Dynamic Price Dispersion of Storable Goods

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Dynamic Dispersion of Storable Good Prices

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Abstract

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1 Introduction

Supermarket retailers adjust retail prices on a weekly basis, and change the set of promoted products time to time. In each product category, a shopper can almost always find some products on promotion, marked with eye-grabbing price labels or piled at the entrance of aisles. Supermarket managers clearly find it more profitable to put different items on sales time to time, in spite of the high menu cost and administrative cost (**?**), than keeping prices constant at some high level over time (like what convenience stores usually do). Nowadays, not only can consumers download on-line next week's promotion brochures of local super markets, but also smart phone applications designed by specific supermarket chains to advertise their in-store sales items are available.

When goods are storable (such as ground beef, laundry detergent, potato chips, etc), consumers purchase more than they will consume in that period and store the rest as inventory. Consumer heterogeneity with respect to "willingness to wait" is well acknowledged by store managers. While the regular price is offered to impatient consumers, the manager would reduce the price in order to "clear out" the demand of those who have been patiently waiting for the next sale to occur. Price promotion serves as a means of intertemporal price discrimination. On the other hand, because the current sales induce stockpiling, future demand would be jeopardised. Therefore, store managers tend to offer medium-size price reductions in order to preserve future sales. Temporary price reductions also serve the role of price discrimination between informed and uninformed consumers. Informed consumers with small search costs purchase at the low price occasionally offered, while uninformed consumers with high search cost purchase at the regular price, unless they encounter the low price by chance.

Empirical studies show that several price variation patterns are widely observed. Using retail price data over 20 categories for 5 years from 30 US Metropolitan areas, **?** find that most items carried can be characterized as having a high regular price, and most deviations from that price are below that level. Price variations mostly results from store promotion, with insignificant impact from wholesale price changes. Therefore, price reductions are likely to be temporary, and will go back to the regular level soon.

Despite the ubiquitous nature of price promotions, there is little common ground among economists as to why supermarket retailers occasionally offer products at discounted prices, or even how and why such price dispersion can exist as an equilibrium phenomenon. To better understand retailers' strategic pricing behavior given consumer heterogeneities in store loyalty, willingness to wait or inventory cost, and knowledge in prices, I construct a model of an oligopoly retailers selling a homogeneous storable good. The good in the model are assumed to be consumed for multiple periods, and therefore does not need purchased frequently. Stores can sell the good at a *regular* price, or hold sales, selling the good at lower prices. Under the infinite horizon setting, the *High* type consumers are assumed to be loyal to a specific store, have infinite inventory cost and therefore never stockpile, and do not search for price reductions; they purchase only when the good is ran out, at the store they are loyal to. The *Low* type consumers search for price at zero search cost, purchase if the lowest price offered is below some critical price, and store it at some inventory cost.

In this paper a symmetric Markov-perfect equilibrium (MPE) is found. As in the classic search models, the competing stores face a trade-off between selling only to its own loyals at the regular price and to both loyals and shoppers at some sale price. Retailers randomize prices, and the *cdf* of the equilibrium price distributions have a mass point at the regular price p^R. Moreover, the equilibrium price distribution is a function of the shoppers' inventory. The mixed strategy equilibrium is characterized by a critical price depend upon which purchase decision is made in each period. The realized price evolution consists of several consecutive regular-price periods, where no sales are offered, and occasionally one-time price reductions.

The endogenous price evolution exhibits non-absorbing Markov transition of states: when shoppers hold high inventory, the probability of holding a sale is low, which means inventory will more likely to drop down.

2 Literature

Models in the literature aim to explain the strategic pricing interactions among competing retailers and consumer purchase behaviors. Specifically, models are developed to generate price distributions, which characterize equilibrium, that have similar patterns to empirical observations. Two classes of models have been constructed. Both examine the pricing decision of single product retailers, and show how consumer heterogeneity can lead to retail price variation over time. Their basic setups in consumer heterogeneity consider the following factors: whether the two types (High and Low) differ in reservation price, willingness to wait, inventory cost, store loyalty, and search cost. The market structure considered is typically an oligopoly. The equilibria of the models are characterized by a price distribution, continuous or not, implying price is drawn from that distribution and therefore varies every period.

The first class assumes consumers differ in their knowledge. Since sellers face a tradeoff between selling to only non-searchers at high price and selling to both searcher and non-searchers at the lowest price among all sellers, the symmetric mixed-strategy equilibrium features a continuous distribution of price. **?** model is the seminal contribution of this class. It describes a monopolistically competitive equilibrium in which sales are the outcome of mixed strategy equilibrium among retailers who compete over cohorts of informed and uninformed consumers. **?** characterize competitive promotional strategies by their depth and frequency within a mixed strategy equilibrium similar to **?**.

The second class views sales as means of price discrimination. **???** fall into this type. Consumers differ in their reservation prices, willingness to wait for sales, and/or inventory costs (analytically equivalent to willingness to wait). Since the high types with high reservation price are not willing to wait for sales, while the low types only purchase at low prices, the equilibrium is characterized by purchases of high types in all periods, and periodically reduced price that is to "clear out" the low types.

? is a combination of the two classes. In this model, not only do consumers differ in store loyalty and searching costs, but also only the low-type consumer store for inventory. Under this setup, oligopoly stores have an incentive to reduce price, both to sell to searchers and to consumer inventory. Observing consumer inventory, each store draw his own prices from a common distribution, and the low type consumers make purchase decision given the lowest price offered, taking into account the next-period state and payoffs predicted by her action and the transition rule. Because of the existence of store loyals, the Markov equilibrium price distribution has a mass point. Moreover, the equilibrium price distribution varies over time.

Though Varian's model can explain price variations of both perishable and nonperishable goods, it fails to predict the fact that most goods have a regular price as the price distribution has no mass point. The random price behavior that emerges from a mixed strategy equilibrium is fundamentally inconsistent with observed prices that tend to stay fixed for a long period of time and then fall temporarily, returning to the previous level after one period or two. This static model also fails to provide intuition of purchase at sales for inventory. The price discrimination type models (**???**) succeed in predicting a mass point in price distribution, yet they o not assume price searching behavior. Moreover, although these models are of infinite horizon, many have restrictions on the number of packages purchased, consumption amount in each period, or the maximum capacity of storage. For example, to fit into a Logit estimation, **?** assume that a consumer can purchase at most one package of laundry detergent. **?** assumes that a consumer can store at most one package, and must consume one package every period.

Besides consumer heterogeneities, another dynamic explanation for incentive of price promotion is provided by **?** in which price reductions are necessary to restore brand loyalty given its tendency to degrade over time. However, this loyalty restoring could be more likely actions of manufactures rather than retailers. If retail markets do have the incentive to restore loyalty of some brands, then it means that elasticities must vary across brands, and that the promoted brand probably has great impact on store revenue.

Finally, many authors argue that price promotions on a limited set of products may serve as a tool of "luring" consumers come into store and shop other products within store at regular prices (**???**). Since supermarkets virtually offer a diversity of products (20,000+ items), consumers with a purchase bundle face a trade-off between being loyal to one store and switching to another one for promotions at a transportation cost. Effective price promotions must increase store traffic. Furthermore, price promotion of one product is likely to cannibalize sales from other products within store or attracts customers from a rival retailer, if the degree of heterogeneity among store is low, but heterogeneity among products is high (**?**).

3 Model

There are N identical retailers selling a homogeneous good in each time period $t = 0, 1, 2, ...$ in an isolated city of population 1. The good is assumed to be storable, and the package size is L. In each period, each consumer must consume 1 unit of the good, thus one package will last for L periods. A proportion γ of the population have zero search cost and will be referred to as *shoppers*, who will buy from the lowest price retailer, if they buy at all. A proportion $1-\gamma$ do not search, and each of them is loyal to a fixed and specific retailer. They will be referred to as *loyals*. I assume that the retailers have equal share of the non-searchers, and that the number of loyal customers who buy a package lasting L periods is the same in every period. In each period, a consumer, shopper or loyal, can purchase at most one package of the good. Shoppers can store the new package as inventory at a unit cost c. Loyals never purchase if they still hold inventory, and only purchase when they run out. Both shoppers and loyals value the per period consumption of the good at v. All agents share a common discount factor δ. The good has a regular price p^R that satisfies $p^R = \sum_{t=0}^{L-1} \delta^t v - \sum_{t=0}^{L-1} \delta^t (L-t)c$, which is the total consumption utility net off the total inventory costs. Define any price strictly lower than p^R as a *sales price*. A retailer *holds a sale* in period t if she offers a sales price. Therefore, in each period, each retailer can sell to at least $(1-\gamma)/NL$ consumers at any price no greater than p^R. A loyal's surplus is zero if she buys at p^R, and positive if she buys at a sale.
I make three assumptions on shoppers' purchase behavior.

Assumption 1. The inventories of all shoppers are the same at the starting point.

Assumption 2. If a shopper runs out of stock, she must purchase at any price no greater than p^R.

Assumption 3. A shopper does not purchase if her inventory is greater than a positive integer K, where $K < L$. It implies that a shopper stores no more than one package of the good.

In each period, retailers simultaneously make pricing decisions p_{it}, and the set of prices offered in period t is denoted by $P_t = \{p_{it} | i = 1, ..., N\}$. The marginal cost for retailers (wholesale price) is assumed zero for simplicity. Observing P_t, a shopper j maximizes the total expected discounted utility by making purchase decision, taking into account future states and prices. Denote the purchase decision of shopper j by $d_{jt} = (d^i_{jt})'$ satisfying $d^i_{jt} \in \{0,1\}$ and $\sum_i d^i_{jt} \leq 1$, and her inventory in period t by I_{jt}. If the shopper makes a purchase at retailer i, then $d^i_{jt} = 1$. The good not consumed in the current period will be stored at a unit cost c. Denote the state variable of this infinite-horizon problem by ι_t, and in equilibrium it equals the synchronized shopper inventory, because as shown below the purchase decisions of all shoppers will be synchronized in equilibrium, $d_t = d^i_{jt}, \forall j$. It follows that the state transition rule is $\iota_{t+1} = \iota_t - 1 + d_t L$.

The shopper's problem is represented by

$$\max_{\{\{d^i_{jt}\}^\infty_{t=0}\}} \; E\left[\sum_{t=0}^\infty \delta^t u_{jt}(P_t, I_{jt}, d^i_{jt})|\iota_1\right]$$

$$\text{subject to} \quad \text{d}^i_{jt} \in \{0,1\}, \; \forall i,$$

$$\text{I}_{jt} \geq 0,$$

$$\sum_i d^i_{jt} = 1 \text{ if } I_{it} = 0,$$

$$\sum_i d^i_{jt} = 0 \text{ if } I_{it} > K,$$

$$\text{I}_{j,t+1} = I_{jt} - 1 + \sum_i d^i_{jt} L.$$

(1)

The expectation is taken with respect to future states. The flow utility u_{jt} is given by

$$\text{u}_{jt}(P_t, I_{j,t+1}, \iota_t, d_{jt}) = -cI_{j,t+1} - d_{jt}p_{t,min} + v,$$

(1)

where $p_{t,min} = \min P_t$, the lowest price offered in period t; $I_{j,t+1} = I_{j,t} - 1 + \sum_i d^i_{jt} L$ is the low of motion. Define the value function $V(\cdot)$ that represents a shopper's maximized payoff under her action rule:

$$\text{V}(\text{P}_t, I_{jt}, \iota_t) = \max_{\{d_{jt}\}} \left\{E \sum_{t=0}^\infty \delta^t u_{jt}(P_t, I_{j,t+1}, d_{jt}, \iota_t)|d_{jt}, d_{-jt}\right\}.$$

(1)

Define shopper j's choice-specific value function $\tilde{V}(\cdot)$ as

$(\mathrm{P}_t, I_{jt}, \iota_t, d_{jt}) = u_{jt}(P_t, I_{j,t+1}, \iota_t, d_{jt}) + \delta E\left[V(P_{t+1}, I_{j,t+1}, \iota_{t+1})|d_{jt}, d_{-jt}\right].$
(1)

Retailers simultaneously make pricing decisions $\{\{p_{it}\}_{t=0}^{\infty}\}$ to maximize the expected total payoff:

$\max_{\{\{p_{it}\}_{t=0}^{\infty}\}} \sum_{t=0}^{\infty} \delta^t \pi_{it},$
(1)

where π_{it} is the per-period profit and δ is the discount factor. In period t, a retailer i can always sell to its royals who ran out of stock at any price no greater than p^R. And it sells to shoppers if it offers the lowest price and if the shoppers buy at all. Retailer i's per-period payoff is given by

$\pi_{it} = p_{it}\left(\frac{1-\gamma}{NL} + \gamma d_t^i(p_{it}, p_{-it}, \iota_t)\right),$
(1)

where p_{it} is the price; $\frac{1-\gamma}{NL}$ is the quantity sold to its loyals; $d_t^i(p_{it}, p_{-it}, \iota_t)$ is the synchronized purchase decision of all shoppers, as a function of its price p_{it}, rivals' prices p_{-it}, and consumer inventory ι_t; and γd_t^i is the quantity demanded of retailer i (assuming all shoppers' inventories are the same at the starting point, their equilibrium purchase decisions will synchronize as shown later). It is straightforward that $d_t^i = 1$ only if $p_{it} = p_{t,min}$, given that shoppers buy at all.

I seek for a subgame-perfect MPE that satisfies several conditions. First, I assume that the current inventory levels, shoppers' and loyals', are sufficient for the decision making of all agents. Second, the current decision making does not depend on states or actions taken in previous periods. Said differently, retailers' behavior is predicted by the current shopper inventory only.

Bounding the state space. Since the number of states in a MPE must be finite, we would like to have an upper bound of the inventory level upon which stockpiling behavior is endogenously irrational (*Assumption 3*). For simplicity, such upper bound is assumed to be less than the size of one package, $K < L$. Thus, the highest possible inventory level is $K - 1 + L$, and the number of states is $K + L$.[1]

Critical prices. The MPE is characterized by a series of critical prices, $\{p_k\}_{k=0}^{K}$, where $p_k \leq p^R$. By *Assumption 2*, the critical price at $k = 0$ is simply p^R. In each state $k > 0$, shoppers will purchase, if the lowest price offered is no greater than p_k, and not purchase otherwise, i.e, $\sum_i d_t^i(P_t, \iota_t = k) = 1$ if $p_t \leq p_k$.

[1] The upper bound could also be, for example, $L < K < 2L$, but it implicitly allows purchase with storage, which complicates the transition paths of states and the calculation of continuation value.

3.1 Pricing Strategy

The retailers in the model will randomize prices, because it always pays to break ties, provided shoppers will purchase. Denote the distribution of prices in state k by F_k. Since the pricing strategy is state-dependent, for ease of notation I drop the subscript t. First notice that retailers will not choose any price greater than p^R because such prices produces zero sales. In the event that shoppers will purchase, it would be profitable to slightly undercut other retailers; such undercutting will not result in Bertrand consequence because it it more profitable to charge simply p^R.

Next, consider retailer i's pricing strategy at state k. If p_i is greater than the critical price, then retailer i sells to its loyal consumers only in that period; the next period state depends on other retailer's prices: if there exists at least one retailer that offers a price no greater than p_k, which occurs with probability $1-(1-F_k(p_k))^{N-1}$, then the state transfers to $k-1+L$; if no retailers offer such price, then the state transfers to $k-1$ with probability $(1-F_k(p_k))^{N-1}$. If p_i is no greater than p_k and happens to be the lowest price among all retailers with probability $(1-F_k(p_i))^{N-1}$, then it sells to both loyals and shoppers, and next period state transfers to $k-1+L$ with certainty. If p_i is not the lowest price though lower than p^k, then it profits from loyals only.

From the retail's problem in equation (1), the value function of a retailer i is given by

$$\mathrm{W}^k \equiv \max_{\{\{p_{it}\}_{t=0}^{\infty}\}} \textstyle\sum_{t=0}^{\infty} \delta^t \pi_i(p_{it}, P_{-it}, \iota_t = k). \quad (1)$$

Retailer i's choice-specific value function in state $k \neq 0$ can be written as

$$\mathrm{W}_i(p, p_{-i}, \iota = k) = \begin{cases} p\frac{(1-\gamma)}{NL} + \delta W^{k-1+L} \times (1-(1-F_k(p_k))^{N-1}) \\ +\delta W^{k-1} \times (1-F_k(p_k))^{N-1}, \text{ if } p > p_k, \\ \\ p\frac{(1-\gamma)}{NL} + p\gamma \times (1-F_k(p))^{N-1} \\ +\delta W^{k-1+L}, \text{ if } p \leq p_k. \end{cases} \quad (1)$$

The mixed-strategy equilibrium requires that a retailer makes equal profits at any price drawn from F_k. This means that, when shoppers hold inventory, no retailer would charge a price in the interval of (p_k, p^R): a price slightly less than p^R will induce no greater sales, therefore there will be no loss from raising price to p^R. In other words,the probability of drawing p^R at state $k>0$ is strictly positive.

The pricing strategy in state $k=0$ is slightly different. The reason is that, according to *Assumption*

2, when inventory drops to zero all shoppers must purchase at any price no greater than p^R, and the subsequent state will be $L-1$ with certainty. The price distribution from which retailers randomize their prices will be continuous, since any price lower than p^R would attract shoppers with positive probability. The choice-specific value function of retailer i is

$$\mathrm{W}_i(p, p_{-i}, \iota = 0) = p\frac{(1-\gamma)}{NL} + p\gamma(1 - F_0(p))^{N-1} + \delta W^{L-1}. \tag{1}$$

Lemma 1

For all $k \geq 0$,

$$\mathrm{W}^k = \frac{p^R}{1-\delta}\frac{(1-\gamma)}{NL}. \tag{1}$$

Proof.

Because a retailer randomizes prices, then undercutting other retailers must bring equal profit as charging p^R. Thus,

$$\mathrm{W}(\mathrm{k},\ \mathrm{p}^R) = p^R\frac{(1-\gamma)}{NL} + \delta W^{k-1+L}(1 - (1 - F^k(p^k))^{N-1})$$

$$+ \delta W^{k-1}(1 - F^k(p^k))^{N-1}$$

$$= \mathrm{W}(\mathrm{I}_t = k, p_{it} \leq p_k)$$

$$= \mathrm{p}\ (1\text{-}\gamma)\frac{}{NL + p\gamma(1-F_k(p))^{N-1} + \delta W^{k-1+L}}. \tag{1}$$

Denote $(1 - F^k(p^k))^{N-1}$ by $prob_k$. Then,

$$\mathrm{W}^0(0, p < p^R) = W_0(0, p^R) = p^R\frac{(1-\gamma)}{NL} + \delta W_{-1+L}$$

$$W^1 = p^R \frac{(1-\gamma)}{NL} + \delta W_L (1 - prob_1) + \delta W_0 prob_1$$

$\vdots$

$$W_{L-1} = p^R \frac{(1-\gamma)}{NL} + \delta W_{2L-2}(1 - prob_{L-1}) + \delta W_{L-2} prob_{L-1}.$$

(1)

So,

$$W^0 = p^R \frac{(1-\gamma)}{NL} + \delta p^R \frac{(1-\gamma)}{NL} + \delta(\delta W^{2L-2}(1 - prob_{L-1}) + \delta W_{L-1} prob_{L-1})$$

$$= p^R \frac{(1-\gamma)}{NL} + \delta p^R \frac{(1-\gamma)}{NL} + \delta^2 p^R \frac{(1-\gamma)}{NL} + \cdots$$

$$= p^R \frac{1}{1-\delta} \frac{(1-\gamma)}{NL}.$$

(1)

Similarly, it can be sequentially shown that the total payoff of a retailer in all states is equal to the payoff as if she charged one single price p^R in all periods.

$$W^k = p^R \frac{(1-\gamma)}{NL} + \delta W^{k-1+L}(1 - (1 - F^k(p^k))^{N-1}) + \delta W^{k-1}(1 - F^k(p^k))^{N-1}$$

$$= \ p^R \frac{(1-\gamma)}{NL} + \delta(p^R \frac{(1-\gamma)}{NL} + \delta W^{k-2+2}(1 - (1 - F^k(p^k))^{N-1})$$

$$+ \ \delta W^{k-2+L}(1 - F^k(p^k))^{N-1})$$

$$= \ \cdots$$

$$= \ p^R \frac{1}{1-\delta} \frac{(1-\gamma)}{NL}.$$

(1)

Lemma 1 permits the direct calculation of price distribution in state k:

$$(1\text{-}F_0(p))^{N-1} p\gamma = \frac{1-\gamma}{NL}(p^R - p), \ \text{ for } \underline{P}_0 \le p \le p^R,$$

(1)

and

$$
\begin{cases} (1-F_k(p))^{N-1}p\gamma = \frac{1-\gamma}{NL}(p^R - p), \text{ for } \underline{P}_k \le p \le p_k. \\ F_k(p) = 1 - F_k(p_k), \text{ for } p = p^R \end{cases} \tag{1}
$$

where $\underline{P}_0$ and $\underline{P}_k$ are the lower bounds of the support of price distribution, satisfying $F_0(\underline{P}_0) = 0$ and $F_k(\underline{P}_k) = 0$, respectively. It is clear that the supports of price distribution for different k have the same lower bound:

$$
\underline{\mathrm{P}} = \underline{\mathrm{P}}_k = p^R \frac{1-\gamma}{1-\gamma+NL\gamma}, \forall\ 0 \le k \le K. \tag{1}
$$

While F_0 is continuous on its support, F_k has a mass point on p^R. When $k > 0$, the probability of choosing p^R is strictly positive and is given by $1 - F_k(p_k)$. This difference is due to the increment in sales when slightly reducing price from p^R: since shoppers must purchase at $k = 0$, a price slightly lower than p^R results in a positive probability of sales. In contrast, at $k > 0$, the is no gain in sales if the reduced price is not as low as p_k.

A retailer's equilibrium pricing strategy is that, when shoppers' inventory is zero, $p \sim F_0$; when shoppers hold inventory, $0 < k \le K$, $p \sim F_k$; when $k > K$, according to *Assumption 3*, $p = p^R$ with probability one.

3.2 Purchase Strategy

A rational shopper takes into account the current lowest price $p_{t,min}$, her own next period inventory I_{jt+1}, and future state ι_{t+1}. Buying today at a low price means the inventory cost will immediately occur, while postponing purchase will risk losing the good deal but avoiding this cost.

The purchase strategy is characterized by a critical price p_k at each state. Given retailers' symmetric pricing strategy, suppose in period t at state k the lowest price available is $p_{t,min} \le p_k$. Shoppers will purchase a new package for inventory, and the state will transit into $\iota_{t+1} = k - 1 + L$. By the assumption that $K < L$ is the highest state where sales prices could occur, the succeeding states will be $k - 2 + L, \cdots, K, \cdots$, and prices will stay at p^R for $k + L - K$ periods until $\iota = K$ where the next sale would occur.

Now consider a potential deviater who does not purchase at $p_{t,min} \le p_k$. According to *Assumption 2* and *3*, she is certain about the states and price distribution in the subsequent $K + L - k$ periods. Because all prices will be staying at p^R for at least $k + L - K$ periods, and the deviater will run out in $k < k + L - K$

periods, she will have to pay p^R for the next package. Thus, she would rather postpone the next purchase as late as possible. Said differently, if postpone purchase, then it will be postponed for k periods. After this very late purchase, her inventory will again synchronize with the rest.

In order to prevent deviation, $p_{t,min}$ must be small enough such that buying today is no worse than postponing it. Recall that $V(P_t, I_{j,t}, \iota_{t+1})$ is the value function which can denote the continuation value of a potential deviater j, whose inventory is $I_{j,t}$ at the beginning of period t at state ι_{t+1}. Expressing the deviater's tradeoff using the choice-specific value functions, we obtain the upper bound of $p_{t,min}$.

Suppose $\mathrm{p}_{t,min} \leq p_k,$ then p_k satisfies

$$(\mathrm{P}_t, k, k, 1) \geq \tilde{V}(P_t, k, k, 0)$$

$$\Downarrow$$

$$\mathrm{p}_{t,min} \leq -cL + \delta\Big(V(P_{t+1}, k-1+L, k-1+L) - V(P_{t+1}, k-1, k-1+L)\Big). \tag{1}$$

$V(P_{t+1}, k-1+L, k-1+L)$ is the expected continuation value of a non-deviater with inventory $k-1+L$, where the expectation is taken with respect to P_{t+1} in which all $p_{i,t+1} \sim F_{k-1+L}$. Her inventory will be $k-1+L$ after this purchase and gradually declined to K along with consumption. She needs not consider her purchase decision until the next sale, which occurs after $k-1+L-K$ periods. Thus, it can be written in the sum of the total inventory costs and total consumption utilities in the next $k-1+L-K$ periods, and the expected value at state K.

$$\mathrm{V}(\mathrm{P}_{t+1}, k-1+L, k-1+L)$$

$$= \sum_{i=1}^{k-1+L-K} -c(k-1+L-i)\delta^{i-1} + v\left(\sum_{i=1}^{k-1+L-K} \delta^{i-1}\right)$$

$$+ \delta^{k-1+L-K} V(P, K, K). \tag{1}$$

Similarly, $V(P_{t+1}, k-1, k-1+L)$, the deviater's continuation value, can be written as the sum of cumulative inventory costs and consumption utilities in the next $k-1$ periods, and the continuation value when she runs out of stock and is forced to purchase at p^R. The latter continuation value can be decomposed into three parts, consumption utilities and inventory costs before inventory declines to K, and the discounted continuation value at state K with inventory K (see **Appendix A** for details):

$$V(P_{t+1}, k-1, k-1+L)$$

$$= \sum_{i=1}^{k-1} -c(k-1-i)\delta^{i-1} + \sum_{i=0}^{L-1+K} (-c(L-1-i))\delta^{k-1+i}$$

$$+ v \left(\sum_{i=1}^{k-1+L-K} \delta^{i-1}\right) + \delta^{k-1+L-K} V(P, K, K)$$

(1)

Not surprisingly, the two terms in the parenthesis of inequality (3.2), the continuation values of a non-deviater and a deviater, contain a common component, $\delta^{k-1+L-K}V(P, K, K)$. This is consistent with the fact that the deviater's inventory will synchronize with the rest after $k-1+L-K$ periods. The infinite-horizon optimizing problem boils down to a $k+L-K$-period problem. Plugging the above two expressions back, we obtain

$$p_{t,min} \leq -cL \left(\sum_{i=0}^{k-1} \delta^{i}\right) + \delta^k p^R$$
$$= \text{-cL 1-}\delta^k \frac{}{1-\delta+\delta^k p^R}$$

(1)

The deviater postpones her purchase till k periods later and pays p^R, while the rest of shoppers purchase now at $p_{t,min}$, but have to store the new package for k periods. Therefore, there would be no incentive to deviate, if the cumulated inventory costs of the new package do not offset the difference between $\delta^k p^R$ and $p_{t,min}$. To rationalize shoppers' purchase behavior, the critical price is

$$p_k = -cL\frac{1-\delta^k}{1-\delta} + \delta^k p^R.$$

(1)

There are two things worth noticing about the critical price. First, the value of the right hand side of the above equation is the payoff of purchasing a new package at the regular price when the current package is ran out (k periods later), a reservation value that a shopper can always get by postponing purchase. Second, the critical price monotonically decreases on k; when shoppers possess higher inventory, the price that induce purchase has to be lower.

If $p_{t,min} > p_k$, or $p_{it} = p^R$, for all i, the rational decision is "not buy". This is a trivial case. The deviater cannot gain by making a purchase. First notice that there would be sales held in the subsequent periods as long as no retailer offers a sale price in any of the previous periods, during which the inventory of all shoppers declines by one in each period. If she buy in t at p^R, the new package will incur inventory cost, and, if there would be a sale in the subsequent periods, she would have to miss it.

4 Equilibrium

First of all, an equilibrium series of critical prices $\{p_k\}_{k=0}^K$ (p_0 is trivially p^R) satisfies

- $F_k(p_k) > 0$, which is equivalent to $\underline{P} < p_k$, $\forall\ 1 \leq k \leq K$.

Recall that p_k monotonically decreases on k and the lower bounds of price supports are $\underline{P}$, the above condition will be violated when k is sufficiently big. This ides is used to rationalize *Assumption 3*. If the highest possible inventory level at which the probability of sales is strictly positive is K, the above condition implies

$$\mathrm{p}^R \tfrac{1-\gamma}{1-\gamma+NL\gamma} < -cL\tfrac{1-\delta^k}{1-\delta} + \delta^k p^R, \text{ for } 0 \leq k \leq K,$$

$$\mathrm{p}^R \tfrac{1-\gamma}{1-\gamma+NL\gamma} > -cL\tfrac{1-\delta^k}{1-\delta} + \delta^k p^R, \text{ for } K+1 \leq k. \tag{1}$$

Since the series of critical prices monotonically decreases on k and the starting value $p_0 = p^R > \underline{P}$ and the limit of the series $\lim_{k\to\infty} p_k = \frac{cL}{1-\delta} < \underline{P}$, there exists an integer $K > 0$ that satisfies the above inequalities. To rationalize *Assumption 3*, we would like the parameters satisfy $K < L$. A sufficient and necessary parameter condition is

$$\mathrm{p}^R \tfrac{1-\gamma}{1-\gamma+NL\gamma} > -cL\tfrac{1-\delta^L}{1-\delta} + \delta^L p^R, \tag{1}$$

which can be satisfied as long as δ is sufficiently small.

In the states where $k > K$, the critical price is lower than $\underline{P}$, all retailers will charge p^R with probability one at those states.
A number of observations immediately follow.

1.Because p_k decreases on k and each F_k is identical to F_0 within the interval $[\underline{P}, p_k]$, $F_k(p_k)$ also decreases on k. This means that the probability of holding a sale is low at high inventories. Though retailers have to offer lower prices in order to induce purchase at high inventories, the prices are more likely to be at the regular level. The oligopoly competition is the most rigorous at state zero, where all retailers offer sales prices.

2. Given observation 1, the transition of states is non-absorbing, in a sense that high states are more likely to decrease than low states, and low states($\leq K$) are more likely to jump up by $L-1$ than high states.

The MPE requires that the decision making depends on the current state only. One would argue that the history affects agents' expectation, because, for example, a shopper anticipates a sale occurs with high probability if there has been no sales in the past long period of time. This intuitive hazard rate of holding a sale is actually consistent with our equilibrium outcome: at a low state, which implicates that no sales have occurred in the past several periods and therefore no purchase has been made by shoppers, the probability of holding a sale is high.

3. The realized price evolution consists of several consecutive periods where price stays at p^R followed by temporary one-time price reductions. During the periods where inventory is low enough for retailers to offer sales, either one or more retailers simultaneously hold sales, or none of them do so. If any one of the stores offers a sale price, the Low type consumers will stock up, and this sale will be followed by another several consecutive non-sale periods.

Suppose the system starts from a "low" state with $k < K$ and the lowest price p_t is less than the critical level, purchase by shoppers will occur and the state transits to $k - 1 + L$ where all prices will be p^R. The price will stay at p^R for at least $k - 1 + L - K$ periods until the state falls back to K. The next sale will occur within K periods, at a state $0 \leq k \leq K$. This sale is again followed by several consecutive periods in which all prices stay at p^R. Since an equilibrium sale must induce purchase resulting in inventory levels higher than K, the sales are one-time and temporary.

5 Conclusion

I construct a model of oligopoly retailers selling a homogeneous storable good. The good in the model are assumed to be consumed for multiple periods, and therefore does not need purchased frequently. Under the infinite horizon setup, the High type consumers are assumed to be loyal to a specific store, never purchase for inventory cost, and do not search for price; they purchase only when run out at the store she is loyal to. The Low type consumers do search for price at zero cost, purchase if the best deal price offered among all stores is below some cut-off level, and store it as inventory at some cost.

One equilibrium among the continuum of equilibria is characterized by a critical price that decreases on inventory. At each state, retailers simultaneously draw prices from F_k, and shoppers purchase if the lowest price is no greater than the critical price. The critical price and the probability of holding a sale are low at high inventories. The pattern of state transition is non-absorbing: high states are more likely to decrease by one, and low states($\leq K$) are likely to jump up by $L - 1$.

This model assumes consumer heterogeneities with respect to store loyalty, search cost, and inventory cost. The predicted patterns of price variation is consistent with empirical observation that there is a strictly positive probability of charging the regular price. The model also predicts that the probability of holding a sale is low when shoppers hold a high inventory; and no shopper would store more than one packages, apart from the package that is currently in use. The predicted price panel consists of several consecutive periods where all prices stay at the regular level and occasional one-period price reductions.

The model relies on the assumption that the inventories and therefore purchase behaviors of all shoppers are synchronized. Relaxing this assumption would result in a complex of equilibria in which only some shoppers whose inventories below some cut-off level would purchase.