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Exporting Delays and Exporting Failures

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Demand Uncertainty: Exporting Delays and Exporting Failures*

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Abstract

This paper presents a model of trade that explains why firms wait to export and why many exporters fail. Firms face uncertain demands that are only realized after the firm enters the destination. The model retools the timing of uncertainty resolution found in productivity heterogeneity models. This retooling addresses several shortcomings. First, the imperfect correlation of demands reconciles the sales variation observed in and across destinations. Second, since demands for the firm's output are correlated across destinations, a firm can use previously realized demands to forecast unknown demands in untested destinations. The option to forecast demands causes firms to *delay* exporting in order to gather more information about foreign demand. Third, since uncertainty is resolved after entry, many firms enter a destination and then exit after learning that they cannot profit. This prediction reconciles the high rate of exit seen in the first years of exporting. Finally, when faced with multiple countries in which to export, some firms will choose to sequentially export in order to slowly learn more about its chances for success in untested markets.

Keywords: firm heterogeneity, exporting, trade failures, trade delay

JEL Codes: F12

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1 Introduction

Productivity heterogeneity models of international trade have gained some traction in recent years[1]. Inspired by empirical works documenting the differences between firms that do and do not export, models such as Melitz (2003) plausibly explain why only a fraction of firms export[2]. In these models, high fixed and variable costs of exporting prevent all but the most productive firms from exporting. This self-selection mechanism has empirical support over other explanations for firm exporting, such as learning-by-exporting.[3]

However, productivity heterogeneity models cannot reconcile several recently uncovered facts. For instance, productivity heterogeneity cannot fully explain the variation of firms sales within a destination. Since productivity is anchored to the firm and translates monotonically to firm sales, these models predict that variation in productivities for a set of firms selling to a destination should fully explain the variation in sales for that set. Recent works have found that firm-specific variation accounts for less than a third of total sales variation.[4]

Since productivities in Melitz (2003) are realized before the firm supplies to any destination, a firm that begins exporting should export immediately to all destinations and forever. This prediction is inconsistent with evidence that most firms delay entry into exporting[5], and that many firms stop exporting almost immediately after they begin.[6]. As Figure 1 shows, over a third of Colombian firm that exported in the 1980's stopped after only one year, and that the exporting hazard rate decreases with time length of exporting[7]. Melitz (2003) is also inconsistent with the pattern of export expansion of Colombian

[1] Notable examples include Eaton and Kortum (2002), Bernard, Eaton, Jensen, and Kortum (2003), Melitz (2003), Bernard, Jensen, and Schott (2006), and Bernard, Jensen, Redding, and Schott (2007).

[2] Notable works include Aw, Chung, and Roberts (2000), Bernard and Jensen (1995, 1999) and Clerides, Lach, and Tybout (1998).

[3] See survey by Greenaway and Kneller (2007)

[4] See Eaton, Kortum, Kramarz (2008), Lawless and Whelan (2008), and Munch and Nguyen (2008)

[5] Damijan, Kostevc, and Polanec (2006) find that Slovenian firms supply domestically for two to four years before they start exporting.

[6] Eaton, Enslava, Kugler, and Tybout (2007) find that nearly half of Colombian firms who started exporting in 1997 stopped the following year.

[7] Colombian firm data statistics calculated by author from dataset generously provided by Jim Tybout.

firms, in which firms slowly expand the set of destinations to which they export.[8]

Insert Figure 1 here

To reconcile these patterns, I introduce a model of trade akin to Melitz (2003) with two novel contributions. The first is that I allow for imperfect correlation of firm heterogeneity across destinations. I do this by interpreting the heterogeneity in demand space: firms face firm-destination specific perceived quality draws. In Melitz (2003), a model where the heterogeneity is perfectly correlated across destinations, firms enter all profitable markets simultaneously. In this new model, firms use realized demands in supplied destinations to forecast demands in unsupplied destinations. In a free entry equilibirium, the ability to forecast demands causes firms to *delay* exporting in order to gather more information about foreign demand. This feature of the model reconciles the observed delays in exporting (Damijan, Kostevc, and Polanec 2006). In a multi-country setting, this forecasting ability results in some firms slowly adding countries to their set of exporting destinations, reconciling the pattern of sequential export expansion (Eaton, Enslava, Kugler, and Tybout 2007).

The second difference between this model and Melitz (2003) is the uncertainty resolution timing. A firm in Melitz (2003) realizes its productivity before any supply decisions are made; the firm perfectly forecasts profits as soon as it is born. Those firms that "fail" in Melitz never supply to any destination; they are not firms that we can see in the sales data. The current model moves the resolution timing until after the firm enters the destination. This results in some firms garnering negative profits. Demands are time-invariant so once the firm supplies to the destination once, it can forecast profits in that market forever. Those who garner negative profits exit the destination the following period. I term an exit after a single period a *Failure*. If the destination is a foreign country, I term it an *Exporting Failure*. This feature of the model reconciles the high initial exporting failures seen in Figure 1.

See Clerides, Lach, and Tybout (1998) for data details.

[8] See Eaton, Enslava, Kugler, and Tybout (2007)

Marketing research points to demand uncertainty as the driver of failures. Table 1 summarizes the results of marketing studies of product failures. Only one of the eight studies points to unexpected high cost as a cause of failures while all of them attribute failures to over-optimistic forecasts of market demand[9].

Table 1: Reasons for New Product Failures

	Study							
	1	2	3	4	5	6	7	8
Unexpected high cost						**X**		
Value to potential buyers was overestimated	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**
Poor planning	X	X		X	X	X	X	
Timing Wrong	X	X	X	X		X		
Enthusiasm crowded on facts				X	X	X	X	X
Product failed	X	X				X		
Product lacked a Champion					X			
Company politics					X			

1.Abrams (1974), 2.Angelus (1969), 3.Booz, Allen, & Hamilton (1968), 4.Constandse (1971), 5.Diehl (1972), 6.Hopkins & Bailey (1971), 7.MacDonald (1967), 8.Miles (1974)
Reproduced from Crawford (1977)

Other papers have overcome the imperfect correlation of sales variation across destinations by layering additional sources of firm-destination-specific heterogeneity on top of the firm-specific productivity heterogeneity[10] or by layering both quality and productivity heterogeneity together[11]. This model is able to reconcile this pattern with a single source of heterogeneity.

The Melitz (2003) model cannot explain the exporting delays and exporting failures present in literature. Recent studies[12] have modeled delays and failures by adding time-varying productivity shocks. Firms experiencing these shocks oscillate back and forth

[9] Marketing studies lament the persistence of high failure rates in light of 75 years of marketing advances. The firm can spend an exorbant amount of money forecasting market demand only to produce a product that the market ultimately rejects. Recent examples include new Coke and HD-DVD. This points towards a mechanism of failure that cannot be overcome by increasing advertising or other fixed costs.

[10] See for example Eaton, Kortum, and Kramarz (2008), Ghironi and Melitz (2005), and Das, Roberts, and Tybout (2007).

[11] e.g. Hallak and Svidasan (2008), and Benedetti Fasil and Borota (2010).

[12] See Luttmer (2004), Ghironi and Melitz (2005), Irarrazabal and Opromolla (2005)

across the exporting productivity cutoff. These firms start and stop exporting based on the direction of the exogenous shock. The time-varying productivity channel certainly explains some of the patterns described above, but has some shortcomings. The channel is one of exogenous shocks, so extracting implications from those models are more difficult. This paper suggests an orthogonal mechanism by which firms decide delay and stop exporting[13]. In addition, time-varying productivity cannot reconcile why some exporters are less productive than nonexporters, which this paper can.

Instead of varying firm heterogeneity across time, this paper varies it across destinations. Exporting failures arise, not from exogenous shocks, but because firms do not know whether they can succeed in a destination before it is actually in that destination. Exporting delays occur, not because firms are waiting for exogenous shocks, but because a zero profit general equilibrium prevents a new firm from supplying both home and foreign destinations immediately. It has to choose one destination. By delaying, the firm learns more about itself before deciding whether to export. In a multicountry setting, this learning process enables some firms to supply to single additional destination, in order to learn more about themselves before expanding further.

One additional contribution of the model is the ability to reconcile the existence of export-only firms, which are firms that export all of their output. Approximately one percent of American firms in 1987 and five percent of Colombian firms in the 1980's were export-only firms[14]. The model shows how this can occur in the cases of both symmetric and asymmetric countries.

The following section presents the structure of the model. It describes an overview of the economy, and decribes the supply decisions firms must make each period. I then restrict the model to two countries in Section 3. I show the general equilibrium testing strategies of firms in a simple symmetric case, and present numerical results. I then extend the base model to a more general assymetric two country case and a multiple

[13] There can be other explanations of exporting delay, e.g. financial constraints by Bellone, Musso, Nesta, & Sahiavo (2010).

[14] For American firms, see Figure 1 in Bernard and Jensen (1995). Columbian firm statistics calculated by author from dataset generously provided by Jim Tybout.

symmetric country case. The assymetric country case shows that when one country is much larger than the other, firms will rather sell to the larger market first. The multiple symmetric country case shows that some firms will export to a subset of available untested destinations, instead of all available destinations, even if those destinations are ex ante identical.

2 Model Structure

The model is an extension of Krugman's (1980) model of trade in varieties and most resembles Melitz (2003). In Melitz (2003), single-variety firms are differentiated by marginal costs of production. These costs are perfectly correlated across destinations and resolved before supply decisions are made. In the current model, firms are differentiated by demand shifters. The shifters are imperfectly correlated across destinations and resolved after initial supply decisions are made. These two changes lead to a richer story of how firms decide when and where to sell.

The world consists J countries and an infinite horizon of discrete time periods t. Consumers in each country $j \in (1, ..., J)$ consume both a homogenous good and a differentiated good. The homogenous good is produced with a constant returns to scale production technology and traded freely. This equalizes wages across countries, which we normalize to one. We focus the rest of the exposition on the differentiated good. In every country, there is a potentially limitless number of firms that produce unique varieties of the differentiated good to sell in one or more destinations[15]. Time-invariant preferences for the differentiated good can be represented by the utility function u_j :

$$u_j(\{q_{j\omega t}\}_{\omega\in\Omega_j t}) = \int_{\omega\in\Omega_{jt}} \exp\left(\frac{X_{j\omega}}{\sigma}\right)(q_{j\omega t})^{\frac{\sigma-1}{\sigma}} d\omega$$

where $q_{j\omega t}$ is the quantity of variety ω consumed in j at time t, $\sigma > 1$ is a measure of the elasticity of substitution between these varieties, and $X_{j\omega}$ is a random variable

[15] This is a one-variety-per-firm model, but the number of varieties a firm produces has no bearing on the model predictions as long as the costs and demands are variety-specific and not firm-specific.

determining $j's$ time-invariant *perceived quality* of ω. $X_{j\omega}$ can also be interpreted as the appeal, or popularity, of ω in j. Given Ω_{jt}, the set of varieties available to consumers in j at t, destination $j's$ demand for variety ω can be expressed as

$$q_{j\omega t} = \frac{p_{j\omega t}^{-\sigma} Y_{jt}}{\Pi_{jt}} \exp(X_{j\omega}) \tag{1a}$$

$$\Pi_{jt} = \int_{\omega \in \Omega_{jt}} \exp(X_{j\omega}) p_{j\omega t}^{1-\sigma} d\omega \tag{1b}$$

where $p_{j\omega}$ is the destination price of variety ω, Y_{jt} is the total expenditure of j on Ω_{jt}, and Π_{jt} is the endogenous level of competition in j. Π_{jt} is large enough to be unaffected by a price change of any single variety ω. It resembles the inverse of the usual CES price index.

Firms produce their unique varieties using a production technology that is identical across varieties. This differs from Melitz (2003), in which firms produce using idiosyncratic technologies. The labor cost l_{ijt} required to produce q_t units of any variety from country i and supply them to destination j in period t is

$$l_{ijt} = \tau_{ij} q_t + f \tag{2}$$

That is, there are fixed costs f and marginal costs τ_{ij} of supply[16]. The firm must pay these fixed costs for each period t and each destination j to which it supplies. The destination fixed costs represent, for example, storefront rent, fixed shipping and port fees, or advertising costs[17]. The marginal costs represent, for example, variable production, transport, and tariff costs. All discussed variables except for $x_{j\omega}$ are known to the firm owner at all times.

Faced with the production costs (2) and demand (1) , the owner of a firm ω in country

[16] In this baseline model, the fixed cost is assumed to be constant for the two destinations, while the marginal costs differ. The model has similar export delay/failure predictions if fixed costs, instead of marginal costs, differed across destinations. Both costs affect the cutoffs in equation 7 similarly.

[17] This paper considers fixed advertising costs to be constant and exogenous, but others have examined endogenous advertising costs in a partial equilibrium setting (e.g. Arkolakis 2009, Gormsen 2009)

i sets the price $p_{j\omega t}$ as a constant mark-up over marginal cost:

$$p_{j\omega t} = \frac{\sigma}{\sigma - 1}\tau_{ij}. \tag{3}$$

Since the optimal price is the same across all periods and varieties produced in i and sold in j, I simplify notation by defining $p_{ij} = \frac{\sigma}{\sigma-1}\tau_{ij}$. This optimal pricing results in the period t profits of

$$\pi_{ijt}\left(X_{j\omega}\right) = \frac{p_{ij}^{1-\sigma} Y_{jt}}{\sigma \Pi_{jt}} \exp(X_{j\omega}) - f \tag{4}$$

The firm owner observes $X_{j\omega}$ only if she supplies ω to j at least once. If $X_{j\omega}$ is known, then $\pi_{ijt}\left(X_{j\omega}\right)$ is perfectly forecasted for all future periods. If not, the firm owner must decide whether to supply ω to j based on her beliefs about $X_{j\omega}$. If the distribution from which $X_{j\omega}$ is drawn is degenerate, $\exp(X_{j\omega})$ is constant for all varieties and can be factored out, reducing demand equation (1) to that presented in Krugman (1980), although destination profit equation 4 stills differs from Krugman (1980) by the $-f$ term.

I examine steady state equilibria in which aggregate market conditions do not change over time. Therefore, Y_{jt} and Π_{jt} can be characterized by Y_j and Π_j. I drop the t subscript whereever it is superfluous.

2.1 The Distribution of Perceived Qualities

This section discusses the multivariate random vector $X_\omega = (X_{1\omega}, .., X_{j\omega}, ..X_{J\omega})$ consisting of J random variables each corresponding to the perceived quality of ω in a destination. It is a continuous random vector with a joint multivariate normal pdf[18] denote by $g\left(\cdot\right)$[19].

[18] This is not a critical assumption for the qualitative results of the model. A normal distribution is used so that the resultant sales are lognormally distributed. The lognormal distribution more closely matches firm size patterns in Bernard, Eaton, Jensen, and Kortum (2003) and Eaton, Kortum, and Kramarz (2004) than the commonly used Pareto distribution. Cabral and Mata (2003) show that firm sizes are distributed lognormally. Axtell (2001) argues that firm sizes are Pareto distributed, but concedes that the tails of the distribution are not Pareto. A truncated lognormal may fit the data best.

[19] for example, $g\left(x_{j\omega}, x_{i\omega}\right) = \left(2\pi s^2\sqrt{1-\rho^2}\right)^{-1} \exp\left(-\left(2s^2\left(1-\rho^2\right)\right)^{-1}\left(x_{j\omega}^2 - 2\rho x_{j\omega}x_{i\omega} + x_{i\omega}^2\right)\right)$

For $j \neq k$, $X_{j\omega}$ and $X_{k\omega}$ have the properties

$$E\left[X_{j\omega}\right] = E\left[X_{k\omega}\right] = 0 \tag{5a}$$

$$Var\left[X_{j\omega}\right] = Var\left[X_{k\omega}\right] = s^2 \tag{5b}$$

$$\frac{Cov\left(X_{j\omega}, X_{k\omega}\right)}{s^2} = \rho \tag{5c}$$

where $s^2 > 0$ is the variance of each of the marginal distributions and $0 < \rho < 1$ is the correlation coefficient. The restriction on ρ implies that the demands in any two destinations are positively and imperfectly correlated.

Using Bayesian updating, I determine how the firm owner forecasts perceived qualities in untested markets. To minimize confusion, I number the destinations so that firm ω has observed perceived qualities in the first I destinations. Then the conditional distribution of any unknown $X_{j\omega}$, given the vector of known perceived qualities $\vec{x}^I_\omega \equiv (X_{1\omega} = x_{1\omega}, ..., X_{I\omega} = x_{I\omega})$, is normal with moments[20]:

$$E\left[X_{j\omega}|\vec{x}^I_\omega\right] = \mu_{I\omega} = \frac{\rho\sum_{i=1}^{I} x_{i\omega}}{I\rho + (1-\rho)} \tag{6a}$$

$$VAR\left[X_{j\omega}|\vec{x}^I_\omega\right] = \chi^2_I = s^2\left(1 - \frac{I\rho^2}{I\rho + (1-\rho)}\right). \tag{6b}$$

Note that if $I = 0$, the moments in Equation 6 collapse to those in Equation 5. As the number of known observations I increases, the forecasted perceived quality $\mu_{I\omega}$ approaches the sample mean of the observed perceived qualities; the forecast gets closer to the firm average and away from the prior. Also, as I increases, the conditional variance χ^2_I decreases: the forecast becomes more precise with each observation.

The density of $X_{j\omega}$ given $\vec{x}^I_\omega$ is completely characterized by a normal distribution with a mean of $\mu_{I\omega}$ and variance determined by I. I denote the conditional pdf as $g_I\left(\cdot|\mu_{I\omega}\right)$.

In summary, the firm owner has beliefs about her perceived quality in a destination. These beliefs depend on both the number of destinations previsouly tested, as well as the

[20] I derive this in Appendix A

average of perceived qualities observed in those destinations. In each time period, the firm owner uses these beliefs to make decisions about which destinations to supply.

2.2 Firm Decisions

At the beginning of each period t, the firm owner must decide whether to supply to each of the $j \in (1, ..., J)$ destinations, using the information gleaned from having previously supplied to I of the destinations. These J decisions fall into one of two categories. First, for each of the I previously tested destinations, the firm owner decides whether to *stay* and continue supplying there. Second, for each of the $J - I$ untested destinations, she decides whether to *test* her variety there. We examine each of these two sets of decisions.

2.2.1 Decision to Stay

Consider the firm owner's decision for a destination j that she supplied to in a period before t and thus previously realized $X_{j\omega} = x_{j\omega}$. She will supply ω to j in t if the known profits $\pi_{ij}(x_{j\omega})$ are positive. Rearranging equation (4) shows that $\pi_{ij}(x_{j\omega}) > 0$ only if $x_{j\omega} > x^*_{ij}$, where

$$x^*_{ij} = \ln\left(\frac{f\sigma\Pi_j}{p_{ij}^{1-\sigma}Y_j}\right) \tag{7}$$

If $x_{j\omega} > x^*_{ij}$, the firm makes positive profits by supplying ω to j again. I term this the decision to *stay* in j, since the firm has previously supplied to j. If $x_{j\omega} < x^*_{ij}$, the firm will not supply ω to j in t or in any period after t. I term this a *failure* of ω in j. The *stay cutoff* x^*_{ij} is the minimum perceived quality required by firms in country i to not fail in destination j.

2.2.2 Decision to Test

If the variety ω has not previously been supplied to j, the firm owner must decide whether to supply ω to j for the first time. I term this the decision to *test* ω in destination j.

The firm owner will test j based on her expectations of the lifetime discounted sum of per-period-profits. The profits are discounted because the firm faces an exogenous death rate of δ. For a firm from destination i testing destination j with I known perceived qualities and a forecasted perceived quality of $\mu_{I\omega}$, the expected testing profit $v_{ijI}\left(\mu_{I\omega}\right)$ is

$$v_{ijI}\left(\mu_{I\omega}\right)=\overbrace{\int_{-\infty}^{\infty}\pi_{ij}\left(x\right)g_{I}\left(x|\mu_{I\omega}\right)dx}^{\text{Expected testing period profits}}+\underbrace{\sum_{t=1}^{\infty}\left(1-\delta\right)^{t}}_{\text{Discounted Sum}}\underbrace{\left(\int_{x_{ij}^{*}}^{\infty}\pi_{ij}\left(x\right)g_{I}\left(x|\mu_{I\omega}\right)dx\right)}_{\text{Expected future period profits}} \tag{8}$$

and comprises the profits the firm expects in the first supply period and the discounted sum of all future expected profits, should the firm realize a favorable perceived quality. It is easier to work with the expected testing profit by combining Equations 4, 7 and 8 to produce

$$\begin{aligned}v_{ijI}\left(\mu_{I\omega}\right) &= f\int_{-\infty}^{\infty}\exp\left(x-x_{ij}^{*}\right)g_{I}\left(x|\mu_{I\omega}\right)dx-f \\ &\quad+\frac{(1-\delta)}{\delta}f\int_{x_{ij}^{*}}^{\infty}\left(\exp\left(x-x_{ij}^{*}\right)-1\right)g_{I}\left(x|\mu_{I\omega}\right)dx. \end{aligned}\tag{9}$$

Equation 9 shows that the endogenous variable x_{ij}^{*} completely characterizes the aggregate destination market conditions for firm ω. The interactions between destination specific characteristics p_{ij}, Y_j and Π_j in equations 4 and 7 determine x_{ij}^{*}.

It is straightforward to show that $v_{ijI}\left(\mu\right)$ increases with μ and a unique $\mu_{ijI}^{+}<x_{ij}^{*}$ exists for each ijI triplet such that the expected testing profit is positive only if $\mu>\mu_{ijI}^{+}$[21]:

$$\mu>\mu_{ijI}^{+}\Rightarrow v_{ijI}\left(\mu\right)>0 \tag{10}$$

[21] For proof, see Appendix B

.

I term μ_{ijI}^{+} the *testing cutoff*. It is the minimum forecasted perceived quality that a firm from i with I previous observations requires in order to profitably test j.

A firm from country i with I previous observations has J supply decisions determined by the endogenous aggregate state variables represented by the vector $\Theta_{iI} = \left(x_{ij}^{*}, \mu_{ijI}^{+}\right)_{j\in\{1,\ldots,J\}}$. We can combine these state variables for all firms with I observations into $\Theta_{I} = \left(\Theta_{I}\right)_{i\in\{1,\ldots,J\}}$. Finally, we can combine all state variables for all possible I's in this economy to produce a vector characterizing this entire economy:

$$\Theta = \left(\Theta_{I}\right)_{I\in\{0,\ldots,J\}}. \tag{11}$$

To be clear, in the vector Θ, i and j denotes specific countries, while I denotes the number of previously tested countries.

3 Two Symmetric Countries

Now that the structure of the economy has been outlined, I look at how a zero-expected-lifetime-profit steady-state general equilibrium will affect a new firm's testing strategy. In this section, I examine a world consisting of two symmetric countries $H\,(ome)$ and $F\,(oreign)$. I normalize marginal costs to reflect iceberg trade costs between the two countries:

$$\tau_{ij} = \tau_{ji} = \tau > \tau_{ii} = 1 \forall j, i \in \{H, F\}, j \neq i \tag{12}$$

Due to the structure of these trade costs, the stay cutoff for firms exporting to destination j is greater than the stay cutoff those firms located in j and selling domestically:

$$x_{ij}^{*} = x_{jj}^{*} + (\sigma - 1)\ln(\tau) > x_{jj}^{*} \tag{13}$$

In this symmetric country setup, I show that the zero-expected lifetime profit condition will induce firms to delay exporting. Later, I show that if one country is much larger than the other, this will entice all firms in both countries to test the larger one first.

3.1 Testing Order

Consider firm ω located in country $i \in \{H, F\}$ at time t, which hasn't tested in either destination. The firm owner can sequentially or simultaneously the two destinations. She tests ω following one of three strategies (strategy nomenclature in parenthesis):

1(SB). Test ω in both the $Home$ and $Foreign$ destinations in period t.

2(SH). Test ω in only the $Home$ destination in period t. If $x_{H\omega} > \frac{1}{\rho}\mu^{+}_{HF1}$, then test the $Foreign$ destination the following period.

3(SF). Test ω in only the $Foreign$ destination in period t. If $x_{F\omega} > \frac{1}{\rho}\mu^{+}_{FH1}$, then test the $Home$ destination the following period.

In a steady state equilibrium, the value of these strategies are determined by the aggregate state vector Θ in Equation 11. The value of each strategy can be expressed as $V^i(S, \Theta), S \in \{SB, SH, SF\}$, where

$$V^i(SB, \Theta) = v_{iH0}(0) + v_{iF0}(0) \tag{14a}$$

$$V^i(SH, \Theta) = v_{iH0}(0) + (1-\delta)\int_{\frac{1}{\rho}\mu^{+}_{iF1}}^{\infty} v_{iF1}(\rho x)\, g_0(x)\, dx \tag{14b}$$

$$V^i(SF, \Theta) = v_{iF0}(0) + (1-\delta)\int_{\frac{1}{\rho}\mu^{+}_{iH1}}^{\infty} v_{iH1}(\rho x)\, g_0(x)\, dx \tag{14c}$$

The second terms in (14b) and (14c) reflect the discounted expectation of the value of testing the second market should the firm owner draw a high enough perceived quality in the first destination.

3.1.1 Equilibrium

A zero-expected profit steady-state two-symmetric-countries equilibrium is defined as the aggregate variable vector $\tilde{\Theta}$ and firm testing strategy $\tilde{S}$ such that for $i = H, F$:

1. $\tilde{S}$ maximizes $\max_S V^i\left(S, \tilde{\Theta}\right)$

2. $V^i(\tilde{S}, \tilde{\Theta}) = 0$

The first condition states that a new firm will choose the strategy that maximizes its lifetime expected profits from testing in H and F. The second condition states that this maximum value must be zero, due to free entry. For the rest of this section, I show this equilibrium exists and is unique through a series of proofs.

Lemma 1 *The destination with the lower stay cutoff will have a higher strategy value of testing there first. For example,* $x^*_{iF} > x^*_{iH} \Rightarrow V^i(SF, \Theta) < V^i(SH, \Theta)\, \forall \Theta$.

Proof. See Appendix C. ■

The probability of success in a destination, which monotonically decreases with the stay cutoff, completely determines the expected testing profit in that destination, which in turn determines the strategy value of testing that destination first. When comparing the strategy values of two destinations, we do not have to concern ourselves with differential trade costs, aggregate expenditures, or the average price competition from other varieties, as long as we know the stay cutoffs.

Proposition 2 *In an zero-expected-lifetime-profit symmetric equilibrium where* $\max_S V^i\left(S, \tilde{\Theta}\right) = 0$*, firms will not test both markets simultaneously:* $V^i(SB, \Theta) < 0$.

Proof. By construction, $v_{HH1}(\rho x) > 0 \forall x > \frac{1}{\rho}\mu^+_{HH1}$, so $(1-\delta)\int_{\frac{1}{\rho}\mu^+_{HH1}}^{\infty} v_{HH1}(\rho z)\, g(z)\, dz > 0$. Assume by way of contradiction that $v_{iF0}(0) \geq 0$. Therefore, $V^i\left(SF, \tilde{\Theta}\right) > 0$ so $\max_S V^i\left(S, \tilde{\Theta}\right) = 0$ is violated. Therefore, $v_{iF0}(0) < 0$. The symmetric argument applies for why $v_{iH0}(0) < 0$. Thus, $V^i\left(SB, \tilde{\Theta}\right) = v_{iH0}(0) + v_{iF0}(0) < 0$. ■

If the lifetime value of testing a market is nonnegative, the owner would certainly test that market first and only test the second market if the resulting conditional lifetime value of testing the second market is positive. Since zero-expected-lifetime-profit is imposed, this scenario is non-existent. Firms will not sell to all markets simultaneously.

Proposition 3 *In an zero-expected-lifetime-profit symmetric equilibrium where* $\max_S V^i\left(S, \tilde{\Theta}\right) = 0$*, firms will not test the export market first:* $V^H\left(SF, \tilde{\Theta}\right) < 0$ *and* $V^F\left(SH, \tilde{\Theta}\right) < 0$.

Proof. Since $x^*_{HF} > x^*_{iF}$ and lemma 1, $V^H\left(SF, \tilde{\Theta}\right) < V^H\left(SH, \tilde{\Theta}\right)$. Therefore, $V^H\left(SF, \tilde{\Theta}\right)$ cannot maximize $\max_S V^H\left(S, \tilde{\Theta}\right)$, and so $V^H\left(SF, \tilde{\Theta}\right) < 0$. The symmetric argument applies for $V^F\left(SH, \tilde{\Theta}\right) < 0$. ■

Firms will not test the overseas export market first. This is due to the added costs of exporting, which depresses expected profits. Since from Proposition 2, firms will also not test both destinations simultaneously, this leaves only one strategy: test ω in the same destination the firm is located. I term this an *exporting delay.* My model provides a channel to explain why firms delay their exporting: since exporting is so risky, firms need to start in the safer home market to learn about their perceived quality.

Proposition 4 *There exists a unique* Θ^* *such that equililbrium is achieved.*

Proof. See Appendix D ■

If $V(SH, \Theta) > 0$, firms will introduce more new varieties. These new varieties will increase competition and lower the chances of success in both destinations. The values of testing the destinations will decrease and thus decrease the value of a new variety. This cycle will occur until the value of a new variety is driven to zero.

The model characterization of the two symmetric country equilibrium is now complete.[22] In the steady state equilibrium, new firms arise in each period t. New firms in H test their varieties in $Home$ in the initial period t. In the next period $t+1$, firm owners make two parallel decisions. First, they choose to continue supplying profitable varieties to the $Home$ destination. Unprofitable varieties fail. Second, firm owners choose whether to test their varieties in the foreign destination, based on their forecast of perceived quality in $Foreign$. In periods $t+2$ and onwards, firms have all the information they need to decide to which destination(s) they will supply. Those varieties that were tested in both markets will be continued to be supplied to where they are profitable. This process is presented in the top half of Figure 2.

Insert Figure 2.

[22] The equilibrium mass of incoming firms can be determined by clearing the final goods market.

For comparison, the process in Melitz (2003) is presented in the bottom half of Figure 2. Using his notation, firms pay a development cost to observe the productivity φ_ω. Since φ_ω is known at the beginning of t, decisions for all destinations are made immediately. Firms with high productivities will supply to both markets. Firms with mediocre productivities will supply to only the domestic market. Firms with low productivities will not supply at all - the market never sees these last firms.

3.2 Numerical Predictions

The two-symmetric country case generates predictions consistent with the testing and failure rates of exporters. However, the normal distribution does not lend itself to closed-form expressions for these rates. Therefore, I determine equilibrium conditions numerically using model parameters taken from established empirical studies[23]. The parameters of the baseline simulation are summarized in Table 2.

Table 2: Baseline Model Parameters

Parameter	**Value**
τ	1.5
σ	4
ρ	0.7
δ	0.11

I set $\tau = 1.5$ to match typical average trade costs found in the literature (Anderson & van Wincoop 2004). Hummels (2001) find σ to vary between 2 and 5; I set $\sigma = 4$. The hazard rate δ baseline was set to match those of US imports at a 10-digit Harmonized System aggregation (Besedes & Prusa 2006). I choose a baseline value of $\rho = 0.7$ but show that results are robust to the choice of ρ. As seen in equation 9, the fixed cost f can be factored out of all value equations and does not affect the equilibrium cutoffs, but

[23] The Matlab Code is available from the author. The numerical integration of the bivariate normal distribution used 1,000,000 Monte Carlo evaluation points.

instead scales the economy by determining, along with the labor endowment, the total number of firms. I use $f = 1$, but changing f does not change the solutions. I use a baseline $s^2 = 2$, but this just scales the cutoff parameters. As such, I report equilibrium cutoff parameters as multiples of the standard deviation $s = \sqrt{s^2}$.

Using only these few parameters, I can compare the results of my model to reported statistics in recent literature, as summarized in Table 3. I repeat the exercise for $\tau = 1.3$, in case $\tau = 1.5$ is an overestimate of trade costs.

Table 3: Baseline results compared to recent empirical findings

Statistic	Model Predictions ($\tau = 1.5$)	Model Predictions ($\tau = 1.3$)	Observation	Source
Percent Exporters	6%	15%	4-18% 21%	BJRS BEJK
Product Failures	94%	94%	50-80%	CMC
Export Failures	90%	89%	50% the first year 80% the first four years	BP
Export Only Firms	0%	1.6%	1-5%	AU

Percent Exporters: Number of exporters/Number of firms
Product Failures: Percentage of firms that withdraw from the domestic market after the first period
Export Failures: Percentage of firms that exit the foreign market after the first period
Export Only: Percentage of firms that export but do not sell to the domestic market.
BJRS: Bernard, Jensen, Redding, Schott, 2007, BEJK: Bernard Eaton, Jensen, Kortum, 2003
CMC: C. Merle Crawford, 1977, BP: 10-digit HS in Besedes & Prusa, 2006a
AU: Author calculated, as discussed in the introduction

As Table 3 shows, the model matches previous empirical findings well; each prediction is within the range or near the reported point estimate. Firm-level failure rates as predicted in the model are higher than product-level failure rates in Besedes & Prusa (2006a, 2006b). Export Only Firms exist in the model because, for the baseline parameters, $\frac{1}{\rho}\mu^{+}_{HF1} < x^{*}_{HH}$. Some firms draw an $x_{H\omega}$ such that $\frac{1}{\rho}\mu^{+}_{HF1} < x_{H\omega} < x^{*}_{HH}$, so they start exporting even though they fail the domestic market.

Table 3 shows that trade costs affect the number of exporters in equilibrium. To show the how τ affects the equilibrium, I vary τ over the range $[1.1, 2]$. As seen in the right two graphs in Figure 3, changing trade costs significantly affect the rates of firms entering and exiting the foreign market.

First, we examine the top right graph to see the effects of τ on the cutoffs. Increasing trade costs increase the stay cutoff in the Foreign destination, which then increases the test cutoff a firm needs in order to start exporting. The general equilibrium effect of this is that fewer firms become and stay exporters. The decrease in import competition helps new firms, which now face a lower stay cutoff in the Home destination. As the top right graph shows, this decrease is a second-order effect - the decrease in x^*_{HH} over the range of τ is much less than the increases of x^*_{HF} and $\frac{1}{\rho}\mu^+_{HF1}$.

Insert Figure 3 here

As expected, increases in trade costs decrease the fraction of firms willing to test the foreign market from a quarter of new firms at $\tau = 1.1$ to less than one percent at $\tau = 1.9$. This decrease can be seen in the lower right graph of Figure 3. In the case of a plausible $\tau = 1.3$, the model predicts that 14% of new firms test export markets. The percents of exporters and export-only firms also decrease with increasing trade costs, with the number of export-only firms hitting zero percent at a $\tau = 1.5$. The model predictions are in line with the observed values in Table 3.

The domestic failure rate of new firms always stays above 90% in the bottom right of Figure 3. The model suggests that the failure rates of new firms are underreported in the literature. This may be due to a censoring issue: firms with very low demand draws may never show up in the data, as they exit before reporting any sales.

The correlation of demands across destinations also matters for the predictions of the model. The left hand side of Figure 3 illlustrates the effects of varying ρ on the equilibrium. The stay cutoffs x^*_{HH} and x^*_{HF} do not change considerably, but the testing cutoff, which is graphed as $\frac{1}{\rho}\mu^+_{HF1}$, changes dramatically as ρ varies between 0 and 1. This occurs because of two counteracting effects. First, increasing ρ reduces the minimum domestic perceived quality directly. To test the Foreign market, firm ω requires that $x_{H\omega} > \frac{1}{\rho}\mu^+_{HF1}$. For a given μ^{+}_{HF1}, this cutoff obviously decreases with ρ. However, ρ has an opposite effect on μ^{+}_{HF1}. As the demands in the two destinations become more and more correlated, firms trust their domestic draws more. As seen in the conditional distribution 6, the expected value of $X_{F\omega}$ get closer to the observed $x_{H\omega}$ as ρ increases. In addition, the variance of $X_{F\omega}$ decreases; firms are more sure that $\rho x_{H\omega}$ predicts $X_{F\omega}$. This serves to move the testing cutoff μ^{+}_{HF1} closer to the stay cutoff x^{*}_{HF}. Since $\mu^{+}_{HF1} < x^{*}_{HF}$, this direction is always positive. To see the convergence, imagine if $\rho = 1$. In this extreme case, firms that tested in the Home market know exactly their perceived quality in the Foreign. They would then only test the foreign market if $x_{H\omega} > x^{*}_{HF}$. The two effects of ρ show up in the top left graph: for low ρ, the first effect is stronger. As ρ gets closer to 1, the distance between $\frac{1}{\rho}\mu^{+}_{HF1}$ and x^{*}_{HF} must decrease in order to disappear at 1, so the second effect takes over.

These counteracting effects of ρ show up in the fraction of firms that test and fail in the destinations, as seen in the bottom left graph of Figure 3. The percentage of firms that test the Foreign destination increases for low ρ and decreases for high ρ, as foreshadowed by the path of $\frac{1}{\rho}\mu^{+}_{HF1}$ the in top left graph. While x^{*}_{HF} does not change much, the percent of firms that fail the foreign market drops from 95% to 75% as ρ increase from 0.1 to 0.9. This reduction occurs mainly at the high end because as ρ increases, marginal firms with low probabilities of export success are deciding not to export, leaving only those firms with higher export stay probabilities.

As seen in the bottom left graph, values of ρ between 0.4 and 0.5 produce the highest number of exporters, around 15%, when the decreasing failure rates of new exporters is met by an increasing number of firms that begin to export. The model predictions are in line with the observed values. The positive number of export-nly firms comes from the fact that $\frac{1}{\rho}\mu^{+}_{HF1}$ is below x^{*}_{HH},in the top left graph. In this case, firms that failed the domestic market may still have incentives to export. Whether $\frac{1}{\rho}\mu^{+}_{HF1}$ is below x^{*}_{HH} depends on both ρ and τ, as seen in the two top figures. In fact, at $\tau > 1.5$, $\frac{1}{\rho}\mu^{+}_{HF1}$ does not dip below x^{*}_{HH} for any value of ρ.

This exercise shows the power of the model to both reconcile old and new facts concerning exporting, as well as provide counterfactuals showing the equilibrium effects of both trade costs and demand correlations across destinations.

4 Two Asymmetric Countries

Next I extend the baseline model to an economy of two assymetric countries. I show that if one market is much larger than the other, all firms will test there first.

We analyze the testing decisions of a new firm located in country i and introducing the variety ω in time t. The firm has the same three testing strategies outlined in Section 3.1 with the same functional forms. Proposition 2 still applies: no firm will test both destinations simultaneously. However, in the assymetric case, it is not certain that $V^i(SH, \Theta) > V^i(SF, \Theta)$ in equilibrium. The order of the four stay cutoffs is crucial to determining equilibrium because by Lemma 1, they determine the strategies of the firms. From Equation 7, we know $x^*_{HF} > x^*_{FF}$ and $x^*_{FH} > x^*_{HH}$. Therefore, the three possible cutoff rankings are[24]

1. $x^*_{FH} > x^*_{HH} > x^*_{HF} > x^*_{FF}$,

2. $x^*_{FH} > x^*_{HF} = x^*_{HH} > x^*_{FF}$,

3. $x^*_{FH} > x^*_{HF} > x^*_{HH} > x^*_{FF}$

If an equilibrium of type 3 occurs, the firms employ the same strategies as in the symmetric country case: i.e. H firms employ $S = SH$ and F firms employ $S = SF$. This equilibrium is solved just as in the symmetric case above. By examining Equation 7, we see that an equilibrium of type 2 occurs only in the very special case where $\frac{Y_F}{Y_H} = \tau^{\sigma-1}\frac{\Pi_F}{\Pi_H}$. As we will see, $\frac{\Pi_F}{\Pi_H}$ is bounded from above, so this equilibrium requires all parameters $\sigma, \tau, \delta, s^2$, *etc.*.to line up exactly for a given $\frac{Y_F}{Y_H}$. This event occurs with zero probability, so we do not discuss it further.

[24] There are also three additional possible rankings, which are mirror images of the three listed (Just switch H and F). Without loss of generalization, we consider only the listed three.

All firms begin testing the same market Let's discuss the interesting equilibrium of type 1: the firm faces lower stay cutoffs in F than in H, no matter where the firm is located. Testing is more profitable in F, so all firms in both countries will employ strategy $S = SF$. By Equation 7, this scenario exists only if

$$\frac{Y_F}{Y_H} > \tau^{\sigma-1}\frac{\Pi_F}{\Pi_H}. \tag{15}$$

That is, the scenario in which all firms test the *Foreign* destination first occurs only if the relative aggregate demand in the *Foreign* country is larger than the relative level of competition , even for H firms that have to incur added exporting costs. Of course, the level of competition depends on the size of the countries, and the relationship is not linear. We can show that $\frac{\Pi_F}{\Pi_H}$ is bounded from above, and thus there exists an economy where $\frac{Y_F}{Y_H} > \tau^{\sigma-1}\frac{\Pi_F}{\Pi_H}$.

Proposition 5 *There exists* $\left(\frac{Y_F}{Y_H}\right)^*$ *such that in economies with* $\left(\frac{Y_F}{Y_H}\right) > \left(\frac{Y_F}{Y_H}\right)^*$, *all firms will test* F *first.*

Proof. See Appendix.E ■

In this scenario, equilibrium is characterized by all firms testing the foreign market first. The demand for varieties in the foreign destination is so large that even firms in H are willing to risk exporting first. This may be the case for small countries exporting to the US. Firms could start up in those countries with the sole purpose of selling to US customers. Since τ increases the upper bound, this scenario is less likely for further away countries.

5 More than two Countries: Sequential Entry

I now examine predictions of my model for $J > 2$ symmetric countries with no trade costs. Even without trade costs to discourage immediate exporting, I show that firms will often expand their set of supply destinations one country at a time. The structure laid out in Section 2 remains the same, except I now restrict $\tau_{ij} = 1$. Since there is no difference between the costs of supplying different destinations, the term "exporting" is adjusted slightly. Therefore, I define a firm's first destination to which it supplies as its domestic destination, and any additional destinations as export destinations. For exposition purposes, I consider a representative firm ω using the labor from country H who has not yet supplied to any destinations in period t.

Given our multiple destination setup, firm ω has exponentially more testing strategies than the three presented in Section 3.1. The firm can test up to J destinations in period t. If it tests $J-1$ destinations in period t, it has two strategies for period $t+1$: test or don't test the last destination. If instead it tests $J-2$ destinations in period t, it has four strategies for future periods: test zero, one, or both of remaining destinations in period $t+1$ and then, if it tests one in $t+1$, then test or don't test the final destination in $t+2$. All in all, firm ω has 2^J-1 total possible strategies it can pursue given J potential destinations. We have to examine this problem recursively. In the beginning of each period, the firm knows its state variables μ_{I_ω} and I. The firm will choose K additional destinations to test to maximize the lifetime testing profits in the remaining destinations. I define $\gamma\left(\mu_{I\omega}, I\right)$ as the recursive value function for firm ω:

$$\gamma\left(\mu_{I\omega}, I\right) = \max_{0<K<J-I} \left\{K v_j\left(\mu_{I\omega}, I\right) + (1-\delta) E\left[\gamma\left(\mu_{(I+K)\omega}, I+K\right)\right]\right\} \tag{16}$$

Since all destinations are identical, ex ante, firms entering K new destinations choose those K destinations randomly among the remaining $J-I$ destinations. A recursive zero profit steady state symmetric equilibrium is defined as the set of value functions $\tilde{\gamma}$ which satisfy (16) and the aggregate state vector $\tilde{\Theta}$ which produces the initial value

$$\tilde{\gamma}(0,0) = 0. \tag{17}$$

That is, new firms have a zero expected value of introducing a new product.

In this equilibrium, some firms will choose $K < J - I$ additional destinations each period. They do not immediately export to all remaining destinations, even though these destinations are ex ante identical. I term this expansion *sequential exporting.* This matches the pattern of export expansion described in Eaton, Eslava, Kugler, and Tybout (2007). I show this in a series of proofs.

Lemma 6 *If a firm has a positive forecast for the expected testing profit, then its recursive value function is positive:* $\gamma(\mu, I) > 0 \forall \mu > \mu_{jI}^{+}$.

Proof. The firm always has the option of testing all remaining $J-I$ destinations. Suppose the firm has $\mu_{I\omega} = \grave{\mu} > \mu_{HjI}^{+}$. Since $v_j(\grave{\mu}, I) > 0$, then $\gamma(\grave{\mu}, I) > (J-I)\, v_j(\grave{\mu}, I) > 0$ ■

If the firm owner expects positive profits in the remaining destinations, he can test all of them this period. Since $\mu_{HjI}^{+} < \infty$, there is a mass off firms that have a positive recursive value function. However, it is not certain whether all these firms with $\mu_{I\omega} > \mu_{HjI}^{+}$ will test all remaining destinations. For the general case, we cannot determine whether $\gamma(\mu_{I\omega}, I) = (J-I)\, v_j(\mu_{I\omega}, I) \forall \mu_{I\omega} > \mu_{HjI}^{+}$. However, we at least know that all these firms have positive recursive value functions. This is key because I can now show that some firms with $\mu_{I\omega} < \mu_{HjI}^{+}$ also have positive recursive value functions. These firms test only a subset of remaining destinations.

Proposition 7 *In any given period, there exist firms that will not test all remaining destinations, but instead will test a subset of the remaining destinations. That is,* $\exists \mu$: $\gamma(\mu, I) > 0 > (J-1)\, v_j(\mu, I)$

Proof. Consider firm ω having tested I destinations by period t and realizing a perceived quality forecast of $\mu_{I\omega} < \mu_{HjI}^{+}$. Firm ω will not test all remaining destinations because $(J-I)\, v_j(\mu_{I\omega}, I) < 0$. However, suppose the firm takes the following strategy: 1. test 1 additional destination j and obtain new perceived qualities $x_{j\omega}$. 2. If $\mu_{(I+1)\omega} = \frac{(I\rho+(1-\rho)\mu_{I\omega}+x_{j\omega})}{(I+1)\rho+1-\rho} > \mu_{Hj(I+1)}^{+}$, then test all remaining destinations. The lifetime value of this unique strategy is

$$\begin{aligned}\gamma_1\left(\mu_{I\omega}, I\right) &= v_j\left(\mu_{I\omega}, I\right) + (1-\delta)\,\Psi \\ \Psi &= \int_{\mu^+_{Hj(I+1)}}^{\infty} (J-I-1)\, v_j\left(z, I+1\right) h\left(z|\mu_{I\omega}, I\right) dz\end{aligned}$$

where $h\left(z|\mu_{I\omega}, I\right)$ is the probability that the random variable $\mu_{(I+1)\omega} = z$ given $\mu_{I\omega}, I$ and K. By construction, $\Psi > 0$. Since $v_j\left(\mu^+_{HjI}, I\right) = 0$, $v_j\left(\mu, I\right) < 0 \forall \mu < \mu^+_{HjI}$, and $\frac{dv_j(\mu,I)}{d\mu} > 0$, there exists a cutoff $\mu^*_{I\omega}$ such that $\gamma_1\left(\mu, I\right) > 0 \forall \mu > \mu^*_{I\omega}$. For those firms, we can generate the following ranking:

$$\gamma\left(\mu_{I\omega}, I\right) \geq \gamma_1\left(\mu_{I\omega}, I\right) > \gamma\left(0|\mu, I\right) = 0 > (J-1)\, v_j\left(\mu_{I\omega}, I\right) \forall \mu^*_{I\omega} < \mu < \mu^+_{HjI}.$$

Therefore, those firms with $\mu^*_{I\omega} < \mu < \mu^+_{HjI}$ maximizes their recursive value functions by testing at least one additional destination, but not all remaining destinations. ■

There are a mass of firms that, after testing I destinations, forecast negative lifetime profits in every remaining destination, so testing all remaining destination is not a value-added strategy. However, some firms garner positive recursive value functions by *sequentially exporting* in order to update their beliefs. Even if firms project failures in all export destinations, the promise of future profit incentivises some of them to test at least one additional destination. Since destinations are ex ante identical, the firm chooses it's next destination at random. Therefore, two firms with identical $\mu_{I\omega}$ and I may choose different destinations to test next.

6 Conclusions

In this paper, I propose a model of heterogeneous firms that reconcile two new patterns of trade: firms wait to export, and firms fail at exporting. To do so, I retool the standard firm heterogeneity model to allow for imperfect correlation of firm heterogeneity across destinations. This retooling endogenous the delay in exporting and the failures of exporters. When demand is imperfectly correlated across destinations, firms will use known demands in tested destinations to forecast unknown demands in untested destinations.

Because the exporting success of a firm is not guaranteed by its domestic sales, this model has different policy implications than the Melitz (2003) model. Policy makers strictly adhering to Melitz (2003) would focus on getting the best domestic firms to export. My model suggests that even the best domestic firms may be the worst exporting firms, and vice versa. Policy makers should aim to get as many firms to test the foreign market as possible, or help firms forecast foreign demands better.

This is a model of learning, as opposed to a model of evolution like the time-varying productiivty models discussed in the introducation. Firms are static in the sense that their destination-specific perceived qualities are time-invariant. In models of productivity evolution, firms will receive their next period productivity shock no matter what they do. In this model, firms make the endogenous choice whether to learn more about themselves. Because this model is static, its predicted failure rate of firms in their first period is much higher than in later periods. An amalgam of this learning model and a productivity evolution model would have firm heterogeneity change with time and destinations. This amalgamation would smooth out the drop in hazard rate so that the model predictions would more resemble that of Colombian firms. However, even without time-varying demand, this model is able to endogenize the drop in hazard rates not seen in any other model.

When faced with more than two possible destinations, firms will slowly expand their set of export destinations to take advantage of this slow learning. Another extension would heterogenize the demand correlations across destinations. This would make some destinations more attractive than others. Even without an a priori ranking of destinations, many firms will test a subset of untested destinations even though they forecast negative profits in that destination. Firm owners know they have a tiny chance of success. But the hope of future profits entices firms to enter destination markets even though they know they will probably fail. This is the motivation for many new business ventures.

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APPENDICES

A Derivation: Moments of $X_{j\omega}$

The vector X_ω is normally distributed:

$$\begin{aligned} X_\omega \tilde{} N_J\left(0_k, \Xi\right) \\ \Xi &= s^2 \begin{bmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \cdots & \cdots & \cdots & \cdots \\ \rho & \rho & \cdots & 1 \end{bmatrix}. \end{aligned}$$

The marginal distribution of any two elements can be described by Equation 5. Let's partition X_ω by defining $X_\omega = \left[X_\omega^1, X_\omega^I\right]$, with corresponding $\Xi^{I+1} = \begin{bmatrix} 1 & \Xi_{jI} \\ \Xi_{Ij} & \Xi_{II} \end{bmatrix}$ where $X_\omega^I = \vec{a}$ is known and X_ω^1 is a single element. Greene (2008) shows that The conditional distribution of X_ω^1 given X_ω^I is normal with

$$\begin{aligned} E\left[X_{j\omega}|X_{I\omega}\right] &= \Xi_{jI}\Xi_{II}^{-1}\vec{a} \\ VAR[X_{j\omega}, X_{I\omega}] &= s^2 - \Xi_{jI}\Xi_{II}^{-1}\Xi_{Ij} \end{aligned}$$

It is simpler if we simplify $\Xi_{jI}\Xi_{II}^{-1}$ as done in Paltseva (2010).

$$\begin{aligned} \Xi_{jI}\Xi_{II}^{-1} &= \frac{\begin{bmatrix} \rho & \cdots & \cdots & \rho \end{bmatrix}}{(1-\rho)\left(1+(I-1)\rho\right)} \begin{bmatrix} 1+(I-2)\rho & -\rho & \cdots & -\rho \\ -\rho & 1+(I-2)\rho & \cdots & -\rho \\ \cdots & \cdots & \cdots & \cdots \\ -\rho & -\rho & \cdots & 1+(I-2)\rho \end{bmatrix} \\ &= \frac{\rho}{\left(1+(I-1)\rho\right)} \begin{bmatrix} 1 & \cdots & \cdots & 1 \end{bmatrix} \end{aligned}$$

so that

$$\begin{aligned}
\Xi_{jI}\Xi_{II}^{-1}\vec{a} &= \frac{\rho\sum_{i=1}^{I} a_i}{(1+(I-1)\rho)} \\
s^2 - \Xi_{jI}\Xi_{II}^{-1}\Xi_{Ij} &= s^2\left(1-\frac{I\rho^2}{(1+(I-1)\rho)}\right)
\end{aligned}$$

B Proof: A unique $\mu_{ijI}^{+} < x_{ij}^{*}$ exists for each ijI triplet such that the expected testing profit is positive only if $\mu > \mu_{ijI}^{+}$

We need to first show that $\frac{dv_{ijI}(\mu)}{d\mu} > 0$. Then we show that there exists a $\mu_{ijI}^{+} < x_{ij}^{*}$ such that $v_{ijI}\left(\mu_{ijI}^{+}\right) = 0$. Then we show that μ_{ijI}^{+}.

We can use the moments of truncated distributions in Greene (2008, p. 866-867) to rewrite Equation 9 as

$$v_{ijI}(\mu) = f\exp\left(\mu + \frac{\chi_I^2}{2} - x_{ij}^{*}\right) - f + \frac{(1-\delta)}{\delta} f\vartheta \tag{18}$$

where

$$\vartheta = \int_{x_{ij}^{*}}^{\infty} \left(\exp\left(x - x_{ij}^{*}\right) - 1\right) g_I(x|\mu)\, dx.$$

Now, $\frac{dv_{ijI}(\mu)}{d\mu} = f\exp\left(\mu + \frac{\chi_I^2}{2} - x_{ij}^{*}\right) + \frac{(1-\delta)}{\delta} f \frac{d\vartheta}{d\mu}$. Since $\exp(\cdot) > 0$, we only have to show that $\frac{d\vartheta}{d\mu} > 0$. Define $z = \frac{x-\mu}{\chi_I}$. Now

$$\vartheta = \int_{\frac{x_{ij}^{*}-\mu}{\chi_I}}^{\infty} \left(\exp\left(\chi_I z + \mu - x_{ij}^{*}\right) - 1\right)\phi(z)\, dz$$

where ϕ is the standard normal pdf. Now since $\chi_I z + \mu - x^*_{ij} > 0 \forall z > \frac{x^*_{ij} - \mu}{\chi_I}$,

$$\frac{d\vartheta}{d\mu} = \int\limits_{\frac{x^*_{ij}-\mu}{\chi_I}}^{\infty} \exp\left(\chi_I z + \mu - x^*_{ij}\right) \phi\left(z\right) dz > 0 \tag{19}$$

We can plug in $\mu = x^*_{ij}$ and $\mu - \infty$ to show that $v_{ijI}\left(x^*_{ij}\right) = f\left(\exp\left(\frac{\chi_I^2}{2}\right) - 1\right) > 0$ and $\lim_{\mu \to -\infty} v_{ijI}\left(\mu\right) = -f$. Therefore, by the intermediate value theorem, there exists a $\mu^+_{ijI} < x^*_j$ such that $v_{ijI}\left(\mu^+_{ijI}\right) = 0$. Since $\frac{dv_{ijI}(\mu)}{d\mu} > 0$, μ^+_{ijI} must be unique.

C Proof: The ranking of stay cutoffs determines the ranking of conditional values of testing.

Assume $x^*_{iH} > x^*_{iF}$. We will show that $\Delta \equiv V^i\left(SF, \Theta\right) - V^i\left(SH, \Theta\right) > 0$. First, take the difference between equations 14b and 14c:

$$\begin{aligned}
\Delta &= \Delta_1 + \left(1 - \delta\right) \Delta_2 \\
\Delta_1 &= v_{iF0}\left(0\right) - v_{iH0}\left(0\right) \\
\Delta_2 &= \int_{\frac{1}{\rho}\mu^+_{HF1}}^{\infty} v_{iF1}\left(\rho x\right) g_0\left(x\right) dx - \int_{\frac{1}{\rho}\mu^+_{HH1}}^{\infty} v_{iH1}\left(\rho x\right) g_0\left(x\right) dx
\end{aligned}$$

To show that both Δ_1 and Δ_2 are positive, we need to show that $\frac{dv_{ijI}(\mu)}{dx^*_{ij}} < 0$. Using the notation of Appendix B, we see that

$$\frac{dv_{ijI}\left(\mu\right)}{dx^*_{ij}} = -f \exp\left(\mu + \frac{\chi_I^2}{2} - x^*_{ij}\right) - \frac{\left(1 - \delta\right)}{\delta} f \int\limits_{x^*_{ij}}^{\infty} \exp\left(x - x^*_{ij}\right) g_I\left(x|\mu\right) dx < 0.$$

The monotonicity of v_{ijI} with respect to x^*_{ij} ensures that if $x^*_{iH} > x^*_{iF}$, then $v_{iF0}\left(0\right) - v_{iH0}\left(0\right) > 0$. Thus $\Delta_1 > 0$.

The monotonicity of v_{ijI} with respect to both x^*_{ij} and μ ensures $x^*_{iH} > x^*_{iF} \implies \mu^+_{iH1} > \mu^+_{iF1}$. To show this, suppose by way of contradiction that $x^*_{iH} > x^*_{iF}$, but $\mu^+_{iH1} \leq \mu^+_{iF1}$.

By construction, $v_{iF1}\left(\mu^+_{iF1}\right) = 0$, and by the monotonicity of v_{ijI} with respect to x^*_{iF}, $v_{iH1}\left(\mu^+_{iF1}\right) < 0$. Since $v_{iH1}\left(\mu^+_{iF1}\right) < 0$, $\frac{dv_{iH1}(\mu)}{d\mu} > 0$, and $v_{iH1}\left(\mu^+_{iH1}\right) = 0$, $\mu^+_{iH1} > \mu^+_{iF1}$, which is a contradiction.

Since $\mu^+_{iH1} > \mu^+_{iF1}$, we can rewrite Δ_2 as

$$\Delta_2 = \int_{\frac{1}{\rho}\mu^+_{iH1}}^{\infty} \left(v_{iF1}\left(\rho x\right) - v_{iH1}\left(\rho x\right)\right) g_0\left(x\right) dx + \int_{\frac{1}{\rho}\mu^+_{iH1}}^{\frac{1}{\rho}\mu^+_{iF1}} v_{iH1}\left(\rho x\right) g_0\left(x\right) dx \qquad (20)$$

Since we have shown that $v_{iF1}\left(\rho x\right) - v_{iH1}\left(\rho x\right) > 0$, and $v_{iH1}\left(\rho x\right) > 0$ for $x > \frac{1}{\rho}\mu^+_{iH1}$, it is easy to see that $\Delta_2 > 0$.

D Proof: There exists a unique steady state equilibrium

By symmetry, $V^H\left(SH, \Theta\right) = V^F\left(SF, \Theta\right)$. Here I show that $V^H\left(SH, \Theta\right) = 0$ for a single $\Theta = \tilde{\Theta}$. By rearranging equation 7, I can define Π_j as a function of x^*_{HH} :

$$\Pi_j\left(x^*_{HH}\right) = \frac{\exp\left(x^*_{HH}\right) p_{Hj}^{1-\sigma} Y}{f\sigma} \qquad (21)$$

. Likewise, x^*_F, μ^+_{ijI}, and μ^+_{ijI} are all subsequently defined by x^*_{HH}. I can then redefine $\bar{V}\left(x^*_{HH}\right) \equiv V\left(SH, \Theta\right)$ as

$$\bar{V}\left(x^*_{HH}\right) = V\left(SH, \Theta\left(x^*_{HH}\right)\right) = V\left(SH, \left(\Pi_j\left(x^*_{HH}\right), x^*_{ij}\left(x^*_{HH}\right), \mu^+_{ijI}\left(x^*_{HH}\right)\right)_{i\in\{H\ F\}, j\in\{H,F\}, I\in\{0,1\}}\right) \qquad (22)$$

since x^*_{HH} sufficiently characterizes all market variables in the set Θ. Therefore, proving that $\bar{V}\left(x^*_{HH}\right) = 0$ for a single $x^*_{HH} = \tilde{x}$ is sufficient. Which is what I will do, via the intermediate value theorem.

Part 1: $\bar{V}(x^*_{HH}<0)>0$ Using equations 14b and 8, I can decompose $\bar{V}(x^*_{HH})$:

$$\begin{aligned}
\bar{V}(x^*_{HH}) &= \bar{V}_a(x^*_{HH})+\bar{V}_b(x^*_{HH})+\bar{V}_c(x^*_{HH}) \\
\bar{V}_a(x^*_{HH}) &= \left.\int_{-\infty}^{\infty}\pi_{HHt}(u)\,g_0(u|0)\,du\right|_{x^*_{HH}} \\
\bar{V}_b(x^*_{HH}) &= \left.\frac{1-\delta}{\delta}\int_{x^*_{HH}}^{\infty}\pi_{HHt}(u)\,g_0(u|0)\,du\right|_{x^*_{HH}} \\
\bar{V}_c(x^*_{HH}) &= \left.(1-\delta)\int_{\frac{\mu^+_{HF1}}{\rho}}^{\infty}v_{HF1}(\rho u)\,g_0(u|0)\,du\right|_{x^*_{HH}}.
\end{aligned}$$

By construction, $\bar{V}_b(x)>0,\bar{V}_c(x)>0$. Keeping in mind that Π, x^*_F, x^+_H, and x^+_F are all functions of x^*_{HH}, I can rewrite of $\bar{V}_a(x^*_{HH})$ as

$$\bar{V}_a(x^*_{HH})=f\exp\left(\frac{s^2}{2}-x^*_{HH}\right)-f$$

If $x^*_{HH}<0$, $\bar{V}(x^*_{HH})=\bar{V}_a(x^*_{HH})+\bar{V}_b(x^*_{HH})+\bar{V}_c(x^*_{HH})>0$.[25]

Part 2: $\lim_{x^*_H\to\infty}\bar{V}(x^*_{HH})<0$. Now I show that $\bar{V}(x^*_{HH})$ is negative at large, positive values of x^*_{HH}. :

$$\begin{aligned}
\lim_{x^*_{HH}\to\infty}\bar{V}_a(x^*_{HH}) &= \lim_{x^*_{HH}\to\infty}f\exp\left(\frac{s^2}{2}-x^*_{HH}\right)-f=-f \\
\lim_{x^*_{HH}\to\infty}\bar{V}_b(x^*_{HH}) &= \lim_{x^*_{HH}\to\infty}\frac{1-\delta}{\delta}\int_{x^*_{HH}}^{\infty}\pi_{Ht}(u)\,g_0(u|0)\,du=0 \\
\lim_{x^*_{HH}\to\infty}\bar{V}_c(x^*_{HH}) &= \lim_{x^*_{HH}\to\infty}(1-\delta)\int_{\frac{\mu^+_{HF1}}{\rho}}^{\infty}v_{HF1}(\rho u)\,g_0(u|0)\,du=0.
\end{aligned}$$

It is straightforward to see that the $\lim_{x^*_{HH}\to\infty}\bar{V}(x^*_{HH})=-f<0$.

[25] In fact, the last equation showing $\bar{V}_a(x^*_{HH})$ implies that, in order to satisfy our zero profit condition, the equilibrium x^*_{HH} must be greater than $\frac{s^2}{2}$!.

Part 3. $\bar{V}\left(x^*_{HH}\right)$ **is decreasing with** x^*_{HH}. First:

$$\begin{aligned}\frac{\partial \bar{V}_a}{\partial x^*_{HH}} &= -f\exp\left(\frac{s^2}{2}-x^*_{HH}\right)\\ &< 0\end{aligned}$$

The derivative of $\bar{V}_b$ requires Leibnitz's Rule:

$$\frac{\partial \bar{V}_b}{\partial x^*_{HH}} = -\frac{1-\delta}{\delta}\left(\pi_{Ht}\left(x^*_{HH}\right)g_0\left(x^*_{HH}|0\right)+\frac{f}{\exp\left(x^*_{HH}\right)}\int_{x^*_{HH}}^{\infty}\exp\left(u\right)g_0\left(u|0\right)du\right).$$

Since $\pi_{Ht}\left(x^*_{HH}\right)=0$, and $\int_{x^*_{HH}}^{\infty}\exp\left(u\right)g_0\left(u|0\right)du>0$ by construction,

$$\begin{aligned}\frac{\partial \bar{V}_b}{\partial x^*_{HH}} &= -\frac{(1-\delta)}{\delta}\frac{f}{\exp\left(x^*_{HH}\right)}\int_{x^*_{HH}}^{\infty}\exp\left(u\right)g_0\left(u|0\right)du\\ &< 0\end{aligned}$$

We can also use Leibnitz's rule to find $\frac{\partial \bar{V}_c}{\partial x^*_{HH}}$:

$$\frac{\partial \bar{V}_c}{\partial x^*_{HH}} = -\left(1-\delta\right)\left(v_{HF1}\left(\mu^+_{HF1}\right)g_0\left(\mu^+_{HF1}|0\right)\frac{d\mu^+_{HF1}}{\rho d_{X^*_H}}-\int_{\frac{\mu^+_{HF1}}{\rho}}^{\infty}\frac{\partial v_{HF1}\left(u\right)}{\partial x^*_{HH}}g_0\left(u|0\right)du\right)$$

Again, $v_{HF1}\left(\mu^+_{HF1}\right)=0$, so we now have

$$\frac{\partial \bar{V}_c}{\partial x^*_{HH}} = \left(1-\delta\right)\int_{\frac{\mu^+_{HF1}}{\rho}}^{\infty}\frac{\partial v_{HF1}\left(u\right)}{\partial x^*_{HH}}g_0\left(u|0\right)du.$$

Now, I need to decompose $\frac{\partial v_{HF1}(u)}{\partial x^*_{HH}}$ into current period expected profits and expect future profits:

$$\begin{aligned}
\frac{\partial v_{HF1}(u)}{\partial x^*_{HH}} &= \frac{\partial v_{HF11}(u)}{\partial x^*_{HH}} + \frac{\partial v_{HF12}(u)}{\partial x^*_{HH}} \\
v_{HF11}(u) &= \int_{-\infty}^{\infty} \pi_{HF}(u)\, g_0(u|0)\, du \\
v_{HF12}(u) &= \frac{1-\delta}{\delta} \int_{x^*_{HF}}^{\infty} \pi_{HF}(u)\, g_1(u|x)\, du.
\end{aligned}$$

Following exactly our procedure to find $\frac{\partial \bar{V}_a}{\partial x^*_{HH}}$ and $\frac{\partial \bar{V}_b}{\partial x^*_{HH}}$, We can find $\frac{\partial v_{HF1}(u)}{\partial x^*_{HH}} < 0$, which makes $\frac{\partial \bar{V}_c}{\partial x^*_{HH}} < 0$. I then combine all three so that

$$\frac{\partial \bar{V}}{\partial x^*_{HH}} = \frac{\partial \bar{V}_a}{\partial x^*_{HH}} + \frac{\partial \bar{V}_b}{\partial x^*_{HH}} + \frac{\partial \bar{V}_c}{\partial x^*_{HH}} < 0. \tag{23}$$

The intermediate value theorem Since $\bar{V}(x^*_{HH} < 0) > 0$, $\lim_{x^*_{HH}\to\infty} \bar{V}(x^*_{HH}) = -f < 0$, and $\bar{V}(x^*_{HH})$ is decreasing over that range.The intermediate value theorem states that there exists a unique $\tilde{x}$ such that $\bar{V}(x^*_{HH}) = 0$ iff $x^*_{HH} = \tilde{x}$.

E Derivation: Country Size Cutoff

Y_F and Y_H are exogenous parameters of the model, so $\frac{Y_F}{Y_H} \in (0, \infty)$. We will show that there exists a sufficient cutoff $\frac{\Pi_{F,UB}}{\Pi_{H,LB}}$ such that $\frac{\Pi_F}{\Pi_H} < \frac{\Pi_{F,UB}}{\Pi_{H,LB}}$. Therefore, there exists $\left(\frac{Y_F}{Y_H}\right)^* = \tau^{\sigma-1}\frac{\Pi_{F,UB}}{\Pi_{H,LB}}$ such that if $\frac{Y_F}{Y_H} > \left(\frac{Y_F}{Y_H}\right)^*$, $\frac{Y_F}{Y_H} > \frac{\Pi_F}{\Pi_H}$.

First, we can define $\Pi_{F,UB}$, an upper bound for Π_F. Since all firms test F first, the level of competition in F is

$$\begin{aligned}
\Pi_F &= M_F p_{FF}^{1-\sigma}\left(\int_{-\infty}^{\infty} \exp(x)\, g(x)\, dx + \frac{1-\delta}{\delta}\int_{x^*_{FF}}^{\infty} \exp(x)\, g_0(x,0)\, dx\right) \\
&\quad + M_H p_{HF}^{1-\sigma}\left(\int_{-\infty}^{\infty} \exp(x)\, g(x)\, dx + \frac{1-\delta}{\delta}\int_{x^*_{HF}}^{\infty} \exp(x)\, g_0(x,0)\, dx\right)
\end{aligned}$$

where M_j is the mass of new varieties from country j each period. By noticing that $p_{HF}^{1-\sigma} < p_{FF}^{1-\sigma}$, we can show that

$$\Pi_F < \Pi_{F,UB} = p_{FF}^{1-\sigma} \left(M_F + M_H\right) \frac{1}{\delta} \exp\left(\frac{s^2}{2}\right).$$

Similarly. We can define a lower bound for Π_H. Only firms that obtain an $X_{F\omega} > \frac{1}{\rho}\mu_{iH1}^{+}$ will test the H destination. Therefore, the level of competition in H is

$$\begin{aligned}
\Pi_H &= M_F p_{FF}^{1-\sigma} (1-\delta) \int_{\frac{1}{\rho}\mu_{FH1}^{+}}^{\infty} \begin{pmatrix} \int_{-\infty}^{\infty} \exp(z)\, g_1(z|\rho x)\, dz \\ + \frac{1-\delta}{\delta} \int_{x_{FH}^{*}}^{\infty} \exp(z)\, g_1(z|\rho x)\, dz \end{pmatrix} g(x)\, dx \\
&\quad + M_H p_{HF}^{1-\sigma} (1-\delta) \int_{\frac{1}{\rho}\mu_{HH1}^{+}}^{\infty} \begin{pmatrix} \int_{-\infty}^{\infty} \exp(z)\, g_1(z|\rho x)\, dz \\ + \frac{1-\delta}{\delta} \int_{x_{HH}^{*}}^{\infty} \exp(z)\, g_1(z|\rho x, 1)\, dz \end{pmatrix} g(x)\, dx
\end{aligned}$$

and we can define a lower bound for it:

$$\Pi_H > \left(M_F p_{HF}^{1-\sigma} + M_H p_{HF}^{1-\sigma}\right) (1-\delta) \int_{\frac{1}{\rho}\mu_{FH1}^{+}}^{\infty} \left(\int_{-\infty}^{\infty} \exp(z)\, g_1(z|\rho x)\, dz\right) g(x)\, dx.$$

Using the properties of the lognormal, we find that

$$\begin{aligned}
\int_{-a}^{\infty} \exp(u)\, g(z|\rho x, 1)\, dz &= \exp(\rho x) \int_{-a}^{\infty} \exp\left(z\sqrt{(1-\rho^2)}\right) g(z)\, dz \\
&= \exp(\rho x) \exp\left(\frac{(1-\rho^2)\, s^2}{2}\right) \Phi\left(\left(1-\rho^2\right) s - \frac{a}{s}\right)
\end{aligned}$$

and use that results to continute manipulating the inequality:

$$\begin{aligned}
\Pi_H &> \left(M_F p_{FF}^{1-\sigma} + M_H p_{HF}^{1-\sigma}\right)(1-\delta)\int_{\frac{1}{\rho}\mu_{H1}^{+}}^{\infty} \exp(\rho x)\exp\left(\frac{\left(1-\rho^2\right)s^2}{2}\right) g(x)\,dx \\
&> \left(M_F p_{FF}^{1-\sigma} + M_H p_{HF}^{1-\sigma}\right)(1-\delta)\exp\left(\frac{\left(1-\rho^2\right)s^2}{2}\right)\int_{\frac{1}{\rho}\mu_{H1}^{+}}^{\infty} \exp(\rho x) g(x)\,dx \\
&> \left(M_F p_{FF}^{1-\sigma} + M_H p_{HF}^{1-\sigma}\right)(1-\delta)\exp\left(\frac{\left(1-\rho^2\right)s^2}{2}\right)\exp\left(\frac{\rho^2 s^2}{2}\right)\Phi\left(\left(1-\rho^2\right)s - \frac{\mu_{H1}^{+}}{\rho s}\right) \\
&> \left(M_F p_{FF}^{1-\sigma} + M_H p_{HF}^{1-\sigma}\right)(1-\delta)\exp\left(\frac{s^2}{2}\right)\Phi\left(\left(1-\rho^2\right)s - \frac{\mu_{H1}^{+}}{\rho s}\right) \\
&> \left(M_F p_{FF}^{1-\sigma} + M_H p_{HF}^{1-\sigma}\right)(1-\delta)\exp\left(\frac{s^2}{2}\right)\Phi\left(\left(1-\rho^2\right)s - \frac{x_{HH}^*}{s}\right) \qquad (24)
\end{aligned}$$

We need to substitute in for the endogenous x_{HH}^*. From the proof to Proposition 2, we know $0 > v_{HH0}(0)$. Therefore,

$$\begin{aligned}
0 > v_{HH0}(0) &= f\left(\begin{array}{c} \int\limits_{-\infty}^{\infty} \exp\left(x - x_j^*\right) g(x)\,dx - 1 \\ +\frac{(1-\delta)}{\delta}\int\limits_{x_j^*}^{\infty}\left(\exp\left(x - x_j^*\right) - 1\right) g(x)\,dx \end{array}\right) \\
&= f\left(\begin{array}{c} \exp\left(\frac{s^2}{2} - x_{HH}^*\right) - 1 + \frac{(1-\delta)}{\delta}\exp\left(\frac{s^2}{2} - x_{HH}^*\right) \\ -\frac{(1-\delta)}{\delta}\Phi\left(s - \frac{x_{HH}^*}{s}\right) \end{array}\right) \\
\exp\left(\frac{s^2}{2} - x_{HH}^*\right) &> \delta + (1-\delta)\Phi\left(s - \frac{x_{HH}^*}{s}\right) \\
\frac{\exp\left(\frac{s^2}{2} - x_{HH}^*\right) - \delta}{(1-\delta)} &> \Phi\left(s - \frac{x_{HH}^*}{s}\right) \\
\exp\left(\frac{s^2}{2} - x_{HH}^*\right) &> \delta \\
\frac{s^2}{2} - x_{HH}^* &> \ln\delta \\
\left(1-\rho^2\right)s - \frac{x_{HH}^*}{s} &> \frac{\ln\delta}{s} + \left(1-\rho^2\right)s - \frac{s}{2}
\end{aligned}$$

We can plug this result into Inequality 24 to define

$$\Pi_H > \Pi_{H,LB} = \left(M_F p_{HF}^{1-\sigma} + M_H p_{HF}^{1-\sigma}\right)(1-\delta)\exp\left(\frac{s^2}{2}\right)\Phi\left(\frac{\ln\delta}{s} + \left(1-\rho^2\right)s - \frac{s}{2}\right)$$

Now we know that

$$\frac{\Pi_F}{\Pi_H} < \frac{\Pi_{F,UB}}{\Pi_{H,LB}} = \frac{\tau^{\sigma-1}}{\delta(1-\delta)\Phi\left(\frac{\ln\delta}{s} + (1-\rho^2)s - \frac{s}{2}\right)} \tag{25}$$

If $\frac{Y_F}{Y_H} > \tau^{\sigma-1}\frac{\Pi_{F,UB}}{\Pi_{H,LB}}$, then it certainly satisfies Condition 15. Therefore, we can define a sufficient condition for an equilibrium of type 1 :

$$\begin{aligned} \frac{Y_F}{Y_H} &> \frac{\tau^{2\sigma-2}}{\delta(1-\delta)\Phi\left(\frac{\ln\delta}{s} + (1-\rho^2)s - \frac{s}{2}\right)} \\ &\Rightarrow \\ \frac{Y_F}{Y_H} &> \tau^{\sigma-1}\frac{\Pi_F}{\Pi_H} \end{aligned} \tag{26}$$

F Figures

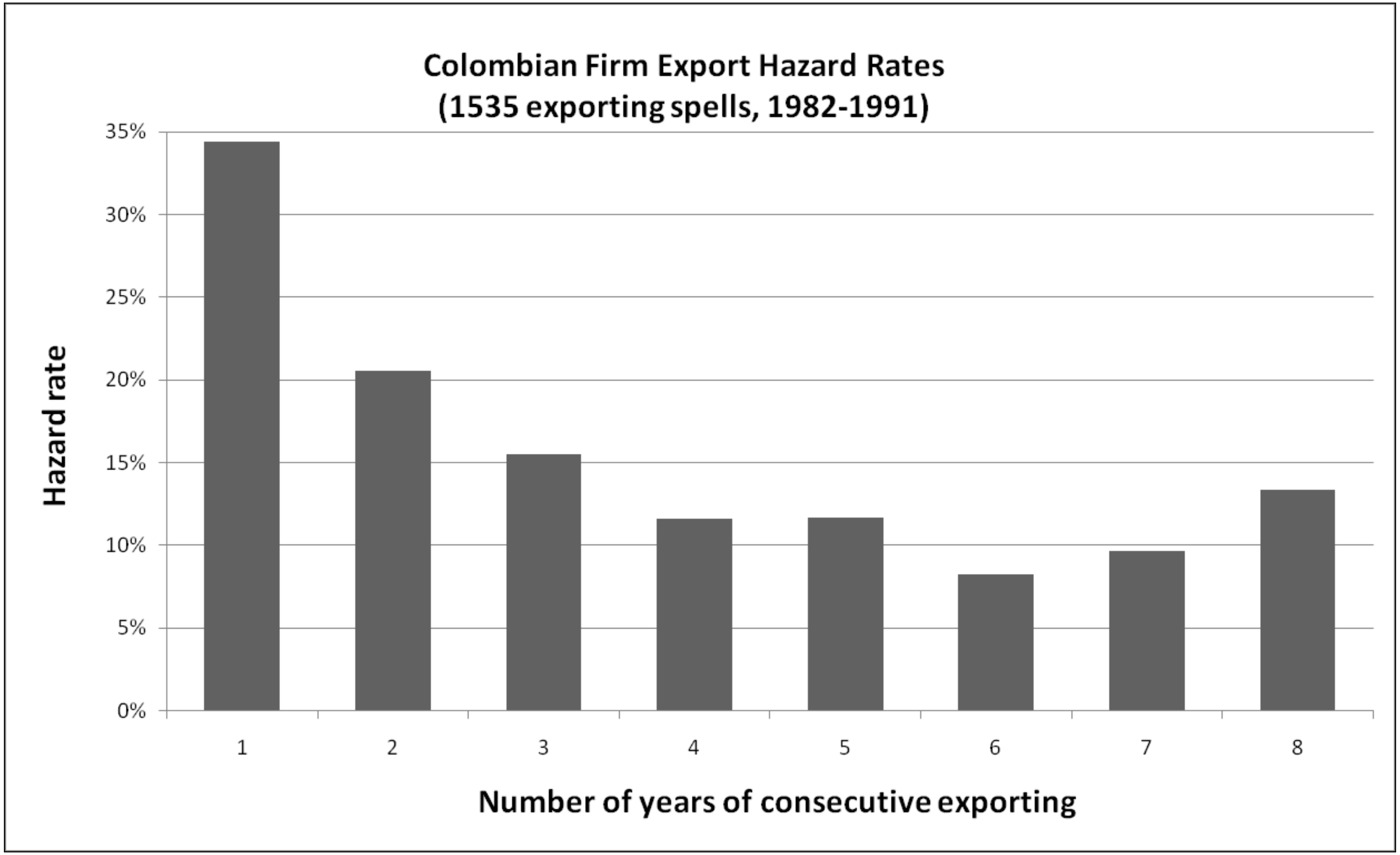

Figure 1: Hazard rates for Colombian firm exporting spells.An exporting spell is defined as the number of years of consecutive exporting.1981 exporting status was used to determine whether a spell started in 1982.1991 exporting status was used to determine the number of firms that survived,but not the numbfer of firms that stopped. For example, there were 166 spells of 2 years that started in 1982 or after and ended in 1990 or before. There were 643 spells of lengths 3 or more that started in 1982 or after, including those that still exported in 1991. Therefore, the hazard rate at spells of 2 years $= \frac{166}{166+643} = 20\%$. The uptick at 8 years could be small sample error; only 6 firms failed after 8 years. No firms had exporting spells of exactly 9 years (1982-1990).

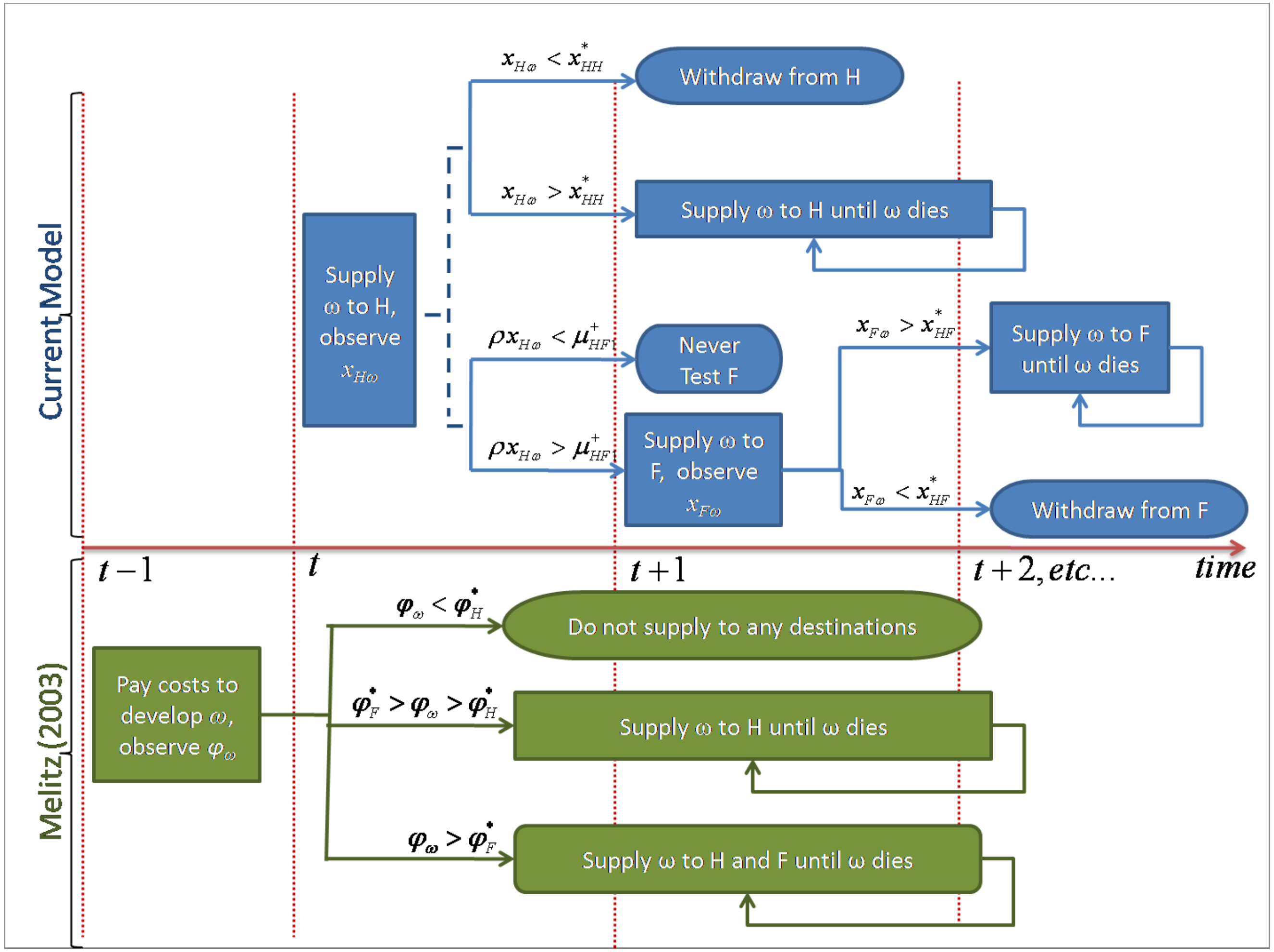

Figure 2: Model Timeline for new firm in H at time t, with Melitz (2003) as reference.

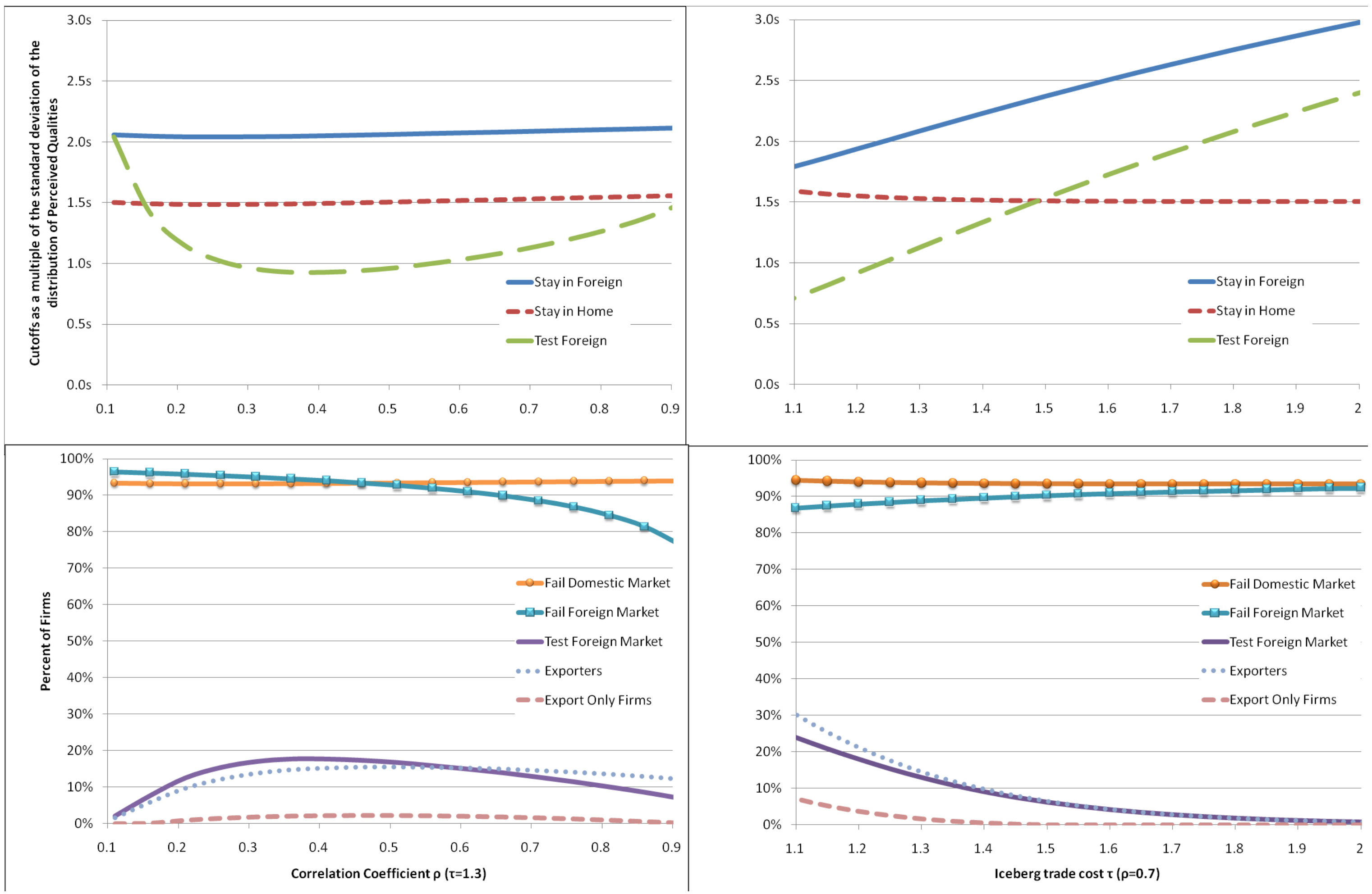

Figure 3: Equilibrium values of stay cutoffs x^*_{HH} and $\frac{1}{\rho}x^*_{HF}$ and test cutoff μ^+_{HF1} (top two graphs) and the percents of firms that test and fail the Home and Foreign markets, and the percent of total firms that are exporters or export only firms (bottom graphs). Cutoff values in the top graphs are expressed as multiples of s, the standard deviation of the exogenous firm perceived quality distribution. For example, at $\tau = 1.75$, a firm needs a perceived quality at least two standard deviations above the mean in order to forecast a positive value of testing the foreign market.