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Paired Structures and other opposites-based models

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Abstract

In this paper we present a new class of fuzzy sets, *paired fuzzy sets*, that tries to overcome any conflict between families of fuzzy sets that share a main characteristic: that they are generated from two basic opposite fuzzy sets. Hence, the first issue is to formalize the notion of *opposition,* that we will assume dependent on a specific *negation,* previously determined. In this way we can define a paired fuzzy set as a couple of opposite valuation fuzzy sets. Then we shall explore what kind of new valuation fuzzy sets can be generated from the semantic tension between those two *poles*, leading to a more complex valuation structure that still keeps the essence of being *paired*. In this way several neutral fuzzy sets can appear, in particular *indeterminacy*, *ambivalence* and *conflict*. Two consequences are then presented: on one hand, we will show how Atanassov´s Intuitionistic Fuzzy Sets can be viewed as a particular *paired structure* when the classical fuzzy *negation* is considered; on the other hand, the relationship of this model with bipolarity is reconsidered from our paired view.

Keywords: Intuitionistic fuzzy sets, bipolar fuzzy sets, paired fuzzy sets.

1. Introduction

Knowledge acquisition indeed is an extremely complex issue where many scientific fields interact. Let simply point out here the relevance of concept representation in our knowledge process (see, e.g., [12], [21]). Our brain is able to produce concepts that mean a compact, reliable and flexible representation of reality, and this representation is the basis for an efficient decision making process, and perhaps more important, the basis for an efficient communication language, when put into words.

But most concepts we use are complex in nature, far for been binary (*hold/do not hold*), and most probably with no associated *objective* measure. Most of the time, in order to understand a concept we need to explore related concept. Only taking into account surrounding concepts we can capture the borders of the concept we are considering, understand what in fact are transition zones from one concept to the other.

Of course concepts in a complex reality do not have a unique related concept. But indeed our knowledge use to start by putting a single concept in front of the concept under study. In Psychology, for example, the importance of bipolar reasoning in human activity has been well stated (see [18], but also [6], [8][7]). But it is relevant to observe that particularly in this context the semantic bipolar scale *positive/negative* comes with a *neutral* valuation stage.

Within fuzzy sets theory we can find several models that fit into the above approach. For example, Atanassov's fuzzy sets (see [3] and [4]) were originally presented in terms of a concept and its negation, allowing some indeterminacy state. And Dubois and Prade offered a unifying view of three kinds of bipolarities (see [8], [9], [10]).

Our proposal here is to focus on modelling how the first pair of concepts is built. We consider that the natural process when a concept is considered is to introduce its negation. And taking into account such a negation, search for somehow *opposite* concepts. Hence, we shall first formalize what an opposite fuzzy set is. The semantic tension between such a pair of opposite fuzzy sets, to be named *paired fuzzy sets,* will bring additional valuation states, that meanwhile they keep their neutral character will conform a *paired structure*. The relation to Atanassov's fuzzy sets will be also stated in this paper.

2. Opposite fuzzy sets

A *negation* function has been traditionally defined (see [22] but also [23]) as a non-increasing function

$$n:[0,1]\rightarrow[0,1]$$

such that $n(0)=1$ and $n(1)=0$. A negation function will be called a *strong* negation if it is in addition a strictly decreasing, continuous negation being also *involutive* (i.e. such that $n(n(v))=v$ for all v in [0,1]). In this paper we shall consider only strong negations. Then, if we denote with $F(X)$ the set of all fuzzy sets (i.e., predicates) over a given universe X, then any strong negation n determines a *negation* operator

$$N:F(X)\rightarrow F(X)$$

such that $N(\mu)(x)=n(\mu(x))$ for any predicate $\mu\in F(X)$ and any object $x\in X$.

Definition 1. A function

$$O:F(X)\rightarrow F(X)$$

will be called an *opposition* operator if the following two properties hold:

A1) $O^2=Id$ (i.e. O is involutive);

A2) $\mu(x) \le \mu(y) \Rightarrow O(\mu)(y) \le O(\mu)(x)$ for all $\mu \in F(X)$ and $x, y \in X$;

The above definition generalizes the following definition given in [24], where a particular negation operator $N: F(X) \to F(X)$ is being assumed:

Definition 2. An *antonym* operator is a mapping

$$A: F(X) \to F(X)$$

verifying the following properties:

A1) $A^2 = Id$;

A2) $\mu(x) \le \mu(y) \Rightarrow A(\mu)(y) \le A(\mu)(x)$ for all $\mu \in F(X)$ and $x, y \in X$;

A3) $A \le N$.

Hence, the following definition seems also natural, as another family of relevant opposites:

Definition 3. An *antagonism* operator is a mapping

$$A: F(X) \to F(X)$$

fulfilling the following properties:

A1) $A^2 = Id$;

A2) $\mu(x) \le \mu(y) \Rightarrow A(\mu)(y) \le A(\mu)(x)$ for any $\mu \in F(X)$ and $x, y \in X$.

A4) $A \ge N$.

In this way, any chosen negation N is an opposite, and it determines two main families of opposites, antonym and antagonism, names that should be usually assigned to any antonym or antagonism different than N. But notice that there are opposites not being antonym or antagonism.

3. Paired fuzzy sets and paired fuzzy structures

Previous definitions offer in our opinion an appropriate range for defining opposites, depending on a previous negation that acts as a reference. In this way we can represent both the case in which two opposite fuzzy sets overlap and the case in which two opposite fuzzy sets remain short to explain reality.

Definition 4. Two predicates (or fuzzy sets) P,Q are *paired* if and only if $P = O(Q)$, and thus also $Q = O(P)$, holds for a certain semantic *opposition* operator O.

In other words, a paired fuzzy set is a couple of opposite fuzzy sets.

Our point is that neutral predicates will naturally emerge from opposites: as in classification context (see, e.g., [1], [2]), two opposite predicates (e.g., *tall/short*) that refer to the same notion (*height*) and, depending on their semantics, can generate different neutral concepts. When opposites overlap (e.g., *more or less tall/more or less short*), both opposites are perhaps too wide and *ambivalence* appears as neutral predicate (to some extent both opposite predicates hold). But if opposites do not overlap (e.g., *very tall/very short*), we find that both opposites are perhaps too strict, and *indeterminacy* appears. And of course both situations might hold, depending on the object under consideration. *Indeterminacy* and *ambivalence* therefore appear as two main neutral valuation concepts. Alternatively, specific intermediate predicates might appear, in some cases leading to a non-paired structure perhaps by modifying the definition of the two basic opposite predicates and/or introducing new non-neutral intermediate predicates, defining perhaps a linear scale (see, e.g., [13]).

But a third main neutrality can appear when opposites are *complex* predicates (e.g., *good/bad*), mainly due to the underlying multidimensional nature of the problem. In this context it is usually suggested a decomposition in terms of simpler predicates, and then we can easily be faced to a *conflict* between different criteria.

Hence, from a basic predicate and its negation we can define an opposite that might imply, to some extent, the existence of indeterminacy (antonym) and/or ambivalence (antagonism), and also conflict. Hence, we have reached to a qualitative scale L={*concept, opposite, indeterminacy, ambivalence, conflict*}, reaching in this way to our paired structure.

Definition 5. Paired structures are represented through the multidimensional fuzzy set A_L given by

$$A_L = \{\langle x; (\mu_s(x))_{s \in L} \rangle \mid x \in X\},$$

where X is our universe of discourse and each object $x \in X$ is assigned up to a degree $\mu_s(x) \in [0,1]$ to each one of the above five predicates $s \in L$, L={*concept, opposite, indeterminacy, ambivalence, conflict*}.

Although more details will be found in [17], we should stress that, consistently with [16], an appropriate logic (or logics if [14] and [15] are taken into account), should be then associated to this structure (X, L, A_L). It is important to point out that, consistently with [1] and [2], Ruspini's condition [20]

$$\sum_{s \in L} \mu_s(x) = 1 \text{ for all } x \text{ in } X$$

is not being *a priori* imposed, although certain circumstances, constraints or generalizations, might suggest specific adaptations, particularly in the management of predicates (see, e.g., [5]).

4. Paired structures in logics and knowledge representation

In this section, we use the framework of paired structures to analyze different logical and knowledge representation models under a new light. To this aim, we propose an example in which we apply the previous paired approach to the notions of *truth* and *falsehood* of classical logic. We restrict ourselves to a crisp setting for simplicity and clarity of exposition, since the main point of our argument is not affected by the consideration or not of a fuzzy framework.

Therefore, let us consider two crisp poles T = *true* and F = *false*, related through an opposition operator O, and defined for the purpose of this example on a universe of discourse

$$U = \{P(x) = \textit{John is tall}, \neg P(x) = \textit{John is not tall}\}$$

formed by two propositions specified in terms of a single object x in X (*John*), and a single property P (*tall*) and its negation $\neg P$ (to be read as *not-tall*).

Within the paired approach we should consider also the negation of the poles NT = *not-true* and NF = *not-false*, defined from both a negation

$$n:\{0,1\}\to\{0,1\},$$

such that $n(0) = 1$ and $n(1) = 0$, and the membership functions

$$\mu_T,\mu_F:U\to\{0,1\},$$

in such a way that $\mu_{NT} = n \circ \mu_T$ and $\mu_{NF} = n \circ \mu_F$.

Let us remark that we use two (or three) different symbols for negation, i.e. the symbol $\neg$ to refer to the negation of properties at the level of the propositions on which the poles apply, and the symbol N to refer to negation at the level of poles (which is in turn dependent on n). We assume all these negations to be involutive.

Then, within the paired approach, evidence regarding objects u in U is evaluated through pairs

$$\mu_{PAIRED}(u) = (\mu_T(u),\mu_F(u)), \qquad (1)$$

and thus the following valuations of $P(x)$ = *John is tall* are available (we denote them through the symbols before the last double arrow):

a) $P(x)$ is true $\leftrightarrow T_{P(x)} \leftrightarrow \mu_T(P(x)) = 1$ and $\mu_F(P(x)) = 0$;

b) $P(x)$ is false $\leftrightarrow F_{P(x)} \leftrightarrow \mu_F(P(x)) = 1$ and $\mu_T(P(x)) = 0$;

c) $P(x)$ is ambivalent $\leftrightarrow A_{P(x)} \leftrightarrow \mu_T(P(x)) = 1$ and $\mu_F(P(x)) = 1$;

d) $P(x)$ is indeterminate $\leftrightarrow I_{P(x)} \leftrightarrow \mu_T(P(x)) = 0$ and $\mu_F(P(x)) = 0$;

Similarly, regarding $\neg P(x)$ the following valuations are available:

e) $\neg P(x)$ is true $\leftrightarrow T_{\neg P(x)} \leftrightarrow \mu_T(\neg P(x)) = 1$ and $\mu_F(\neg P(x)) = 0$;

f) $\neg P(x)$ is false $\leftrightarrow F_{\neg P(x)} \leftrightarrow \mu_T(\neg P(x)) = 0$ and $\mu_F(\neg P(x)) = 1$;

g) $\neg P(x)$ is ambivalent $\leftrightarrow A_{\neg P(x)} \leftrightarrow \mu_T(\neg P(x)) = 1$ and $\mu_F(\neg P(x)) = 1$;

h) $\neg P(x)$ is indeterminate $\leftrightarrow I_{\neg P(x)} \leftrightarrow \mu_T(\neg P(x)) = 0$ and $\mu_F(\neg P(x)) = 0$.

Here we are neither concerned with interpreting this paired logical framework nor with establishing its soundness. Rather, we use the formal framework it provides to study how different logics and/or formal models for knowledge representation may be obtained by assuming different principles and properties. However, let us observe that different well-founded paraconsistent logics and semantics can be developed from this general approach (see [19],[25]).

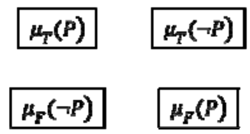

Fig. 1: Logical structure of a general paired approach. Truth and falsehood of a proposition P as well as truth of P and truth of its negation $\neg P$ are not related.

4.1. Paired representation of classical logic

From the general logical framework allowed by a paired representation of truth and falsehood, classical logic can be obtained by assuming just two conditions. First, let us assume

$$\mu_F(P(x)) = \mu_T(\neg P(x)), \qquad (2)$$

that is, falsehood of a proposition is equal to the truth of the $\neg$-negated proposition. And second, assume also

$$\mu_T(\neg P(x)) = \mu_{NT}(P(x)), \qquad (3)$$

that is, negation $\neg$ at the level of propositions is interchangeable with negation N at the level of the poles.

As a consequence of these two assumptions and the equality $\mu_{NT} = n \circ \mu_T$, the falsehood of P is equal to the non-truth of P, which in turn can be obtained from just the truth value of $P(x)$ through negation n, that is

$$\mu_F(P(x)) = n(\mu_T(P(x))). \qquad (4)$$

Notice that (4) entails that the poles T and F are each other complement (i.e. $T = NF$ and $F = NT$). Moreover, the so obtained logic verifies both the *excluded middle principle* (*EMP*)

$$T_{P(x)\vee\neg P(x)} \leftrightarrow (T_{P(x)}\wedge F_{\neg P(x)})\vee(F_{P(x)}\wedge T_{\neg P(x)})\vee(T_{P(x)}\wedge T_{\neg P(x)})$$

and the no contradiction principle (NCP)

$$F_{P(x)\wedge\neg P(x)} \leftrightarrow (T_{P(x)}\wedge F_{\neg P(x)})\vee(F_{P(x)}\wedge T_{\neg P(x)})\vee(F_{P(x)}\wedge F_{\neg P(x)})$$

where $\vee$ and $\wedge$ respectively represent the classical *OR* and *AND* connectives. Particularly, notice that the valuations $T_{P(x)}\wedge T_{\neg P(x)}$ and $F_{P(x)}\wedge F_{\neg P(x)}$ are not allowed in this framework.

In these conditions, a paired logical representation of the evidence available in the framework of classical logic for a proposition u in U is given by pairs

$$\mu_{CL}(u) = (\mu_T(u),\mu_F(u)) \qquad (5)$$

such that $\mu_F(u) = n(\mu_T(u))$. Notice that as a consequence of the complementarity of T and F no neutral valuations are allowed, that is, classical logic does neither admit ambivalent nor indeterminate propositions. Obviously, the pair μ_{CL} is equivalent to the classical representation of the sets P and $\neg P$ through characteristic functions μ_P, $\mu_{\neg P}$:$X \rightarrow \{0,1\}$, in such a way that

$$\mu_P(x) = \mu_T(P(x)) \text{ and } \mu_{\neg P}(x) = \mu_F(P(x)).$$

Let us remark that fuzzy representations of properties is basically grounded on the same principles (2) and (3), although the verification of *EMP* and *NCP* is dependent on the particular choice of fuzzy connectives $\vee$ and $\wedge$ for representing *OR* and *AND*. Particularly, as shown for instance in [11], both principles hold simultaneously if only if n is a strong negation and $\vee$ and $\wedge$ are Lukasiewicz-like operators.

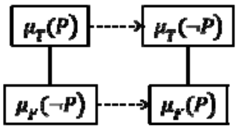

Fig. 2: In classical logic, truth of a proposition P is identified (solid lines) with falsehood of its negation $\neg P$, and the truth of P is related to the truth of $\neg P$ through a negation n (dashed arrows).

4.2. Paired representation of intuitionistic logic

Notice that expression (2) above makes possible to interpret

$$\mu_{CL}(u) = (\mu_T(u),\mu_T(\neg u)) \qquad (6)$$

for any proposition u in U. In a crisp setting, intuitionistic logic (in the sense of [3]) retains condition (2) and the interpretation in (6), but replaces condition (3) by the more general constraint

$$\mu_T(\neg P(x)) \leq \mu_{NT}(P(x)), \qquad (7)$$

which asserts through (2) that the falsehood of a proposition is a more restrictive claim than the non-truth of the same proposition, that is, a proposition may be not-true while at the same time being not-false.

Then, intuitionistic logic can be represented in a paired logical framework through pairs

$$\mu_{INT}(u) = (\mu_T(u),\mu_T(\neg u)) \qquad (8)$$

such that $\mu_T(\neg u) \leq n(\mu_T(u))$ for any u in U.

It is important to notice that this setting allows a proposition $P(x)$ to be evaluated as indeterminate (and then so will be $\neg P(x)$), since now it is allowed that $\mu_T(P(x)) = 0$ and $\mu_F(P(x)) = \mu_T(\neg P(x)) = 0$ can simultaneously hold. This last entails that *EMP* does not hold in general in the framework of intuitionistic logic.

Notice also that, although the presence of indeterminacy violates the previous strong formulation of *NCP*, this principle still holds in the weaker sense of not allowing both $P(x)$ and $\neg P(x)$ to hold simultaneously, that is

$$\mu_{INT}(u) = (1,1)$$

is not an available valuation.

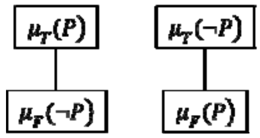

Fig. 3: In intuitionistic logic, truth of a proposition P is still identified (solid lines) with falsehood of its negation $\neg P$, but the truth of $\neg P$ can no more be obtained from that of P.

It is important to remark that in the case of intuitionistic logic, the logical poles T and F are no longer assumed to be each other complement, and thus they can respectively differ of *NF* and *NT*. Let us observe that the basic ideas holding in this crisp framework also hold in the fuzzy setting of [3].

4.3. Bipolar knowledge representation models

Notice that in the previous examples we restricted ourselves to consider a universe of discourse formed by just two complementary propositions, $P(x)$ and $\neg P(x)$. Such a universe contains exclusively the needed propositions in order to study the logical dependence of a proposition and its negation, the first and main issue of any logical analysis as those of classical logic and intuitionistic logic.

However, bipolar models (in the sense of [8]) are not actually logical models developed to analyze the poles T and F of classical logic, but rather bipolar models deal with knowledge representation in the presence of opposing arguments *under* the logical perspective of classical (or fuzzy) logic.

That is, in this case we should consider the alternative universe

$$U = \{P(x) = \textit{John is tall}, \neg P(x) = \textit{John is not tall},$$
$$Q(x) = \textit{John is short}, \neg Q(x) = \textit{John is not short}\}$$

given by four propositions stated in terms of a single object x in X and a pair of properties (and the corresponding negations) sharing a certain kind of opposition.

We may then assume that properties P and Q constitute a pair of poles related through an opposition operator ∂ in the sense of Definition 1, i.e. $Q = \partial(P)$.

Notice that we intentionally use two different symbols to differentiate the opposition operator acting at the level of logical poles (i.e. the operator O such that $F = O(T)$) from that acting at the level of the represented properties or predicates (i.e. the operator ∂). This also allows distinguishing between the neutral valuations arising at the logical level and those arising at the level of knowledge representation.

Particularly, we claim that most bipolar models actually admit neutral valuations at the level of the poles P and Q, but do not admit logical neutralities at the level of the poles T and F, similarly to classical logic.

That is, bipolar models represent evidence through pairs

$$\mu_{BIP}(u) = (\mu_T(u),\mu_T(\partial u)), \qquad (9)$$

but assume expressions (2) and (3) to hold regarding the logical relationships between a property and its negation in terms of logical truth and falsehood.

Thus, bipolar models enable to model opposite arguments at the level of knowledge representation, assuming two separate coordinates or dimensions (usually referred to as positive evidence and negative evidence, respectively) of knowledge representation, each of these dimensions in turn assuming a classical logic framework. That is, both *EMP* and *NCP* hold in each dimension regarding P and Q.

Moreover, these two dimensions may refer to logically independent properties (in the sense that P and Q may not be each other complement). Nevertheless, these two dimensions are not fully independent from a logical perspective since P and Q are related at the logical level through the opposition operator ∂, which relates the logical descriptions of P and Q (i.e. their membership functions μ_P and μ_Q) in order to guarantee that they meet the semantics of opposition.

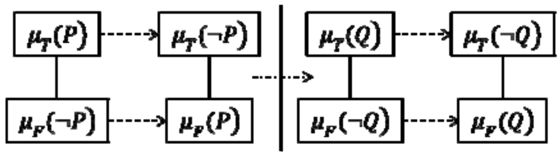

Fig. 4: In bipolar models, two separate dimensions of logical representation are employed to model opposite arguments, and are related through an opposition operator (dot-dashed arrow). Each of these dimensions assumes a classical logic structure.

5. Intuitionistic sets and knowledge representation

Notice that the difference between intuitionistic and bipolar models can now be stated more precisely.

Firstly, intuitionistic models rely on a different set of logical assumptions than bipolar models. The latter models assume a classical logical framework in each coordinate, while the former do not.

Secondly, intuitionistic models are stated in terms of a proposition and its negation, while bipolar models work in terms of opposite properties. In other words, intuitionistic models introduce opposition at the level of logical poles but does not consider opposite (but complementary) properties, while bipolar models allow opposite properties but assumes the logical structure of classical logic for each opposite property.

Notice that these differences make possible to think that both approaches (intuitionistic and bipolar) could be in fact complementary, in the sense that we could allow introducing opposition at *both* levels of knowledge representation and logic, i.e. a paired structure at the level of the properties P to be represented and another paired structure at the level of the logical poles T and F that evaluates the verification of such properties. That is, nothing seems to forbid the formal specification of a model

$$\mu_{BIP\text{-}INT}(u) = ((\mu_T(u),\mu_T(\neg u)),(\mu_T(\partial u),\mu_T(\neg\partial u))), \ (10)$$

in such a way that the evaluations $\mu_T(u)$ and $\mu_T(\neg u)$ as well as $\mu_T(\partial u)$ and $\mu_T(\neg\partial u)$ are related through a constraint similar to (7).

Here we do neither try to interpret this model nor to analyze its soundness, but we just claim it is formally possible in a general paired framework assuming paired structures on both the representational and logical levels.

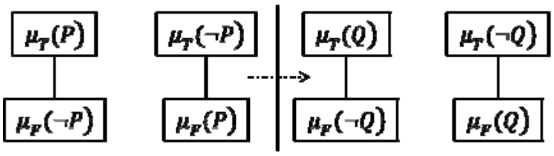

Fig. 5: The structure of a bipolar-intuitionistic model. Two dimensions of logical representation model opposite arguments related through opposition (dot-dashed arrow), but these dimensions assume an intuitionistic logical structure each.

Anyway, we should clarify that our opinion is that the plausibility of intuitionistic models for knowledge representation is debatable, since they are grounded on debatable logical assumptions that seems more concerned with meta-mathematical issues (as the dialectic of intuitionism vs. formalism) than with everyday-knowledge representation.

In this sense, when intuitionistic models are used for knowledge representation, we rather interpret them as bipolar models in the form (9) using an antonym (in the sense of Definition 2) as the opposition operator ∂, in such a way that

$$\mu_T(\partial P(x)) \le \mu_{NT}(P(x)) = n(\mu_T(P(x))) \qquad (11)$$

holds instead of (7). That is, in practice intuitionistic models behave as bipolar models under particular opposition assumptions.

In this way, what intuitionistic models call *negation* (in the sense of $\neg$) is for us an antonym ∂, and thus the indeterminacy allowed by intuitionistic models, which refers in principle to a logical indeterminacy, is in this way recovered and reinterpreted as a representational indeterminacy.

Therefore, our position is that paired structures do represent an adequate framework for discussing about both knowledge representation and logic models and their relationships.

6. About the different classes of bipolarity and paired structures

Another advantage of our proposal is that a natural classification of paired models is quite natural, simply taking into account the specific neutralities generated from opposites. But our classification will be different from the three types of bipolarities proposed by Dubois and Prade [8],[9],[10]:

- *Basic paired concepts* appear when the opposition operator is identified with a negation. A concept and its negation do not generate any additional neutral concept.

- *Simple paired concepts* based upon antonym or antagonism, and allowing *indeterminacy* and/or *ambivalence* neutralities.

- *Complex paired concepts* are associated to multi-dimensional frameworks, where *conflicting* views coming from different criteria can appear.

7. Final comments

The objective of this paper is to introduce *paired fuzzy sets* as an alternative to find a unifying view to all those models based upon the existence of a couple of opposite concepts. In this way we propose to focus our studies on the basic model, taking very much into account the first stages of our standard learning process: how from a seminal concept we generate its opposite, and how the semantic tension between them generates specific neutral elements.

Of course more complex evaluation structures might be suggested, but we consider that having a clarifying view on *paired structures*, limited to a couple of opposites and their associated neutralities, should be the first step to avoid undesired confusions in more complex situations. The notion of *opposite* and the notion of *neutrality* are the cornerstones of our approach, that though simple implies already a number of key issues to be addressed related to semantics, aggregation and decomposition of information.

In addition, we postulate that bringing existing models into our paired structures framework allows a better understanding the relationship of existing knowledge representation and logic models. This is possibly one the main aims of paired structures. Nevertheless, paired structures are also developed having in mind specific practical applications, and thus an objective of our future work is to propose a more simple framework in which some of the valuations μ_s, $s \in L$, may be derived from just the valuations of the poles, thus reducing the complexity of our paired models.

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