered research and teaching in the philosophical and natural sciences as a central unity at the universities. Six years earlier the Prussian politician and administrator Wilhelm von Humboldt had written his famous philosophical book entitled "Theorie der Bildung des Menschen" (Humboldt 1792). In this interesting book he introduced the concept called "Bildung". The German word "Bildung" is in Danish "Dannelse", but in English there is no word meaning exactly the same. Maybe we could say "Educated". But anyway, the main point is that the classic universities at the beginning of the 19th century should research and teach in such a way that the students got so much academic and philosophical knowledge that they were able to interpret all fields in their specific sciences and that they acted as educated persons. That is the content of "Bildung" (Humboldt 1792). This point of view was later supported by the British biologist Thomas Huxley (Huxley 1876), the Danish philologist Johan Nicolai Madvig (Petersen 1993), the American researcher Abraham Flexner (Flexner 1930), and the German philosopher Karl Jaspers (Jaspers 1923 and 1946).

In modern mass university activities it is very difficult (or rather almost impossible) to reach that specific academic standard where the candidates are leaving the universities as specifically educated persons in the classic sense. The courses nowadays are short and the studies have even got an upper time limit such that all time schedules are extremely tightened up. But we have experienced that many students really like to hear something about their subjects seen in a larger and philosophical context. For example one day the lecture started with the so called "Königsberg Problem" from 1735 when people in the old eastern Prussian town of Königsberg were asking if it was possible to make a continuous route passing each of the seven bridges crossing the river Pregel only once. Five of these bridges connecting the island of Kneiphof in the middle of the river. The answer of this riddle was given by the famous Swiss mathematician Leonhard Euler in 1736, and he demonstrated that there doesn't exist any such continuous route (Hopkins 2004, Hopkins 2007, Textbook 12 Olesen 2007). Apparently this had nothing to do with mathematics for economists but when the students also were told that Euler's solution of the specific "Königsberg Problem" was a general method and that this method later has been developed to the modern and advanced theory of graphs that plays a central role in applied mathematics, they got a better understanding of what they were studying and they saw that mathematics also has some important cultural aspects. They became engaged and they became excited and began studying more on their own hand. So they were also personally activated. Actually by the use of storytelling as a pedagogical tool the students had obtained "Bildung" in the old sense.

Therefore some important elements from the old ideals of university teaching and research are now in a new and direct form brought into the teaching at the modern mass university already at the basic first year curriculum.

We believe that these philosophical and cultural aspects of mathematics will give our students at the Department of Economics a very positive attitude to their studies such that they easier than before will understand the abstract mathematical theories they have to learn and apply in other subjects. And furthermore we believe than "Bildung" will also give the students a crucial basis for studying on their own hand such that they are able to supply their knowledge any time in the future.

References

1. Damberg, Erik et al.: *Gymnasiepædagogik.* Hans Reitzels Forlag 2006.
2. Flexner, Abraham: *Universities: American, English, German*. 1930.
3. Gibbons, Michael: *The New Production of Knowledge*. 1994.

4. Hopkins, Brian and Robin J. Wilson: *The Truth about Königsberg*. College Math. Journal. Vol. 55 (2004).
5. Hopkins, Brian and Robin J. Wilson: *The Truth about Königsberg. The Genius of Euler*. The MAA Tercentenary Euler Celebration 2007.
6. Humboldt, Wilhelm von: *Theorie der Bildung des Menschen*. 1792.
7. Huxley, Thomas: *Address on University Education*. 1876.
8. Jaspers, Karl: *Die Idee der Universität*. 1923, 1946.
9. Kant Immanuel: *Der Streit der Fakultäten*. 1798.
10. Korsgaard, Ole: *Kampen om folket*. Gyldendal 2004.
11. Käufer, Katrin & Claus Otto Scharmer: *Universität als Schauplatz für den unternehmenden Menschen. Hochschulen als "Landestation" für das In-die-Welt-Kommen des Neuen.* Rainer Hampp Verlag 2000.
12. Olesen, Mogens Noergaard: *Introduktion til sandsynlighedsregning og statistik.* Forlaget Nautilus 2007.
13. Olesen, Mogens Noergaard: *Undersøgelse af eksamensbesvarelserne fra juni 2007*. Københavns Universitets Økonomiske Institut 2007.
14. Olesen, Mogens Noergaard: *Undersøgelse af eksamensbesvarelserne fra juni 2008.* Københavns Universitets Økonomiske Institut 2008.
15. Petersen, Niels: *Københavns Universitet 1848 – 1902*. Københavns Universitet 1479 – 1979 II. G. E. C. Gads Forlag 1993.

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