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Paired Fuzzy Sets: a unifying model for early knowledged acquisition

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***Abstract*—In this paper we want to stress the relevance of paired fuzzy sets, as already proposed in previous works of the authors, as a family of fuzzy sets that offers a unifying view for different models based upon the opposition of two fuzzy sets, simply allowing the existence of different types of neutrality associated to the different semantic relationships that may hold between opposite references. This scheme should be seen as a basic model for knowledge acquisition, which eventually will lead to a better understanding of the relationship of different knowledge representation models and to the acquisition of more complex valuation scales.**

Keywords—Atanassov's intuitionistic fuzzy sets, bipolar fuzzy sets, paired fuzzy sets.

I. INTRODUCTION

Several models can be found in the fuzzy literature sharing a simplified view of reality. Acknowledging that our brain understands reality by means of concepts (see, e.g., [18]), and that most of these concepts are fuzzy in nature, we should be aware of the difficulty of understanding isolate concepts. Concepts indeed represent very efficient tools for representing reality, and once we find a proper word for them they can be easily communicated to others. But most concepts we use, being compact and flexible, are complex in nature, difficult to capture, and still need learning in order to be used. And this learning cannot be developed without taking into account surrounding concepts. We rarely look at reality taking into account a unique concept.

In Probability Theory, for example, the simplest crisp (0-1) experiment is explained in terms of what is being called the Bernoulli trial, i.e., one single (in some way random) observation that ends with checking whether certain even has occurred or not. If we are not able to differentiate whether a certain event has happened or has not happened, the experiment, at least in this crisp framework, is considered unacceptable as an experiment. It is a must in these Bernoulli trials to be able to answer yes-no to the question *does this concept holds?* And the answer *no* implies that we know what the negation of such a concept means. We cannot define Bernoulli experiments without considering a predicate and its negation (using the term *failure* to refer to the negation of *success* is absolutely misleading in our opinion, since it suggests a different predicate from negation). More in general, in order to understand a predicate we need a comparison with the maximum possible number of linked or related predicates. Among these related predicates, *negation* should be the first to be considered, since it comes with the logic we are using to design our experiment.

But one of the terrible illusions that crisp thinking produces is a consequence of the fact that within such a framework there is only one negation, and that this negation is the unique existing opposite. Such a restricted approach does not give enough room to explain how our brain works, being as it is an indeed complex machinery (see e.g. [5] and [6]). Outside binary logic things are not so simple (see e.g. [21], [22] and [23]). For example, bipolarity appeared in Psychology somehow to claim the importance of considering the simultaneous views of reality that come from different perspectives (see e.g. [9] and [10]). But soon it was realized that, even in the case of assuming only two perspectives, the existence of these two perspectives indeed generates the need of certain *neutral* valuation states in between them.

From this point of view, Atanassov's intuitionistic fuzzy sets [3] represent the first attempt to offer a compact model to represent two opposite fuzzy sets, in particular by assuming a Ruspini partition [20] within two opposite fuzzy sets and a third neutral fuzzy set, called *indeterminacy*. Atanassov's proposal has been subject to some criticism (see [11] but also [4]), but the fact is that his model has found extensive diffusion. An alternative proposal is *bipolarity* as proposed by Dubois and Prade (see e.g. [12], [13] and [14]), who conclude that three different types of bipolarity can be distinguished in our knowledge representation models.

This paper pursues to stress the relevance of the notion of paired fuzzy sets, as proposed in a joint work between this team and other researchers at the Public University of Navarra, Spain [16]. Our approach should be considered also a continuation of a paper already published in the Fuzzy Sets and Systems journal [17]. Thus, this paper is organized as follows: in Section 2 we recall the basics of the paired approach by discussing different opposition relationships and the semantic structures they lead to. Section 3 provides a first typology of the so obtained paired structures, focusing on the types of neutrality they allow representing. The representation of some of these structures by means of appropriate scales is discussed on Section 4. Furthermore, the ability of these structures to also represent ignorance or lack of knowledge is analyzed in Section 5. Finally, Section 6 presents some final comments and conclusions.

II. The Paired Approach

In this section we recall the basic definitions proposed in [16]. These definitions were justified on the basis of a semantic argument, which tries to explore our knowledge representation process: from a given predicate (for instance, one we consider relevant in a given context), we are able to generate its negation. But, as discussed in [17], the logical dependence between a predicate and its negation makes them equivalent in terms of knowledge representation. No new information, apart from that already contained in the original predicate, is obtained through negation. In other words, negation does not provide a new, different perspective from that offered by the original predicate. However, negation provides a (both logical and semantic) landmark from which defining opposite predicates to the original one. Although an opposite is semantically related to the original predicate, both predicates are logically independent, and thus the opposite is able to provide an informative, different perspective from the original predicate. Then, the combination of these opposite perspectives leads to a richer representational framework, in which different types of neutrality may arise as a result of the semantic tension between such opposing perspectives, somehow capturing what's *in between* the references provided by a predicate and its opposite.

In particular, consistently with the classification approach proposed in [1][2], our semantic view will suggest three main types of neutrality: *indeterminacy*, when both reference predicates do not fully explain a part of the reality (see e.g. [8]); *ambivalence*, when both concepts overlap, simultaneously applying to a certain reality (see e.g. [7]); and *conflict*, which can appear when reference predicates are conceived as multidimensional notions that depend on lower-level descriptions that may oppose in a conflictive manner.

A. About opposites

As already stated, our point of departure when a predicate has been given is its negation, and from this negation we search for an opposite predicate.

Let us represent by $F(X)$ the set of all fuzzy sets over a given universe *X*. A *negation* operator will be a mapping $N: F(X) \to F(X)$ such that $N(\mu)(x) = n(\mu(x))$ for any predicate $\mu \in F(X)$ and any object $x \in X$, being *n* a strong negation function, i.e., a strictly decreasing continuous function $n:[0,1] \to [0,1]$ such that $n(0) = 1$, $n(1) = 0$ and $n(n(v)) = v$ for all *v* in [0,1] (see also [21], [22] and [23]). The basic definitions proposed in [16] are the following:

Definition 1. A function $O: F(X) \to F(X)$ will be called an *opposition* operator if the following two properties hold:

A1) $O^2 = Id$ (i.e. *O* is involutive);

A2) $\mu(x) \le \mu(y) \Rightarrow O(\mu)(y) \le O(\mu)(x)$ for all $\mu \in F(X)$ and $x, y \in X$.

This definition of opposition generalizes the definition of antonym given in [23], once a particular negation $N: F(X) \to F(X)$ has been already assumed.

Definition 2. An *antonym* operator is a mapping $A: F(X) \to F(X)$ verifying the following properties:

A1) $A^2 = Id$;

A2) $\mu(x) \le \mu(y) \Rightarrow A(\mu)(y) \le A(\mu)(x)$ for all $\mu \in F(X)$ and $x, y \in X$;

A3) $A \le N$.

The following definition of antagonism as a different kind of opposite has been also proposed in [16], noticing that we can find opposites that are neither antonyms nor antagonisms.

Definition 3. An *antagonism* operator is a mapping $A: F(X) \to F(X)$ fulfilling the following properties:

A1) $A^2 = Id$;

A2) $\mu(x) \le \mu(y) \Rightarrow A(\mu)(y) \le A(\mu)(x)$ for any $\mu \in F(X)$ and $x, y \in X$.

A4) $A \ge N$.

B. Paired Fuzzy Sets and Paired Structures

Hence, a couple of paired fuzzy sets are simply a predicate with an opposite, which depends on the given negation, as proposed in [16].

Definition 4. Two predicates (or fuzzy sets) *P*,*Q* are *paired* if and only if *P* = *O*(*Q*), and thus also *Q* = *O*(*P*), holds for a certain semantic opposition operator *O*.

We then postulate with [16] that the semantic tension between opposites generates different types of neutrality: with too extreme opposite predicates some objects can not be associated to any of both predicates, suggesting *indeterminacy*; with too wide opposite predicates we can find that some objects fully fulfil both, suggesting *ambivalence*; and when our predicates have a multidimensional nature some conflicting situations may appear if the object simultaneously meets some lower-level characteristic that belongs to the original predicate but other lower-level characteristic that belongs to its opposite. In this way we are considering a structure of five predicates (two opposite predicates plus three neutral predicates), and we can give the following definition (see again [16]):

Definition 5. Paired structures are represented through a multidimensional fuzzy set A_L given by

$$A_L = \left\{ \left\langle x; (\mu_s(x))_{s\in L} \right\rangle \mid x \in X \right\},$$

where X is our universe of discourse and each object $x \in X$ is assigned up to a degree $\mu_s(x) \in [0,1]$ to each one of the above five predicates $s \in L$, L={*concept*, *opposite*, *indeterminacy*, *ambivalence*, *conflict*}.

It is important to point out that we refer to a paired model meanwhile we only consider neutralities in addition to the original two opposites. Indeed, ambivalence or indeterminacy might suggest in some cases, for example, the existence of a linear scale (see, e.g., [15]). But such a linear scale (or any other more complex scale with no neutral valuation stages or more than two references) will not be a paired structure.

Hence, following [1] and [2], it is clear that our family of predicates does not necessarily defines a Ruspini's partition, i.e., the condition

$$\sum_{s\in L} \mu_s(x) = 1 \text{ for all } x \text{ in } X$$

might not hold. In fact, as pointed out in [1] and [2], Ruspini's fuzzy partition should be, if it is the case, a possible objective in our learning process. In fact, in some cases getting this kind of partition is something desirable, for which we reshape our classes or reference predicates until we get such a fuzzy partition, or any other generalization taking into account alternative connectives in order to assure covering with no overlapping. But quite often there is no fuzzy partition fitting reality, or simply a fuzzy partition does not represent a desirable approach for the decision making we are faced to.

III. CLASSES OF PAIRED FUZZY SETS

The classification we propose of paired fuzzy sets is precisely based upon the different types of neutrality generated from the semantic relationship of the reference predicates:

- In first place, as already pointed out, the first paired couple under consideration should be the one containing the original predicate and its negation. Since such a negation is a consequence of certain logic, and defines a logically dependent opposite predicate, it leaves no room for any neutral predicate in between. A fuzzy set and its negation define what we can call *basic paired fuzzy sets*.

- In second place, when our opposite predicate has been semantically defined, we can find that such an opposite can be an antonym or an antagonism, or a mixture of both, producing indeterminacy and/or ambivalence. A fuzzy set and a proper opposite define what we can call *simple paired fuzzy sets*.

- In a third case, within a multidimensional framework, in which reference predicates are decomposed in a set of lower-level descriptions, a conflict between different lower-level perspectives or criteria can appear. In this case we refer to *complex paired fuzzy sets*.

In this way we pretend to cover most models based upon two opposite fuzzy sets, particularly offering an alternative but unifying view to Atanassov's and Dubois-Prade's models.

IV. THE REPRESENTATION ISSUE

It is interesting to consider the triangular representation for each one of the above neutralities.

For example, if indeterminacy means that opposites do not cover the whole universe and that there exists a region where none of both predicates hold, such a situation suggests that it is necessary to search for more information, or even a re-definition of the considered opposites. In case of a continuous gradable scale, a possible representation of the underlying lattice can be proposed by means of a triangle

$$L_I = \left\{ (x,y) \in [0,1]^2 \mid x + y \le 1 \right\}$$

where the point (0,0) means that none of both poles (1,0) and (0,1) hold (see Figure 1 below).

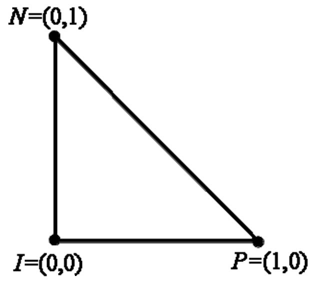

Figure 1: Triangle representation for indeterminacy.

Analogously, if ambivalence means the acknowledgement of overlapping opposites, i.e., the existence of a region where both opposites hold without conflict, this situation might suggest the convenience of two more strict opposites (regions were each opposite holds but not the other opposite). In case of a continuous gradable scale, a possible representation of the underlying lattice can be proposed by means of a similar triangle:

$$L_C = \left\{ (x,y) \in [0,1]^2 \mid x + y \ge 1 \right\}$$

where the point (1,1) means that both poles (1,0) and (0,1) fully hold (see *Figure 2*).

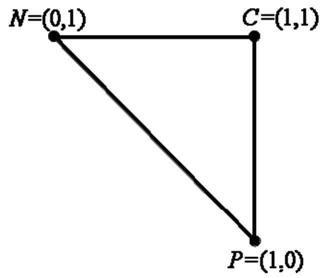

Figure 2: Triangle representation for ambivalence.

When predicates are described in terms of several criteria, opposites can be in conflict besides the possibility of not covering the whole universe of discourse. Again, in case of a continuous gradable scale, and meanwhile such a multidimensional description has not been made explicit in a more complex model, conflict can be also represented in terms of a triangle

$$L_K = \{(x,y) \in [0,1]^2 \mid x+y \geq 1\}$$

where the point (1,1) means now the existence of conflictive arguments in favour of both poles (1,0) and (0,1) (see Figure 3a). In case we need the simultaneous representation of conflict and indeterminacy, for example, a possible representation of the underlying lattice can be proposed by means of a square

$$L = [0,1]^2$$

where the point (0,0) means that there is no clear argument for none of opposites (1,0) and (0,1), and the point (1,1) means the existence of clear arguments for both opposites (see Figure 3b).

But of course our three neutralities can simultaneously appear in practice, so an efficient representation tool is clearly needed.

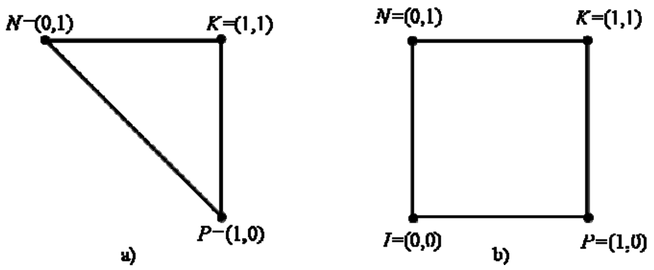

Figure 3: a) triangle representation for conflict; b) square representation for conflict and indeterminacy.

V. The Imprecision Issue

We should nevertheless acknowledge that in addition to the above types of neutrality, there are other non-neutral representational situations that can be wrongly identified as a kind of neutrality, since they produce in the decision maker some kind of symmetry among opposites. However, this symmetry does not actually originate from the semantic tension between reference opposites, but as a result of allowing both such references and the formal tools we use to represent them to be imprecise. Let us discuss here two of them.

On one hand, *points of symmetry* arise whenever the degree of membership to a given predicate equals the degree of membership to its opposite, in such a way that both degrees are each other negation. Decision makers find difficulties in choosing among both opposites in this situation. But a point of symmetry does not constitute a different (neutral) concept that provides a new valuation category as a result of a semantic tension between references. Instead, they arise because of the imprecise nature of the reference predicates, whenever we allow such imprecision to be introduced in our models through a fuzzy modelling of the paired references.

On the other hand, and more importantly, once we are faced to the estimation problem (membership functions should be somehow estimated), we should be aware of a different imprecision issue, now associated to the difficulty of estimating exact membership degrees. Our knowledge may be not so rich to allow estimating exact degrees, and thus imprecise degrees or estimations may be allowed to model lack of knowledge or ignorance. Again, decision makers can find difficulties in choosing among opposites under a significant level of ignorance. However, such ignorance is not semantically generated from a pair of opposing references, but it derives from an insufficient or imperfect knowledge of the objects into consideration.

Therefore, this second kind of imprecision, associated to ignorance, refers to a lack of knowledge about the exact value to be chosen within a valuation scale. In case of a continuous (or linear) scale, a possible representation of this lack of knowledge can be again obtained through another triangle, as shown in Figure 4, in which the vertex *EI* (or estimation ignorance) is associated to a total lack of information about the degree of verification of the references 1 and 0, that occupy the other two vertices. For example, each coordinate μ within such a triangle may be associated to an estimated interval $[\underline{\mu}, \overline{\mu}] \subset [0,1]$ for examining gradualness (and notice that the interval associated with the evaluation *EI* is the whole interval [0,1]). This leads to consider the usual scale

$$L_{EI} = \{(x,y) \in [0,1]^2 \mid x \leq y\},$$

where each pair (x,y) is associated to the interval $[x,y]$. In this way membership is estimated by assigning an interval instead of a single value.

Such an imprecision might apply to each one of the fuzzy predicates that composes a paired structure, leading to a more complex structure.

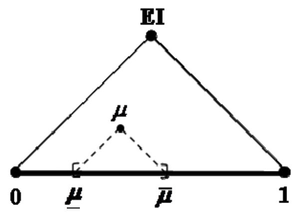

Figure 4: Triangle representation for imprecision.

VI. Final Comments

The main aim of this paper is to make a call towards the interest of paired fuzzy sets and paired structures as the basic model for early learning. We think that the proposal, developed more in detail in [16], allows a unifying alternative for all those models based upon two opposite fuzzy sets and the neutral categories generated from the semantic tension between those two opposites. Once a relevant predicate has been considered, its limits need to be explored by means of its surrounding predicates, being the first one its negation and, more in general, an opposite. The semantic tension between a predicate and its opposite generates additional neutral predicates at a first stage, and eventually may suggest more complex valuation scales or the need of sequentially reshaping the reference predicates under consideration. As shown in this paper, such a basic model is rich enough and its development is far from being simple.

However, our point is that paired fuzzy sets and structures may contribute to shed some light on the relationships and particularities of different knowledge representation formalisms (as e.g. intuitionistic fuzzy sets, bipolarity, interval-valued fuzzy sets and type-2 fuzzy sets) that extends the representational power of Zadeh's fuzzy sets.

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