Massive, massless and ghost modes of gravitational waves from higher-order gravity

Bogdanos, Charalampos; Capozziello, Salvatore; De Laurentis, Mariafelicia; Nesseris, Savvas

Publication date:
2009

Document Version
Early version, also known as pre-print

Citation for published version (APA):
Bogdanos, C., Capozziello, S., De Laurentis, M., & Nesseris, S. (2009). Massive, massless and ghost modes of gravitational waves from higher-order gravity. arXive: 0911.3094.

Massive, massless and ghost modes of gravitational waves from higher-order gravity

Charalampos Bogdanos[1], Salvatore Capozziello[2,3], Mariafelicia De Laurentis[2,3], Savvas Nesseris[4]
[1] *LPT, Université de Paris-Sud-11, Bât. 210, 91405 Orsay CEDEX, France*
[2] *Dipartimento di Scienze Fisiche, Università di Napoli " Federico II" and* [3] *INFN Sez. di Napoli, Compl. Univ. di Monte S. Angelo, Edificio G, Via Cinthia, I-80126, Napoli, Italy*
[4] *The Niels Bohr International Academy, The Niels Bohr Institute, Blegdamsvej 17, DK-2100, Copenhagen Ø, Denmark*
(Dated: January 26, 2010)

We linearize the field equations for higher order theories that contain scalar invariants other than the Ricci scalar. We find that besides a massless spin-2 field (the standard graviton), the theory contains also spin-0 and spin-2 massive modes with the latter being, in general, ghost modes. Then, we investigate the possible detectability of such additional polarization modes of a stochastic gravitational wave by ground-based and space interferometric detectors. Finally, we extend the formalism of the cross-correlation analysis, including the additional polarization modes, and calculate the detectable energy density of the spectrum for a stochastic background of the relic gravity waves that corresponds to our model. For the situation considered here, we find that these massive modes are certainly of interest for direct detection by the LISA experiment.

PACS numbers: 04.30, 04.30.Nk, 04.50.+h, 98.70.Vc
Keywords: gravitational waves; alternative theories of gravity; cosmology

I. INTRODUCTION

Recently, the data analysis of interferometric gravitational wave (GW) detectors has been started (for the current status of GWs interferometers see [1–5]) and the scientific community aims at a first direct detection of GWs in next years. The design and the construction of a number of sensitive detectors for GWs is underway today. There are some laser interferometers like the VIRGO detector, built in Cascina, near Pisa, Italy, by a joint Italian-French collaboration, the GEO 600 detector built in Hannover, Germany, by a joint Anglo-German collaboration, the two LIGO detectors built in the United States (one in Hanford, Washington and the other in Livingston, Louisiana) by a joint Caltech-MIT collaboration, and the TAMA 300 detector, in Tokyo, Japan.

Many detectors are currently in operation too, and several interferometers are in a phase of planning and proposal stages (for the current status of gravitational waves experiments see [6–8]). The results of these detectors will have a fundamental impact on astrophysics and gravitational physics and will be important for a better knowledge of the Universe and either to confirm or rule out the physical consistency of General Relativity or any other theory of gravitation [9]. Several issues coming from Cosmology and Quantum Field Theory suggest to extend the Einstein General Relativity (GR), in order to cure several shortcomings emerging from astrophysical observations and fundamental physics. For example, problems in early time cosmology led to the conclusion that the Standard Cosmological Model could be inadequate to describe the Universe at extreme regimes. In fact, GR does not work at the fundamental level, when one wants to achieve a full quantum description of spacetime (and then of gravity).

Given these facts and the lack of a final self-consistent Quantum Gravity Theory, alternative theories of gravity have been pursued as part of a semi-classical scheme where GR and its positive results should be recovered. The approach of Extended Theories of Gravity (ETGs) based on corrections and enlargements of the Einstein scheme, have become a sort of paradigm in the study of the gravitational interaction. Beside fundamental physics motivations, these theories have received a lot of interest in cosmology since they "naturally" exhibit inflationary behavior which can overcome the shortcomings of standard cosmology. The related cosmological models seem realistic and capable of coping with observations. ETGs are starting to play an interesting role to describe today's observed Universe. In fact, the good quality data of last decade has made it possible to shed new light on the effective picture of the Universe.

From an astrophysical point of view, ETGs do not require finding candidates for dark energy and dark matter at the fundamental level; the approach starts from taking into account only the "observed" ingredients (i.e. gravity, radiation and baryonic matter); it is in full agreement with the early spirit of a GR that could not act in the same way at all scales. For example, it is possible to show that several scalar-tensor and $f(R)$-models (where f is a generic function of the Ricci scalar R) agree with observed cosmology, extragalactic and galactic observations and Solar System tests, and give rise to new effects capable of explaining the observed acceleration of the cosmic fluid and the missing matter effect of self-gravitating structures without considering dark energy and dark matter. For comprehensive reviews on the argument, see [10].

At a fundamental level, detecting new gravitational modes could be a sort of *experimentum crucis* in order to discriminate among theories since this fact would be the "signature" that GR should be enlarged or modified [11, 12].

The outline of the paper is as follows. In Sect. II, the general action of the class of theories under consideration is introduced. Then we will linearize them around a Minkowski background to find the modes of the metric perturbations. In Sect. III, we take into account the various polarizations of the massless and massive modes, while in Sect. IV we investigate the response of a single detector to a GW propagating in certain direction with each polarization mode. In Sect. V, we discuss the spectrum of the GW stochastic background where also further modes are considered. Conclusions are drawn in Sect. VI.

II. HIGHER ORDER GRAVITY

Let us generalize the action of GR by adding curvature invariants other than the Ricci scalar. Specifically, we will consider the action [1]

$$S=\int d^4x\sqrt{-g}f(R,P,Q) \tag{2.1}$$

where

$$\begin{aligned}P&\equiv R_{ab}R^{ab}\\ Q&\equiv R_{abcd}R^{abcd}\end{aligned} \tag{2.2}$$

Varying with respect to the metric one gets the field equations [13]:

$$\begin{aligned}FG_{\mu\nu} &= \frac{1}{2}g_{\mu\nu}\left(f-R\,F\right)-(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)F\\ &-2\left(f_P R^a_\mu R_{a\nu}+f_Q\ R_{abc\mu}R^{abc}{}_\nu\right)\\ &-g_{\mu\nu}\nabla_a\nabla_b(f_P R^{ab})-\Box(f_P R_{\mu\nu})\\ &+2\nabla_a\nabla_b\left(f_P\ R^a{}_{(\mu}\delta^b{}_{\nu)}+2f_Q\ R^a{}_{(\mu\nu)}{}^b\right)\end{aligned} \tag{2.3}$$

where we have set

$$F\equiv\frac{\partial f}{\partial R},\quad f_P\equiv\frac{\partial f}{\partial P},\quad f_Q\equiv\frac{\partial f}{\partial Q} \tag{2.4}$$

and $\Box=g^{ab}\nabla_a\nabla_b$ is the d'Alembert operator while the notation $T_{(ij)}=\frac{1}{2}(T_{ij}+T_{ji})$ denotes symmetrization with respect to the indices (i,j).

Taking the trace of eq. (2.3) we find:

$$\Box\left(F+\frac{f_P}{3}R\right)=$$
$$\frac{1}{3}\left(2f-RF-2\nabla_a\nabla_b((f_P+2f_Q)R^{ab})-2(f_P P+f_Q Q)\right) \tag{2.5}$$

[1] Conventions: $g_{ab}=(-1,1,1,1)$, $R^a_{bcd}=\Gamma^a_{bd,c}-\Gamma^a_{bc,d}+\ldots$, $R_{ab}=R^c_{acb}$, $G_{ab}=8\pi G_N T_{ab}$ and all indices run from 0 to 3.

Expanding the third term on the RHS of (2.5) and using the purely geometrical identity $G^{ab}{}_{;b}=0$ we get:

$$\begin{aligned}&\Box\left(F+\frac{2}{3}(f_P+f_Q)R\right)=\frac{1}{3}\times\\ &[2f-RF-2R^{ab}\nabla_a\nabla_b(f_P+2f_Q)-R\Box(f_P+2f_Q)\\ &-2(f_P P+f_Q Q)]\end{aligned} \tag{2.6}$$

If we define

$$\begin{aligned}\Phi &\equiv F+\frac{2}{3}(f_P+f_Q)R\\ &\text{and}\\ \frac{dV}{d\Phi} &\equiv \text{RHS of (2.6)}\end{aligned} \tag{2.7}$$

then we get a Klein-Gordon equation for the scalar field Φ:

$$\Box\Phi=\frac{dV}{d\Phi} \tag{2.8}$$

In order to find the various modes of the gravity waves of this theory we need to linearize gravity around a Minkowski background:

$$\begin{aligned}g_{\mu\nu}&=\eta_{\mu\nu}+h_{\mu\nu}\\ \Phi&=\Phi_0+\delta\Phi\end{aligned} \tag{2.9}$$

Then from eq. (2.7) we get

$$\delta\Phi=\delta F+\frac{2}{3}(\delta f_P+\delta f_Q)R_0+\frac{2}{3}(f_{P0}+f_{Q0})\delta R \tag{2.10}$$

where $R_0\equiv R(\eta_{\mu\nu})=0$ and similarly $f_{P0}=\frac{\partial f}{\partial P}|_{\eta_{\mu\nu}}$ (note that the 0 indicates evaluation with the Minkowski metric) which is either constant or zero. By δR we denote the first order perturbation on the Ricci scalar which, along with the perturbed parts of the Riemann and Ricci tensors, are given by (see for example Ref.[14]):

$$\begin{aligned}\delta R_{\mu\nu\rho\sigma}&=\frac{1}{2}\left(\partial_\rho\partial_\nu h_{\mu\sigma}+\partial_\sigma\partial_\mu h_{\nu\rho}-\partial_\sigma\partial_\nu h_{\mu\rho}-\partial_\rho\partial_\mu h_{\nu\sigma}\right)\\ \delta R_{\mu\nu}&=\frac{1}{2}\left(\partial_\sigma\partial_\nu h^\sigma{}_\mu+\partial_\sigma\partial_\mu h^\sigma{}_\nu-\partial_\mu\partial_\nu h-\Box h_{\mu\nu}\right)\\ \delta R&=\partial_\mu\partial_\nu h^{\mu\nu}-\Box h\end{aligned}$$

where $h=\eta^{\mu\nu}h_{\mu\nu}$. The first term of eq. (2.10) is

$$\delta F=\frac{\partial F}{\partial R}|_0\ \delta R+\frac{\partial F}{\partial P}|_0\ \delta P+\frac{\partial F}{\partial Q}|_0\ \delta Q \tag{2.11}$$

However, since δP and δQ are second order we get $\delta F\simeq F_{,R0}\ \delta R$ and

$$\delta\Phi=\left(F_{,R0}+\frac{2}{3}(f_{P0}+f_{Q0})\right)\delta R \tag{2.12}$$

Finally, from eq. (2.6) we get the Klein-Gordon equation for the scalar perturbation $\delta\Phi$

$$\begin{aligned}\Box\delta\Phi &= \frac{1}{3}\frac{F_0}{F_{,R0}+\frac{2}{3}(f_{P0}+f_{Q0})}\delta\Phi - \\ &\quad \frac{2}{3}\delta R^{ab}\partial_a\partial_b(f_{P0}+2f_{Q0}) - \frac{1}{3}\delta R\Box(f_{P0}+2f_{Q0}) \\ &= m_s^2\delta\Phi\end{aligned} \tag{2.13}$$

The last two terms in the first line are actually are zero since the terms f_{P0}, f_{Q0} are constants and we have defined the scalar mass as $m_s^2 \equiv \frac{1}{3}\frac{F_0}{F_{,R0}+\frac{2}{3}(f_{P0}+f_{Q0})}$.

Perturbing the field equations (2.3) we get:

$$\begin{aligned}&F_0(\delta R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\delta R) = \\ &-(\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu)(\delta\Phi - \frac{2}{3}(f_{P0}+f_{Q0})\delta R) \\ &-\eta_{\mu\nu}\partial_a\partial_b(f_{P0}\delta R^{ab}) - \Box(f_{P0}\delta R_{\mu\nu}) \\ &+2\partial_a\partial_b(f_{P0}\ \delta R^a{}_{(\mu}\delta^b{}_{\nu)} + 2f_{Q0}\ \delta R^a{}_{(\mu\nu)}{}^b)\end{aligned} \tag{2.14}$$

It is convenient to work in Fourier space so that for example $\partial_\gamma h_{\mu\nu} \to ik_\gamma h_{\mu\nu}$ and $\Box h_{\mu\nu} \to -k^2 h_{\mu\nu}$. Then the above equation becomes

$$\begin{aligned}&F_0(\delta R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\delta R) = \\ &(\eta_{\mu\nu}k^2 - k_\mu k_\nu)(\delta\Phi - \frac{2}{3}(f_{P0}+f_{Q0})\delta R) \\ &+\eta_{\mu\nu}k_a k_b(f_{P0}\delta R^{ab}) + k^2(f_{P0}\delta R_{\mu\nu}) \\ &-2k_a k_b(f_{P0}\ \delta R^a{}_{(\mu}\delta^b{}_{\nu)}) - 4k_a k_b(f_{Q0}\ \delta R^a{}_{(\mu\nu)}{}^b)\end{aligned} \tag{2.15}$$

We can rewrite the metric perturbation as

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{\bar{h}}{2}\ \eta_{\mu\nu} + \eta_{\mu\nu}h_f \tag{2.16}$$

and use our gauge freedom to define to demand that the usual conditions hold $\partial_\mu\bar{h}^{\mu\nu} = 0$ and $\bar{h} = 0$. The first of these conditions implies that $k_\mu\bar{h}^{\mu\nu} = 0$ while the second that

$$\begin{aligned}h_{\mu\nu} &= \bar{h}_{\mu\nu} + \eta_{\mu\nu}h_f \\ h &= 4h_f\end{aligned} \tag{2.17}$$

With these in mind we have:

$$\begin{aligned}\delta R_{\mu\nu} &= \frac{1}{2}\left(2k_\mu k_\nu h_f + k^2\eta_{\mu\nu}h_f + k^2\bar{h}_{\mu\nu}\right) \\ \delta R &= 3k^2 h_f \\ k_\alpha k_\beta\ \delta R^\alpha{}_{(\mu\nu)}{}^\beta &= -\frac{1}{2}\left((k^4\eta_{\mu\nu} - k^2 k_\mu k_\nu)h_f + k^4\bar{h}_{\mu\nu}\right) \\ k_a k_b\ \delta R^a{}_{(\mu}\delta^b{}_{\nu)} &= \frac{3}{2}k^2 k_\mu k_\nu h_f\end{aligned} \tag{2.18}$$

Using equations (2.16)-(2.18) into (2.15) and after some algebra we get:

$$\begin{aligned}&\frac{1}{2}\left(k^2 - k^4\frac{f_{P0}+4f_{Q0}}{F_0}\right)\bar{h}_{\mu\nu} = \\ &(\eta_{\mu\nu}k^2 - k_\mu k_\nu)\frac{\delta\Phi}{F_0} + (\eta_{\mu\nu}k^2 - k_\mu k_\nu)h_f\end{aligned} \tag{2.19}$$

Defining $h_f \equiv -\frac{\delta\Phi}{F_0}$ we find the equation for the perturbations:

$$\left(k^2 + \frac{k^4}{m_{spin2}^2}\right)\bar{h}_{\mu\nu} = 0 \tag{2.20}$$

where we have defined $m_{spin2}^2 \equiv -\frac{F_0}{f_{P0}+4f_{Q0}}$, while from eq. (2.13) we get:

$$\Box h_f = m_s^2 h_f \tag{2.21}$$

From equation (2.20) it is easy to see that we have a modified dispersion relation which corresponds to a massless spin-2 field ($k^2 = 0$) and a massive spin-2 ghost mode $k^2 = \frac{F_0}{\frac{1}{2}f_{P0}+2f_{Q0}} \equiv -m_{spin2}^2$ with mass m_{spin2}^2. To see this, note that the propagator for $\bar{h}_{\mu\nu}$ can be rewritten as

$$G(k) \propto \frac{1}{k^2} - \frac{1}{k^2 + m_{spin2}^2} \tag{2.22}$$

Clearly the second term has the opposite sign, which indicates the presence of a ghost, and this agrees with the results found in the literature for this class of theories [15–17].

Also, as a sanity check, we can see that for the Gauss-Bonnet term $\mathcal{L}_{GB} = Q - 4P + R^2$ we have $f_{P0} = -4$ and $f_{Q0} = 1$. Then, equation (2.20) simplifies to $k^2\bar{h}_{\mu\nu} = 0$ and in this case we have no ghosts as expected.

The solution to eqs. (2.20) and (2.21) can be written in terms of plane waves

$$\bar{h}_{\mu\nu} = A_{\mu\nu}(\overrightarrow{p}) \cdot exp(ik^\alpha x_\alpha) + cc \tag{2.23}$$

$$h_f = a(\overrightarrow{p}) \cdot exp(iq^\alpha x_\alpha) + cc \tag{2.24}$$

where

$$\begin{aligned}k^\alpha &\equiv (\omega_{m_{spin2}}, \overrightarrow{p}) \quad \omega_{m_{spin2}} = \sqrt{m_{spin2}^2 + p^2} \\ q^\alpha &\equiv (\omega_{m_s}, \overrightarrow{p}) \qquad \omega_{m_s} = \sqrt{m_s^2 + p^2}.\end{aligned} \tag{2.25}$$

and where m_{spin2} is zero (non-zero) in the case of massless (massive) spin-2 mode and the polarization tensors $A_{\mu\nu}(\overrightarrow{p})$ can be found in Ref. [18] (see equations (21)-(23)). In eqs. (2.20) and (2.23) the equation and the solution for the standard waves of General Relativity [26] have been obtained, while eqs. (2.21) and (2.24) are respectively the equation and the solution for the massive mode (see also [27]).

The fact that the dispersion law for the modes of the massive field h_f is not linear has to be emphasized. The velocity of every "ordinary" (i.e. which arises from General Relativity) mode $\bar{h}_{\mu\nu}$ is the light speed c, but the dispersion law (the second of eq. (2.25)) for the modes of h_f is that of a massive field which can be discussed like a wave-packet [27]. Also, the group-velocity of a wave-packet of h_f centered in $\overrightarrow{p}$ is

$$\overrightarrow{v_G} = \frac{\overrightarrow{p}}{\omega}, \tag{2.26}$$

which is exactly the velocity of a massive particle with mass m and momentum $\overrightarrow{p}$.

From the second of eqs. (2.25) and eq. (2.26) it is simple to obtain:

$$v_G = \frac{\sqrt{\omega^2 - m^2}}{\omega}. \tag{2.27}$$

Then, wanting a constant speed of the wave-packet, it has to be [27]

$$m = \sqrt{(1 - v_G^2)}\omega. \tag{2.28}$$

Now, before we proceed with the analysis, we should discuss the phenomenological limitations to the mass of the GW [28]. Taking into account the fact that the GW needs a frequency which falls in the range for both of space based and earth based gravitational antennas, that is the interval $10^{-4}Hz \leq f \leq 10KHz$ [1–5, 29–31], a quite strong limitation will arise. For a massive GW, from [32] it is:

$$2\pi f = \omega = \sqrt{m^2 + p^2}, \tag{2.29}$$

were p is the momentum. Thus, it needs

$$0eV \leq m \leq 10^{-11}eV. \tag{2.30}$$

A stronger limitation is given by requirements of cosmology and Solar System tests on extended theories of gravity. In this case it is

$$0eV \leq m \leq 10^{-33}eV. \tag{2.31}$$

For these light scalars, their effect can be still discussed as a coherent GW.

III. POLARIZATION STATES OF GRAVITATIONAL WAVES

Considering the above equations, we can note that there are two conditions for eq. (2.13) that depend on the value of k^2. In fact we can have a $k^2 = 0$ mode that corresponds to a massless spin-2 field with two independent polarizations plus a scalar mode, while if we have $k^2 \neq 0$ we have a massive spin-2 ghost mode and there are five independent polarization tensors plus a scalar mode. First, lets consider the case where the spin-2 field is massless.

Taking $\overrightarrow{p}$ in the z direction, a gauge in which only A_{11}, A_{22}, and $A_{12} = A_{21}$ are different to zero can be chosen. The condition $\bar{h} = 0$ gives $A_{11} = -A_{22}$. In this frame we may take the bases of polarizations defined in this way[2]

$$e_{\mu\nu}^{(+)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad e_{\mu\nu}^{(\times)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e_{\mu\nu}^{(s)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.1}$$

Now, putting these equations in eq. (2.16), it results

$$\begin{aligned} h_{\mu\nu}(t,z) &= A^{+}(t-z)e_{\mu\nu}^{(+)} + A^{\times}(t-z)e_{\mu\nu}^{(\times)} \\ &+ h_s(t - v_G z)e_{\mu\nu}^{s} \end{aligned} \tag{3.2}$$

The terms $A^{+}(t-z)e_{\mu\nu}^{(+)} + A^{\times}(t-z)e_{\mu\nu}^{(\times)}$ describe the two standard polarizations of gravitational waves which arise from General Relativity, while the term $h_s(t - v_G z)\eta_{\mu\nu}$ is the massive field arising from the generic high order $f(R)$ theory.

When the spin-2 field is massive, we have that the bases of the six polarizations are defined by

$$e_{\mu\nu}^{(+)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad e_{\mu\nu}^{(\times)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e_{\mu\nu}^{(B)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad e_{\mu\nu}^{(C)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$e_{\mu\nu}^{(D)} = \frac{\sqrt{2}}{3}\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad e_{\mu\nu}^{(s)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

---

[2] The polarizations are defined in our 3-space, not in a spacetime with extra dimensions. Each polarization mode is orthogonal to one another and is normalized $e_{\mu\nu}e^{\mu\nu} = 2\delta$. Note that other modes are not traceless, in contrast to the ordinary plus and cross polarization modes in GR.

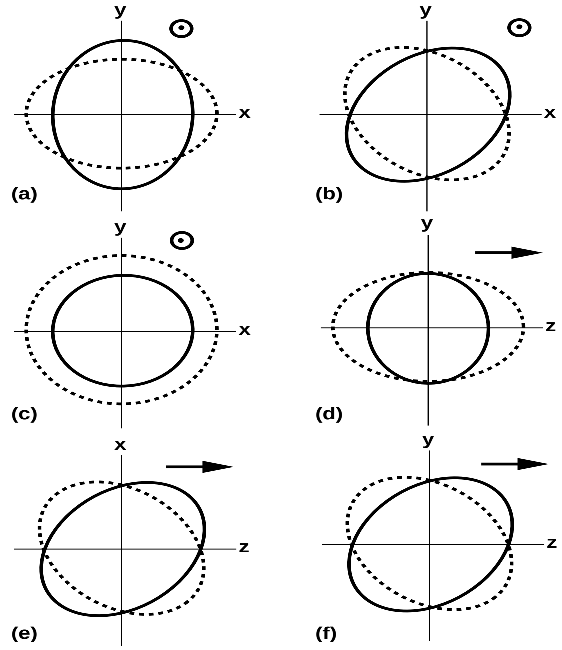

Figure 1: The six polarization modes of gravitational waves. The picture shows the displacement that each mode induces on a sphere of test particles at the moments of different phases by π. The wave propagates out of the plane in (a), (b), (c), and it propagates in the plane in (d), (e) and (f). Where in (a) and (b) we have respectively the plus mode and cross mode, in (c) the scalar mode, in (d), (e) and (f) the D, B and C mode.

and the amplitude can be written in terms of the 6 polarization states as

$$h_{\mu\nu}(t,z) = A^{+}(t - v_{G_{s2}}z)e^{(+)}_{\mu\nu} + A^{\times}(t - v_{G_{s2}}z)e^{(\times)}_{\mu\nu}$$
$$+B^{B}(t - v_{G_{s2}}z)e^{(B)}_{\mu\nu} + C^{C}(t - v_{G_{s2}}z)e^{(C)}_{\mu\nu}$$
$$+D^{D}(t - v_{G_{s2}}z)e^{(D)}_{\mu\nu} + h_s(t - v_G z)e^{s}_{\mu\nu}. \tag{3.3}$$

where $v_{G_{s2}}$ is the group velocity of the massive spin-2 field and is given by

$$v_{G_{s2}} = \frac{\sqrt{\omega^2 - m^2_{s2}}}{\omega}. \tag{3.4}$$

The first two polarizations are the same as in the massless case, inducing tidal deformations on the x-y plane. In Fig.1, we illustrate how each GW polarization affects test masses arranged on a circle.

The presence of the ghost mode may seem as a pathology of the theory from a purely quantum-mechanical approach. There are several reasons to consider such a mode as problematic if we wish to pursuit the particle picture interpretation of the metric perturbations. The ghost mode can be viewed as either a particle state of positive energy and negative probability density, or a positive probability density state with a negative energy. In the first case, allowing the presence of such a particle will quickly induce violation of unitarity. The negative energy scenario leads to a theory where there is no minimum energy and the system thus becomes unstable. The vacuum can decay into pairs of ordinary and ghost gravitons leading to a catastrophic instability.

One way out of such problems is to impose a very weak coupling of the ghost with the rest of the particles in the theory, such that the decay rate of the vacuum will become comparable to the inverse of the Hubble scale. The present vacuum state will then appear to be sufficiently stable. This is not a viable option in our theory, since the ghost state comes in the gravitational sector, which is bound to couple to all kinds of matter present and it seems physically and mathematically unlikely for the ghost graviton to couple differently than the ordinary massless graviton does. Another option is to assume that this picture does not hold up to arbitrarily high energies and that at some cutoff scale M_{cutoff} the theory gets modified appropriately as to ensure a ghost-free behavior and a stable ground state. This can happen for example if we assume that Lorentz invariance is violated at M_{cutoff}, thereby restricting any potentially harmful decay rates [33].

However, there is no guaranty that theories of modified gravity such as the one investigated here are supposed to hold up to arbitrary energies. Such models are plagued at the quantum level by the same problems as ordinary General Relativity, i.e. they are non-renormalizable. It is therefore not necessary for them to be considered as genuine candidates for a quantum gravity theory and the corresponding ghost particle interpretation becomes rather ambiguous. At the purely classical level, the perturbation $h_{\mu\nu}$ should be viewed as nothing more than a tensor representing the "stretching" of spacetime away from flatness. A ghost mode then makes sense as just another way of propagating this perturbation of the spacetime geometry, one which carries the opposite sign in the propagator than an ordinary massive graviton would.

Viewed in this way, the presence of the massive ghost graviton will induce on an interferometer the same effects as an ordinary massive graviton transmitting the perturbation, but with the opposite sign in the displacement. Tidal stretching from a polarized wave on the polarization plane will be turned into shrinking and viceversa. This signal will in the end be a superposition of the displacements coming from the ordinary massless spin-2 graviton and the massive ghost. Since these induce two competing effects, this will lead to a less pronounced signal than the one we would expect if the ghost mode was absent, setting in this way less severe constraints on the theory. However, the presence of the new modes will also affect the total energy density carried by the gravitational waves and this may also appear as a candidate signal in stochastic backgrounds, as we will see in the following.

IV. GRAVITATIONAL WAVES PROPAGATING IN A CERTAIN DIRECTION AND THE POSSIBLE DETECTOR RESPONSE

Let us consider now now the possible response of a detector revealing GWs coming from a certain direction. It is important to stress that the detector output depends on the GW amplitude that is determined by a specific theoretical model. However, one can study the detector response to each GW polarization without specifying, a priori, the theoretical model. Following [19, 22–25, 39] the angular pattern function of a detector to GWs is given by

$$\begin{aligned} F_A(\hat{\mathbf{\Omega}}) &= \mathbf{D} : \mathbf{e}_A(\hat{\mathbf{\Omega}}) \,, \qquad (4.1)\\ \mathbf{D} &= \frac{1}{2}\left[\hat{\mathbf{u}}\otimes\hat{\mathbf{u}} - \hat{\mathbf{v}}\otimes\hat{\mathbf{v}}\right] \,, \end{aligned}$$

here $A = +, \times, B, C, D, s$. The symbol $:$ is contraction between tensors. $\mathbf{D}$ is the *detector tensor* representing the response of a laser-interferometric detector. It maps the metric perturbation in a signal on the detector. The vectors $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ are unitary and orthogonal to each other. They are directed to each detector arm and form an orthonormal coordinate system with the unit vector $\hat{\mathbf{w}}$ (see Fig. 2). $\hat{\mathbf{\Omega}}$ is the vector directed along the GW propagation. Eq. (4.1) holds only when the arm length of the detector is smaller and smaller than the GW wavelength that we are taking into account. This is relevant for dealing with ground-based laser interferometers but this condition could not be valid when dealing with space interferometers like LISA.

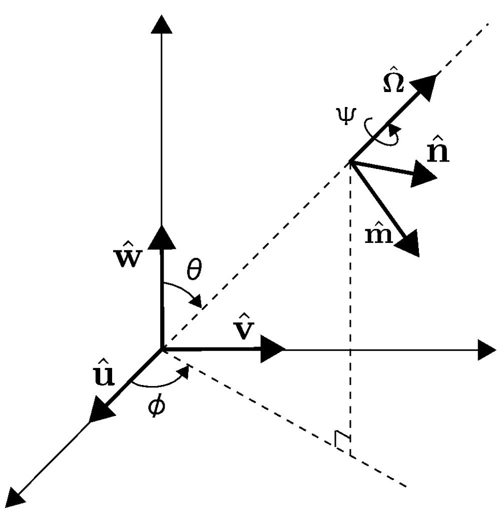

Figure 2: The coordinate systems used to calculate the polarization tensors and the pictorial view of the coordinate transformation.

A standard orthonormal coordinate system for the detector is

$$\begin{cases} \hat{\mathbf{u}} = (1,0,0) \\ \hat{\mathbf{v}} = (0,1,0) \\ \hat{\mathbf{w}} = (0,0,1) \end{cases} .$$

On the other hand, the coordinate system for the GW, rotated by angles (θ, ϕ), is given by

$$\begin{cases} \hat{\mathbf{u}}' = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta) \\ \hat{\mathbf{v}}' = (-\sin\phi, \cos\phi, 0) \\ \hat{\mathbf{w}}' = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \end{cases} .$$

The rotation with respect to the angle ψ, around the GW-propagating axis, gives the most general choice for the coordinate system, that is

$$\begin{cases} \hat{\mathbf{m}} = \hat{\mathbf{u}}'\cos\psi + \hat{\mathbf{v}}'\sin\psi \\ \hat{\mathbf{n}} = -\hat{\mathbf{v}}'\sin\psi + \hat{\mathbf{u}}'\cos\psi \\ \hat{\mathbf{\Omega}} = \hat{\mathbf{w}}' \end{cases} .$$

Coordinates $(\hat{\mathbf{u}}, \hat{\mathbf{v}}, \hat{\mathbf{w}})$ are related to the coordinates $(\hat{\mathbf{m}}, \hat{\mathbf{n}}, \hat{\mathbf{\Omega}})$ by the rotation angles $(\phi,\ \theta,\ \psi)$, as in Fig. 2. By thevectors $\hat{\mathbf{m}}$, $\hat{\mathbf{n}}$, and $\hat{\mathbf{\Omega}}$, the polarization tensors are

$$\begin{aligned} \mathbf{e}_+ &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{m}}\otimes\hat{\mathbf{m}} - \hat{\mathbf{n}}\otimes\hat{\mathbf{n}}\right) \,, \\ \mathbf{e}_\times &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{m}}\otimes\hat{\mathbf{n}} + \hat{\mathbf{n}}\otimes\hat{\mathbf{m}}\right) \,, \\ \mathbf{e}_B &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{m}}\otimes\hat{\mathbf{\Omega}} + \hat{\mathbf{\Omega}}\otimes\hat{\mathbf{m}}\right) \,, \\ \mathbf{e}_C &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{n}}\otimes\hat{\mathbf{\Omega}} + \hat{\mathbf{\Omega}}\otimes\hat{\mathbf{n}}\right) \,. \\ \mathbf{e}_D &= \frac{\sqrt{3}}{2}\left(\frac{\hat{\mathbf{m}}}{\mathbf{2}}\otimes\frac{\hat{\mathbf{m}}}{\mathbf{2}} + \frac{\hat{\mathbf{n}}}{\mathbf{2}}\otimes\frac{\hat{\mathbf{n}}}{\mathbf{2}} + \hat{\mathbf{\Omega}}\otimes\hat{\mathbf{\Omega}}\right) \,, \\ \mathbf{e}_s &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{\Omega}}\otimes\hat{\mathbf{\Omega}}\right) \,, \end{aligned}$$

Taking into account Eqs. (4.1), the angular patterns for each polarization are

$$\begin{aligned} F_+(\theta,\phi,\psi) &= \frac{1}{\sqrt{2}}(1+\cos^2\theta)\cos 2\phi\cos 2\psi \\ &\quad -\cos\theta\sin 2\phi\sin 2\psi \,, \\ F_\times(\theta,\phi,\psi) &= -\frac{1}{\sqrt{2}}(1+\cos^2\theta)\cos 2\phi\sin 2\psi \\ &\quad -\cos\theta\sin 2\phi\cos 2\psi \,, \\ F_B(\theta,\phi,\psi) &= \sin\theta\left(\cos\theta\cos 2\phi\cos\psi - \sin 2\phi\sin\psi\right) \,, \\ F_C(\theta,\phi,\psi) &= \sin\theta\left(\cos\theta\cos 2\phi\sin\psi + \sin 2\phi\cos\psi\right) \,, \\ F_D(\theta,\phi) &= \frac{\sqrt{3}}{32}\cos 2\phi\left(6\sin^2\theta + (\cos 2\theta + 3)\cos 2\psi\right) \,, \\ F_s(\theta,\phi) &= \frac{1}{\sqrt{2}}\sin^2\theta\cos 2\phi \,. \end{aligned}$$

The angular pattern functions for each polarization are plotted in Fig. 3. These results, also if we have considered a different model, are consistent, for example, with those in [19–21]. Another step is now to consider the stochastic background of GWs in order to test the possible detectability of such further contributions in gravitational radiation.

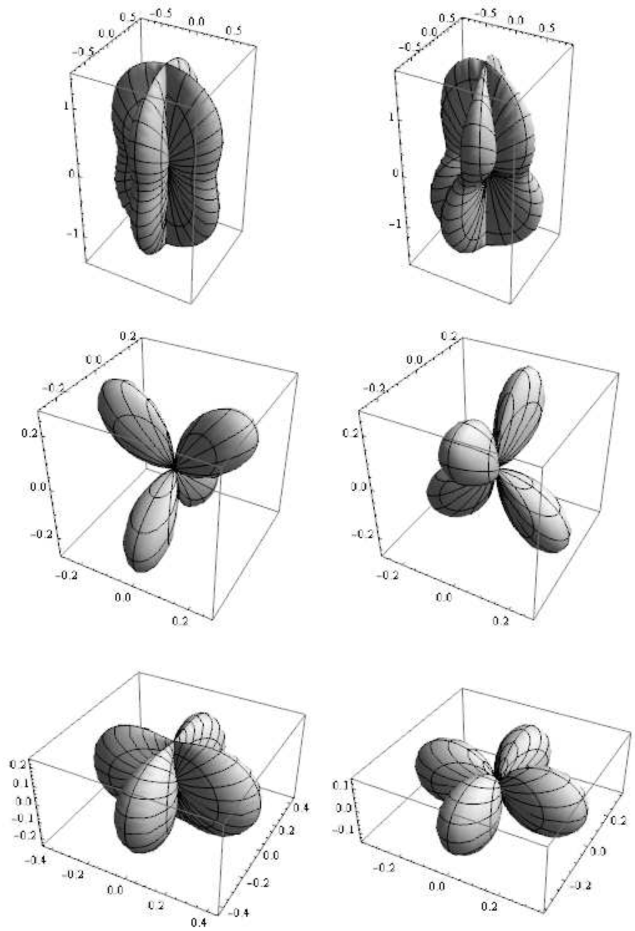

Figure 3: Plots along the panel lines from left to right of angular pattern functions of a detector for each polarization. From left plus mode F_+, cross mode $F_\times$, B mode F_B, C mode F_C, D mode F_D,and scalar mode F_s. The angular pattern function of the F_B and F_C mode is the same except for a rotation.

V. THE STOCHASTIC BACKGROUND OF GRAVITATIONAL WAVES

The contributions to the gravitational radiation coming from higher order gravity could be efficiently selected if it would be possible to investigate gravitational sources in extremely strong field regimes. In such a case, the further polarizations coming from the higher order contributions could be, in principle, investigated by the response of a single GW detector described above. However, this situation seems extremly futuristic at the moment so the only realistic approach to investigate these further contribution seems the cosmological background, in particular, the stochastic background of GWs. Such a GW background can be roughly divided into two classes of phenomena: the background generated by the incoherent superposition of gravitational radiation emitted by large populations of astrophysical sources (hard to be resolved individually [34]), and the primordial GW background generated by processes in the early cosmological eras [35]. Primordial components of such background are interesting, since they carry information on the primordial Universe and, on the other hand, can give information on the gravitational interaction at that epochs [40, 41]. The physical process of GW production has been analyzed, for example, in [36–38] but only for the first two standard tensorial components of Eq. (3.2), that is the GR components. Actually the process can be improved considering all the components that we have considered here. Before starting with the analysis, it has to be emphasized that, considering a stochastic background of GWs, it can be described and characterized by a dimensionless spectrum (see the definition [36, 37, 39, 43])

$$\Omega^A_{gw}(f) = \frac{1}{\rho_c}\frac{d\rho^A_{gw}}{d\ln f}, \tag{5.1}$$

where

$$\rho_c \equiv \frac{3H_0^2}{8\pi G} \tag{5.2}$$

is the (actual) critical energy density of the Universe, H_0 the today observed Hubble expansion rate, and $d\rho_{sgw}$ is the energy density of the part of the gravitational radiation contained in the frequency range f to $f + df$.

$$\rho_{\rm gw} = \int_0^\infty df\, \tilde{\rho}_{\rm gw}(f)\,. \tag{5.3}$$

where $\tilde{\rho}_{\rm GW}$ is the GWs energy density per unit frequency. $\Omega_{\rm gw}(f)$ is related to $S_h(f)$ by [38, 39]

$$\Omega^A_{\rm gw}(f) = \left(\frac{4\pi^2}{3H_0^2}\right) f^3 S^A_h(f)\,. \tag{5.4}$$

Note that the above definition is different from that in the literature [38, 39], by a factor of 2, since it is defined for each polarization. It is convenient to represent the energy density with the form $h_0^2\,\Omega_{\rm gw}(f)$ by parametrizing the Hubble constant as $H_0 = 100\,h_0\,{\rm km\,s^{-1}\,Mpc^{-1}}$. Then, the GW stochastic background energy density of all modes can be written as

$$\Omega^A_{\rm gw} \equiv \Omega^+_{\rm gw} + \Omega^\times_{\rm gw} + \Omega^B_{\rm gw} + \Omega^C_{\rm gw} + \Omega^D_{\rm gw} + \Omega^s_{\rm gw} \tag{5.5}$$

we can split $\Omega^A_{\rm gw}$ as a part arising from GR

$$\Omega^{GR}_{\rm gw} = \Omega^+_{\rm gw} + \Omega^\times_{\rm gw}\,, \qquad \Omega^+_{\rm gw} = \Omega^\times_{\rm gw} \tag{5.6}$$

a part from higher-order-gravity

$$\Omega^{HOG}_{\rm gw} = \Omega^B_{\rm gw} + \Omega^C_{\rm gw} + \Omega^D_{\rm gw}\,, \qquad \Omega^B_{\rm gw} = \Omega^C_{\rm gw} = \Omega^D_{\rm gw} \tag{5.7}$$

and a scalar part $\Omega^s_{\rm gw}$.

We are considering now standard units and study only the modes which arise from higher order theory.

The relic stochastic background of GWs can be derived by considering only general assumptions and basic principles of Quantum Field Theory and GR. The quantum fluctuations of the zero-point energy can be amplified in the early Universe by the large variations of gravity and this mechanism produces GWs. A very interesting by-product of GWs is that they can be used to probe the evolution of the Universe at early times, even up to the Planck epoch and the Big Bang singularity [36, 37, 39, 43]. The mechanism of the GWs is connected to inflationary scenario [44, 45], which fits well the WMAP data and is in particularly good agreement with almost exponential inflation and spectral index ≈ 1, [46, 47].

A remarkable fact about the inflationary scenario is that it contains a natural mechanism which gives rise to perturbations for any field. It is important for our aims that such a mechanism provides also a distinctive spectrum for relic scalar GWs. These perturbations in inflationary cosmology arise from the most basic quantum mechanical effect: the uncertainty principle. In this way, the spectrum of relic GWs that we could detect today is nothing else but the adiabatically-amplified zero-point fluctuations [36, 37]. The calculation for a simple inflationary model can be performed for the scalar field component of eq. (3.2). Let us assume that the early Universe is described an inflationary de Sitter phase emerging in a radiation dominated phase [36, 37, 43]. The conformal metric element is

$$ds^2 = a^2(\eta)[-d\eta^2 + d\overrightarrow{x}^2 + h_{\mu\nu}(\eta, \overrightarrow{x})dx^\mu dx^\nu], \quad (5.8)$$

where, for a purely GW the metric perturbation (3.2) reduces to

$$h_{\mu\nu} = h_A e_{\mu\nu}^{(A)}. \quad (5.9)$$

where $A = +, \times, B, C, D$, and s. Let us assume a phase transition between a de Sitter and a radiation-dominated phase [36, 37], we have: η_1 is the inflation-radiation transition conformal time and η_0 is the value of conformal time today. If we express the scale factor in terms of comoving time $cdt = a(t)d\eta$, we have

$$a(t) \propto \exp(H_{ds}t), \qquad a(t) \propto \sqrt{t} \quad (5.10)$$

for the de Sitter and radiation phases respectively. In order to solve the horizon and flatness problems, the condition $\frac{a(\eta_0)}{a(\eta_1)} > 10^{27}$ has to be satisfied. The relic scalar-tensor GWs are the weak perturbations $h_{\mu\nu}(\eta, \overrightarrow{x})$ of the metric (5.9) which can be written in the form

$$h_{\mu\nu} = e_{\mu\nu}^{(A)}(\hat{k})X(\eta)\exp(i\overrightarrow{k}\cdot\overrightarrow{x}), \quad (5.11)$$

in terms of the conformal time η where $\overrightarrow{k}$ is a constant wavevector. From eq.(5.11), the component is

$$\Phi(\eta, \overrightarrow{k}, \overrightarrow{x}) = X(\eta)\exp(i\overrightarrow{k}\cdot\overrightarrow{x}). \quad (5.12)$$

Assuming $Y(\eta) = a(\eta)X(\eta)$, from the Klein-Gordon equation in the FRW metric, one gets

$$Y'' + \left(|\overrightarrow{k}|^2 - \frac{a''}{a}\right)Y = 0 \quad (5.13)$$

where the prime $'$ denotes derivative with respect to the conformal time. The solutions of eq. (5.13) can be expressed in terms of Hankel functions in both the inflationary and radiation dominated eras, that is:
For $\eta < \eta_1$

$$X(\eta) = \frac{a(\eta_1)}{a(\eta)}[1 + iH_{ds}\omega^{-1}]\exp\left(-ik(\eta-\eta_1)\right), \quad (5.14)$$

for $\eta > \eta_1$

$$X(\eta) = \frac{a(\eta_1)}{a(\eta)}\left[\alpha\exp\left(-ik(\eta-\eta_1)\right) + \beta\exp\left(ik(\eta-\eta_1)\right)\right], \quad (5.15)$$

where $\omega = ck/a$ is the angular frequency of the wave (which is function of the time being $k = |\overrightarrow{k}|$ constant), α and β are time-independent constants which we can obtain demanding that both X and $dX/d\eta$ are continuous at the boundary $\eta = \eta_1$ between the inflationary and the radiation dominated eras. By this constraint, we obtain

$$\alpha = 1 + i\frac{\sqrt{H_{ds}H_0}}{\omega} - \frac{H_{ds}H_0}{2\omega^2}, \qquad \beta = \frac{H_{ds}H_0}{2\omega^2} \quad (5.16)$$

In eqs. (5.16), $\omega = ck/a(\eta_0)$ is the angular frequency as observed today, $H_0 = c/\eta_0$ is the Hubble expansion rate as observed today. Such calculations are referred in literature as the Bogoliubov coefficient methods [36, 37].

In an inflationary scenario, every classical or macroscopic perturbation is damped out by the inflation, i.e. the minimum allowed level of fluctuations is that required by the uncertainty principle. The solution (5.14) corresponds to a de Sitter vacuum state. If the period of inflation is long enough, the today observable properties of the Universe should be indistinguishable from the properties of a Universe started in the de Sitter vacuum state. During the radiation dominated phase, the particles are described by the eigenmodes that correspond to the coefficients of α, while the antiparticles correspond to the coefficients of β. Therefore, the number of particles that have been created at angular frequency ω in the radiation phase is given by

$$N_\omega = |\beta_\omega|^2 = \left(\frac{H_{ds}H_0}{2\omega^2}\right)^2. \quad (5.17)$$

Now it is possible to write an expression for the energy density of the stochastic scalar-tensor relic gravitons background in the frequency interval $(\omega, \omega + d\omega)$ for each mode as

$$d\rho_{gw}^A = \hbar\omega\left(\frac{\omega^2 d\omega}{2\pi^2c^3}\right)N_\omega = \frac{\hbar H_{ds}^2H_0^2}{8\pi^2c^3}\frac{d\omega}{\omega} = \frac{\hbar H_{ds}^2H_0^2}{8\pi^2c^3}\frac{df}{f}, \quad (5.18)$$

where f, as above, is the frequency in standard comoving time. eq. (5.18) can be rewritten in terms of the today and de Sitter value of energy density being

$$H_0^2 = \frac{8\pi G\rho_c}{3c^2}\,, \qquad H_{ds}^2 = \frac{8\pi G\rho_{ds}}{3c^2}. \tag{5.19}$$

Introducing the Planck density $\rho_{Planck} = \frac{c^7}{\hbar G^2}$ the spectrum is given by

$$\Omega_{gw}^A(f) = \frac{1}{\rho_c}\frac{d\rho_{gw}}{d\ln f} = \frac{f}{\rho_c}\frac{d\rho_{gw}}{df} = \frac{8}{9}\frac{\rho_{ds}}{\rho_{Planck}}. \tag{5.20}$$

At this point, some comments are in order. First of all, such a calculation works for a simplified model that does not include the matter dominated era. If we also include such an era, we would also have to take into account the redshift at the equivalence epoch and this results in [38]

$$\Omega_{gw}^A(f) = \frac{8}{9}\frac{\rho_{ds}}{\rho_{Planck}}(1+z_{eq})^{-1}, \tag{5.21}$$

for the waves which, at the epoch in which the Universe becomes matter dominated, have a frequency higher than H_{eq}, the Hubble parameter at equivalence. This situation corresponds to frequencies $f > (1+z_{eq})^{1/2}H_0$. The redshift correction in eq.(5.21) is needed since the today observed Hubble parameter H_0 would result different without a matter dominated contribution. At lower frequencies, the spectrum is given by [36, 37]

$$\Omega_{gw}(f) \propto f^{-2}. \tag{5.22}$$

As a further consideration, let us note that the results (5.20) and (5.21), which are not frequency dependent, do not work correctly in all the range of physical frequencies. Waves that have frequencies less than H_0, the energy density is in a sense not well defined, as their wavelength becomes larger than the Hubble scale of the Universe. In a similar manner, at high frequencies, there is a maximal frequency above which the spectrum rapidly drops to zero. In the above calculation, the simple assumption that the phase transition from the inflationary to the radiation dominated epoch is instantaneous has been made. In the physical Universe, this process occurs over some time scale $\Delta\tau$, being

$$f_{max} = \frac{a(t_1)}{a(t_0)}\frac{1}{\Delta\tau}, \tag{5.23}$$

which is the redshifted rate of the transition. In any case, Ω_{gw}^A drops rapidly. The two cutoffs at low and high frequencies for the spectrum guarantee that the total energy density of the relic gravitons is finite. These results can be quantitatively constrained considering the recent WMAP release. Nevertheless, since the spectrum falls off $\propto f^{-2}$ at low frequencies, this means that today, at LIGO-VIRGO and LISA frequencies, one gets for the GR part [39, 42]

$$\Omega_{gw}^{GR}(f)h_{100}^2 < 2\times 10^{-6}. \tag{5.24}$$

for the higher-order-gravity part

$$\Omega_{gw}^{HOG}(f)h_{100}^2 < 6.7\times 10^{-9}. \tag{5.25}$$

and for the scalar part

$$\Omega_{gw}^{s}(f)h_{100}^2 < 2.3\times 10^{-12}. \tag{5.26}$$

It is interesting to calculate the corresponding strain at $\approx 100Hz$, where interferometers like VIRGO and LIGO reach a maximum in sensitivity [6, 7]. With a minor modification we can use the well known equation for the characteristic amplitude [39] for one of the components of the GWs [3]:

$$h_A(f) \simeq 8.93\times 10^{-19}\left(\frac{1Hz}{f}\right)\sqrt{h_{100}^2\Omega_{gw}(f)}, \tag{5.27}$$

and then we obtain for the GR modes

$$h_{GR}(100Hz) < 1.3\times 10^{-23}. \tag{5.28}$$

while for the higher-order modes

$$h_{HOG}(100Hz) < 7.3\times 10^{-25}. \tag{5.29}$$

and for scalar modes

$$h_s(100Hz) < 2\times 1.410^{-26}. \tag{5.30}$$

Then, since we expect a sensitivity of the order of 10^{-22} for the above interferometers at $\approx 100Hz$, we need to gain at least three orders of magnitude. At smaller frequencies the sensitivity of the VIRGO interferometer is of the order of 10^{-21} at $\approx 10Hz$ and in that case it is for the GR modes

$$h_{GR}(100Hz) < 1.3\times 10^{-22}. \tag{5.31}$$

while for the higher-order modes

$$h_{HOG}(100Hz) < 7.3\times 10^{-24}. \tag{5.32}$$

and for scalar modes

$$h_s(100Hz) < 1.4\times 10^{-25}. \tag{5.33}$$

Still, these effects are below the sensitivity threshold to be observed. The sensitivity of the LISA interferometer will be of the order of 10^{-22} at $\approx 10^{-3}Hz$ (see [8]) and in that case it is

$$h_{GR}(100Hz) < 1.3\times 10^{-18}. \tag{5.34}$$

[3] The difference between our result and eq. (19) in Ref. [39] is due to the fact that the latter did their calculation assuming the two polarization modes of GR while we handle each mode separately, hence the $\frac{1}{\sqrt{2}}$ difference.

while for the higher-order modes

$$h_{HOG}(100Hz) < 7.3 \times 10^{-20}. \tag{5.35}$$

and for scalar modes

$$h_s(100Hz) < 1.4 \times 10^{-21}. \tag{5.36}$$

This means that a stochastic background of relic GWs could be, in principle, detected by the LISA interferometer, including the additional modes.

VI. CONCLUSIONS

Our analysis covers extended gravity models with a generic class of Lagrangian density with higher order and terms of the form $f(R, P, Q)$, where $P \equiv R_{ab}R^{ab}$ and $Q \equiv R_{abcd}R^{abcd}$. We have linearized the field equations for this class of theories around a Minkowski background and found that, besides a massless spin-2 field (the graviton), the theory contains also spin-0 and spin-2 massive modes with the latter being, in general, ghosts. Then, we have investigated the detectability of additional polarization modes of a stochastic GW with ground-based laser-interferometric detectors and space-interferometers. Such polarization modes, in general, appear in the extended theories of gravitation and can be utilized to constrain the theories beyond GR in a model-independent way.

However, a point has to be discussed in detail. If the interferometer is directionally sensitive and we also know the orientation of the source (and of course if the source is coherent) the situation is straightforward. In this case, the massive mode coming from the simplest extension, $f(R)$-gravity, would induce longitudinal displacements along the direction of propagation which should be detectable and only the amplitude due to the scalar mode would be the true, detectable, "new" signal [27]. But even in this case, we could have a second scalar mode inducing a similar effect, coming from the massive ghost, although with a minus sign. So in this case, one has deviations from the prediction of $f(R)$-gravity, even if only the massive modes are considered as new signal.

On the other hand, in the case of the stochastic background, there is no coherent source and no directional detection of the gravitational radiation. What the interferometer picks is just an averaged signal coming from the contributions of all possible modes from (uncorrelated) sources all over the celestial sphere. Since we expect the background to be isotropic, the signal will be the same regardless of the orientation of the interferometer, no matter how or on which plane it is rotated, it would always record the characteristic amplitude h_c. So there is intrinsically no way to disentangle any of the mode in the background, being h_c related to the total energy density of the gravitational radiation, which depends on the number of modes available. Every mode, essentially, contributes in the same manner, at least in the limit where the mass for the massive and ghost modes are very small (as they should be). So, it should be the number of the modes available that makes the difference, not their origin.

Again, even if this does not hold, one should still get into consideration at least the massive ghost mode to get a constraint. This is the why we have considered only h_{GR}, h_{HOG} and h_s in the above cross-correlation analysis without giving further fine details coming from polarization. For the situation considered here, we find that the massive modes are certainly of interest for direct attempts at detection with the LISA experiment. It is, in principle, possible that massive GW modes could be produced in more significant quantities in cosmological or early astrophysical processes in alternative theories of gravity, being this possibility still unexplored. This situation should be kept in mind when looking for a signature distinguishing these theories from GR, and seems to deserve further investigation.

Acknowledgements

We would like to thank H. Collins, R. Boels, C. Charmousis and L. Milano for useful discussions and comments. C.B. is supported by the CNRS and the Université de Paris-Sud XI. M.D. acknowledges the support by VIRGO collaboration. S.N. acknowledges the support by the Niels Bohr International Academy, the EU FP6 Marie Curie Research & Training Network "UniverseNet" under Contract No. MRTN-CT-2006-035863 and the Danish Research Council under FNU Grant No. 272-08-0285.

[1] F. Acernese et al. (the Virgo Collaboration) - Class. Quant. Grav. 24, 19, S381- S388 (2007)
[2] B. Willke et al. - Class. Quant. Grav. 23 8S207-S214 (2006)
[3] D. Sigg (for the LIGO Scientific Collaboration) - www.ligo.org/pdf public/P050036.pdf
[4] B. Abbott et al. (the LIGO Scientific Collaboration) - Phys. Rev. D 72, 042002 (2005)
[5] M. Ando and the TAMA Collaboration - Class. Quant. Grav. 19 7 1615-1621 (2002), D. Tatsumi, Y. Tsunesada and the TAMA Collaboration - Class. Quant. Grav. 21 5 S451-S456 (2004)
[6] http://www.ligo.org/pdf public/camp.pdf.; http://www.ligo.org/pdf public/hough02.pdf.
[7] http://www.virgo.infn.it
[8] www.lisa.nasa.gov; www.lisa.esa.int

[9] C. M. Will, Living Rev. in Relativity **9**, 3 (2006) (http://www.livingreviews.org/lrr-2006-3 or gr-qc/0510072)
[10] E.J. Copeland, M. Sami, S. Tsujikawa, Int. Jou. Mod. Phys. **D 15**, 1753 (2006); S.Nojiri and S.D. Odintsov, Int. J. Meth. Mod. Phys. **4**, 115 (2007); S. Capozziello, M. Francaviglia , Gen. Rel. Grav.**40**, 357 (2008); T. P. Sotiriou, V. Faraoni, arXiv:0805.1726 [gr-qc].
[11] S. Bellucci, S. Capozziello, M. De Laurentis, V. Faraoni, Phys. Rev. **D 79**, 104004 (2009).
[12] S. Capozziello, E. Elizalde, S. Nojiri, S. D. Odintsov. Phys. Lett. **B 671**, 193 (2009).
[13] S. M. Carroll, A. De Felice, V. Duvvuri, D. A. Easson, M. Trodden and M. S. Turner, Phys. Rev. D **71**, 063513 (2005)
[14] S. M. Carroll, arXiv:gr-qc/9712019.
[15] A. Nunez and S. Solganik, Phys. Lett. B **608**, 189 (2005) [arXiv:hep-th/0411102].
[16] T. Chiba, JCAP **0503**, 008 (2005) [arXiv:gr-qc/0502070].
[17] K. S. Stelle, Gen. Rel. Grav. **9**, 353 (1978).
[18] H. van Dam and M. J. G. Veltman, Nucl. Phys. B **22**, 397 (1970).
[19] L. S. Finn and P. J. Sutton, Phys. Rev. D 65, 044022, (2002); B. Abbott et al., *Phys. Rev.* **D 76**, 082003 (2007).
[20] A. Nishizawa, A. Taruya, K. Hayama, S. Kawamura, M. Sakagami, *Phys. Rev.* **D 79**, 082002 (2009).
[21] M. E. Tobar, T. Suzuki, K. Kuroda, *Phys. Rev.* **D 79**, 082002 (2009).
[22] N. Bonasia, M. Gasperini, *Phys. Rev.* **71**, 104020 (2005).
[23] D. Babusci, L. Baciotti, F. Fucito, A Nagar, *Phys. Rev.* **64**, 062001 (2001).
[24] A. Vicere', *Phys. Rev.* **66**, 062002 (2002).
[25] P. Leaci et al. *Class. Quant. Grav.* **25**, 195018 (2008).
[26] C. W. Misner, K. S. Thorne and J. A. Wheeler - "Gravitation" - W.H.Feeman and Company - 1973
[27] S. Capozziello, M. De Laurentis, C. Corda, Phys. Lett. B 699, 255 (2008).
[28] C. Corda, JCAP 0704 (2007) 009.
[29] D. Tatsumi, Y. Tsunesada and the TAMA Collaboration - Class. Quant. Grav. **21** 5 S451 (2004) .
[30] www.lisa.nasa.gov
[31] www.lisa.esa.int
[32] S. Capozziello and C. Corda - Int. J. Mod. Phys. D **15** 1119 (2006)
[33] R. Emparan and J. Garriga, JHEP **0603** (2006) 028 [arXiv:hep-th/0512274].
[34] A.J. Farmer and E.S. Phinney, Mon. Not. Roy. Astron. Soc. **346**, 1197 (2003).
[35] A. Buonanno, Proceedings of the Les Houches Summer School, Particle Physics and Cosmology: The Fabric of Spacetime, Les Houches, France, 31 Jul - 25 Aug 2006, arXiv:0709.4682
[36] B. Allen -Proceedings of the Les Houches School on Astrophysical Sources of Gravitational Waves, eds. Jean-Alain Marck and Jean-Pierre Lasota (Cambridge University Press, Cambridge, England 1998).
[37] L. Grishchuk et al. - Phys. Usp. 44 1-51 (2001); Usp.Fiz.Nauk 171 3-59 (2001).
[38] B. Allen - Phys. Rev. D 3-7,2078 (1988).
[39] M. Maggiore - Phys. Rep. 331, 283-367 (2000).
[40] S. Capozziello, M. De Laurentis, M. Francaviglia, *Astrop. Phys.* **29** 125 (2008).
[41] S. Capozziello, Ch. Corda, M. De Laurentis *Mod. Phys. Lett* **A 22**, 2647 (2007).
[42] A. Buonanno, arXiv:gr-qc/0303085.
[43] B. Allen and A.C. Ottewill - Phys. Rev. D 56, 545-563 (1997).
[44] G.S. Watson- "An exposition on inflationary cosmology" - North Carolina University Press (2000).
[45] A. Guth - Phys. Rev. 23, 347 (1981).
[46] C.L. Bennet et al. - ApJS 148, 1.
[47] D.N. Spergel et al. - ApJS 148, 195.