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Abstract. We develop a simple model of short- and long-term unemployment to study how labor market institutions interact with labor market conditions and personal characteristics of the unemployed. We analyze how the decision to exit unemployment and to mitigate human capital degradation by retraining depends on education, skill degradation, age, labor market tightness, taxes, unemployment insurance benefits and welfare assistance. We extend our analysis by allowing for time-inconsistent choices and demonstrate the possibility of an unemployment trap.

Keywords: Unemployment, Skill Degradation, Retraining, Unemployment Benefits, Welfare Assistance, Present-Biased Preferences.

JEL: J64, J31, J38

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1. Introduction

Unemployment has been a pressing problem in Europe for decades, and the high share of long-term unemployment is a major concern. For example, according to OECD (2005), more than 40 percent of all European unemployment is classified as long-term with a duration of a year and over. However, unemployment is far from uniform in Europe. There is indeed considerable dispersion of both unemployment rates and the prevalence of long-term unemployment within European countries. This variation across European countries, as well as the fact that the share of long-term unemployment is much lower in the U.S. (12 percent) suggest that differences in labor market institutions lie at the heart of the European unemployment problem (e.g. Siebert 1997).

Several empirical studies have emphasized that long-term shifts of OECD unemployment can be best explained by changes in labor market institutions (rather than by macroeconomic shocks or shifts of macroeconomic time-trends). See, in particular, Scarpetta (1996), Nickell (1997), Elmeskov et al. (1998), Belot and van Ours (2001, 2004) and Nickell et al. (2005). The generosity and the duration of eligibility for unemployment benefits as well as labor taxes have been identified as particularly strong predictors of unemployment rates in these studies. To a somewhat lesser degree, employment protection indices and union densities were found to play a role. Furthermore, the literature stresses interaction terms indicating that the effects of a particular institutional feature may differ depending on the overall institutional framework. A related empirical literature emphasizes the role of institutions in amplifying macroeconomic shocks. Blanchard and Wolfers (2000), for example, find that shocks have larger effects on unemployment when unemployment benefits are generous and granted over extended time and when employment protection is strict. Yet, Nickell et al. (2005) demonstrate that shock-interaction terms do not add much explanatory power when they augment the basic model in which unemployment is caused by institutional factors.

A different view is taken by Ljungqvist and Sargent (1998) who argue that many of the labor market institutions deemed to be the cause of high and persistent unemployment were already in place when unemployment was still low in Europe. These authors use simulation techniques to demonstrate the possibility that equilibrium unemployment rates can be similar in a generous welfare state and in a laissez-faire economy in tranquil times with little aggregate fluctuation. Yet, in turbulent times – engendering large skill losses of the unemployed – unemployment rates
can differ substantially because generous benefits discourage search by highly skilled workers who suffered a loss of human capital while unemployed. Thus, Ljungqvist and Sargent also identify institutional characteristics to be responsible for the European unemployment problem. Their explanation of unemployment change, however, is based on increasing skill degradation (turbulence). While Ljungqvist and Sargent’s initial hypothesis is challenged by the observation that there were indeed considerable institutional changes within and across European countries over the last 30 years, the central theoretical argument that faster skill degradation may cause or aggravate long-term unemployment remains of course valid.

We adopt the idea that the endowment with human capital and its degradation when not employed are crucial factors in the determination of long-term unemployment. We then proceed by investigating how particular institutional variables affect the decision to remain short- or long-run unemployed and to mitigate skill degradation by retraining.

Skill degradation is frequently invoked as an intuitive explanation of hysteresis in unemployment. It complements other explanations based on insider-outsider effects of wage determination (Lindbeck and Snower, 1988), habit formation (Vendrik, 1993), and stigmatization or scarring effects. See Roed (1997) and Cahuc and Zylberberg (2004) for overviews and Bean (1994) for a critical comparison of supply- and demand-side mechanisms in explaining European unemployment and its persistence. Demand-side effects of skill loss during unemployment are investigated by Pissarides (1992) and Coles and Masters (2000).

We investigate supply-side effects of human capital degradation on (long-term) unemployment as in Ljungqvist and Sargent (1998). Methodologically, however, we use a different approach. Instead of solving a complicated general equilibrium model of search and matching numerically we focus on the labor supply- and retraining-decisions of currently unemployed persons. This micro-economic focus enables us to solve the problem analytically and to present comparative statics in a simple diagrammatic exposition. Our analytical approach builds on human capital theory and investigates how the unemployed react to incentives with respect to retraining and, in particular, the time spent "on the dole". We assume that workers’ earnings depend on their skills, that these skills deteriorate while unemployed but that skill degradation can be mitigated by retraining. These incentives are shaped by the interplay of labor market institutions, socio-economic characteristics of the unemployed and labor market performance. Our approach enables us to investigate a broad range of factors. The institutional factors we analyze
comprise the generosity and the duration of unemployment benefits, labor income taxation, the cost of retraining and welfare benefits. The socio-economic characteristics of the unemployed taken into account are their age, skill level, and time preference. The effects of labor market performance are captured by exogenous shocks to contemporaneous and expected earnings in the labor market, and these shocks might be temporary or permanent.

Our analysis yields a large number of results that are in line with empirical findings, and some of our results are qualitatively similar to those obtained by Ljungqvist and Sargent (1998). However, the main difference to their results is that we find a generous welfare state, in particular high taxes on labor income and generous welfare assistance, to be important determinants of long-term unemployment, while they identify unemployment insurance as the main culprit. These results are qualitatively similar in that these factors all reduce the attractiveness of working relative to unemployment, but practical policy implications might still differ.

We believe that our approach has several advantages. Our model is rich in that it incorporates a large number of determinants of unemployment; it is simple in that the formal reasoning is tractable and can be graphically displayed in a single diagram, it is flexible in that the model can easily be extended to account for present-biased preferences, and it is accurate in the sense that most of the predictions are in line with stylized empirical facts. These advantages come at the cost of some limitations. Our account is partial as it focuses on the supply-side and is not embedded in a general equilibrium framework; if abstracts from search frictions, and macroeconomic fluctuations are modelled in a highly stylized fashion. On balance, we believe that our model provides an attractive complement to more complex models of unemployment.

The paper proceeds as follows. The model presented in Section 2 builds on the standard assumptions that the unemployed make their decisions to retrain and on how long to remain unemployed exclusively according to monetary criteria and they are assumed to make these decisions in a fully rational and consistent way. Section 3 investigates comparative-statics of labor market institutions, socio-economic characteristics, and labor market conditions. Section 4 relaxes the standard assumptions by allowing for time-inconsistent preferences. This opens up the possibility that agents get stuck in an unemployment trap and remain on the dole permanently, against their best intentions.
2. The Model

Consider a person who loses his job but could get a new job immediately or anytime in the future (we abstract from job search frictions). The person can either accept the job, earn a wage that depends on his productivity and current labor market conditions, and pay taxes on his wage income. Alternatively, the person can remain unemployed. We call a person remaining unemployed for less than $\tau$ periods a short-term unemployed (STU), and a person who remains unemployed beyond $\tau$ periods a long-term unemployed (LTU). The STU receive unemployment benefits depending on their last (net) wage for a limited time of $\tau$ periods. Long-term unemployed persons receive social benefits (welfare assistance) which are independent of their last wage until the end of their working life. The cost of being unemployed is that human capital erodes, i.e. the unemployed face skill degradation. We assume that the unemployed can retrain at some cost to mitigate skill degradation.

The decision maker thus faces two decisions: when to exit unemployment (i.e. to be employed, STU or LTU) and, if unemployed, whether to mitigate human capital degradation by retraining. As shown in detail below, our model predicts that some (those with high productivity) find it profitable to accept the job and to be employed, some may find it attractive to obtain unemployment benefits for some periods and use the STU spell for retraining, and some (often those with low skills), may find it profitable to let human capital erode and remain on the dole permanently. These decisions often depend in subtle ways on labor market institutions, personal characteristics and labor market tightness.

Consider a currently unemployed person with human capital (productivity) $h$ who could get a job at a gross wage $w_t = h - \lambda_t$. The parameter $\lambda_t$ represents current labor market tightness, i.e. exogenous shocks to labor market income. We assume that labor income is taxed at a constant rate $\theta$.

It pays for a person with human capital $h$ to exit unemployment immediately (i.e. at time $t = 0$) if the present value of life-time labor market income

$$ V(h, 0) = \sum_{t=0}^{T-1} \delta^t (1 - \theta)(h - \lambda_t) $$

(1)

dommates all other possible income streams. We assume that future income is discounted at factor $\delta < 1$, and that the person retires from working life after $T$ periods. Equation (1) shows
that life-time labor market income $V(0)$ increases with human capital endowment and the length of remaining working life. This implies that $V(0)$ decreases with the worker’s age.

The cost of being unemployed is that human capital degrades during unemployment. This may, for example, be the case because the unemployed do not benefit from on-the-job training, because human capital erodes when not familiarized with the current state-of-the-art technology, or simply because they are not used to work anymore.\(^1\) The unemployed may mitigate human capital degradation or actually improve skills by incurring costly educational efforts (e.g. by taking a computer course). These efforts are valuable because they increase human capital, and thus future wages. To keep the analysis of skill deterioration and retraining tractable, we assume that a fraction $d$ of human capital depreciates during short-term unemployment and that an unemployed person can spend $c \cdot \bar{e}$ units of income on retraining which increases their human capital by $e > 0$ units. Alternatively, they may let their human capital erode and do not retrain ($e = 0$).

A short-term unemployed (STU) receives unemployment benefits $b \cdot (1 - \theta)h$ from unemployment insurance for $\tau$ periods. STU leave unemployment at time $\tau$ and receive a wage of $w_t = (1 - \theta)(1 - d)h + e - \lambda_t$ per period $t$ for the rest of their working life. Thus the present value of an STUs life-time income is given by

\[
V(h, \tau) = \sum_{t=0}^{\tau-1} \delta^t b(1 - \theta)h - c \cdot e + \sum_{t=\tau}^{T-1} \delta^t (1 - \theta) [h(1 - d) + e - \lambda_t],
\]

where $c$ denotes unit costs of retraining (in present value terms).

If people are still unemployed after $\tau$ periods, they become long-term unemployed (LTU) and receive tax-exempt social benefits $s$ per period until time $T$. The present value of an LTU’s life-time income is

\[
V(h, T) = \sum_{t=0}^{\tau-1} \delta^t b(1 - \theta)h + \sum_{t=\tau}^{T-1} \delta^t s.
\]

A person’s life-time income $V(h, t)$ can be understood as a function of his human capital and the time $t$ at which he leaves unemployment. To keep the analysis simple, we assume that labor

\(^1\) Topel (1990) estimates that in the US in the 1970s and 1980s workers losing a job suffered on average a wage reduction of between 15 and 40 percent by the time they found a new job. Jacobsen et al. (1993) find similar results. Keane and Wolpin (1997) estimate that blue-collar skills depreciate by about 10 percent in a year absent from work, while white-collar workers lose about 30 percent of their skills during one year of unemployment. Thus, a more realistic model would tie the depreciation rate itself to the level of human capital. In any case, however, human capital degradation is of non-negligible magnitude.
market tightness is stable up to time \( \tau \) and may then change. Hence, \( \lambda_t \) can take two values: \( \lambda_0 \), representing current labor market tightness and \( \lambda_\tau \) denoting expected labor market tightness from time \( \tau \) until time \( T \). To simplify the notation, we introduce the following aggregated discount factors.

\[
\delta_\tau = \sum_{t=0}^{\tau-1} \delta^t = \frac{1 - \delta^\tau}{1 - \delta}, \quad \delta_T = \sum_{t=\tau}^{T-1} \delta^t = \delta^\tau \cdot \frac{1 - \delta^{T-\tau}}{1 - \delta}.
\] (4)

Note from inspection of equations (4) that a longer working-life increases \( \delta_T \) and leaves \( \delta_\tau \) unaffected, while longer duration of eligibility for unemployment benefits increases \( \delta_\tau \) and reduces \( \delta_T \) in equal proportions, \( \partial \delta_\tau / \partial \tau = -\partial \delta_T / \partial \tau > 0 \). With these notational simplifications we can analyze career decisions conveniently in a \( h\)–\( V \)-diagram taking the exit time from unemployment \( t \in \{0, \tau, T\} \) parametrically. For this purpose we rewrite (1’) – (3’) as

\[
V(h, 0) = \delta_\tau (1 - \theta)(h - \lambda_0) + \delta_T (1 - \theta)(h - \lambda_\tau)
\] (1’)

\[
V(h, \tau) = \delta_\tau b(1 - \theta)h - c \cdot e + \delta_T (1 - \theta) [(1 - d)h + e - \lambda_\tau]
\] (2’)

\[
V(h, T) = \delta_\tau b(1 - \theta)h + \delta_T s.
\] (3’)

The decision to retrain is straightforward. Equation (2’) reveals that life-time income of the short-term unemployed is higher with retraining \( (e > 0) \) than without \( (e = 0) \) if the present value of the labor income gain exceeds the present value of retraining costs. In particular, \( e > 0 \) is chosen if

\[
c < (1 - \theta)\delta_T.
\] (5)

Otherwise, no retraining takes place, i.e. \( e = 0 \). Thus, equation (5) suggests that high labor taxes discourage retraining and that younger workers (i.e. those with higher \( \delta_T \)) tend to do more retraining than older workers. Mitigating human capital degradation is never optimal for a long-term unemployed because he never reaps the returns of retraining on the labor market.

A natural assumption (which holds for every OECD country, see OECD, 1999) is that the net market wage is higher than unemployment benefits, i.e. that the replacement rate \( b \) is smaller than 100 percent. Under this condition, life-time income is always more steeply increasing in human capital for employed persons than for the short-term unemployed (STU). For the STU, in turn, life-time income increases more steeply in \( h \) than for the long-term unemployed (LTU).
\[ V(h, 0)' = \delta_r(1-\theta) + \delta_T(1-\theta) > V(h, \tau)' = \delta_r(1-\theta)b + \delta_T(1-\theta)(1-d) > V(h, T)' = \delta_r(1-\theta)b. \]  

(6)

Thus, in a \( V - h \) diagram, life-time income intersects in three points yielding critical levels of human capital.\(^2\) These intersections are calculated from (1’) – (3’):

\[
h^A = \frac{\delta_T \left[ s + (1-\theta)\lambda_r \right] - e \left[ (1-\theta)\delta_T - c \right]}{\delta_T(1-\theta)(1-d)}
\]

(7)

\[
h^B = \frac{(1-\theta) \left[ \delta_r \lambda_0 + \delta_T \lambda_\tau \right] + s\delta_T}{(1-\theta) \left[ \delta_r(1-b) + \delta_T \right]}
\]

(8)

\[
h^C = \frac{\delta_r(1-\theta)\lambda_0 + e \left[ (1-\theta)\delta_T - c \right]}{(1-\theta) \left[ \delta_r(1-b) + \delta_T d \right]}
\]

(9)

We call the intersection between \( V(h, \tau) \) and \( V(h, T) \) point \( A \), the intersection between \( V(h, 0) \) and \( V(h, T) \) point \( B \), and the intersection between \( V(h, 0) \) and \( V(h, \tau) \) point \( C \) (see Figure 1).

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\(^2\)It is also possible according to (6) that it is never optimal to be STU. This degenerate case is discussed in the Appendix.
Figure 1 reflects several well-known empirical regularities. First, unemployment decreases with education (see e.g. OECD, 2004). Second, long-term unemployment is more prevalent among low-skill workers. Human capital, i.e. productivity, can be so low that the best available choice is to live on the dole permanently. In our model, this is in particular the case when labor markets currently are and will remain tight (high $\lambda_0$ and $\lambda_\tau$), and when social benefits ($s$) are generous.\footnote{Note that we compare income streams and do not consider how utility is affected by time availability – be it positively via leisure or negatively via reduced self-esteem. See Darity and Goldsmith (1996) for an overview on the effects of joblessness on emotional well-being.}

Education and employment go hand in hand, according to the empirical literature. For example, Nickell (1979) and Machin and Manning (1999) show that unemployment duration falls with education. Yet, in our setup becoming LTU means a complete exit from the labor force. In other words, the decision to participate in the labor market again (triggered, for example, by a change of welfare assistance, labor market tightness, or taxation) affects labor supply at the extensive margin. Our result that LTU is - ceteris paribus - observed for people with low skills and low productivity is in line with the empirical regularity that extensive labor supply elasticities are particularly large at the low end of the income distribution (Eissa and Liebman, 1996 and Meyer and Rosenbaum, 2001). A positive relationship between education and participation rates is also documented in OECD (2004).

On the upper end of the education spectrum we find those workers who never prefer to be unemployed because the opportunity costs of unemployment (in terms of human capital degradation and future wage loss) are too high. For intermediate skill-levels short-term unemployment is optimal if $h^A$ is located to the left of $h^C$ – as shown in Figure 1 (see appendix for the case without STU). For the STU it pays to let human capital erode if unemployment benefits are relatively generous, if human capital does not erode much, or if the unemployment spell is actually used to retrain and to upgrade human capital.

STU can be optimal for both persons who intend to retrain as well as for those who intend to let human capital erode. STU always prevails when retraining pays off, i.e. when (5) holds. These persons find it optimal to retreat from the labor market to acquire additional human capital (think of someone taking an MBA). In order to obtain the condition for STU without...
retraining, insert (7) and (9) and \( e = 0 \) into \( h^A < h^C \) to get:

\[
\lambda_\tau + \frac{s}{1 - \theta} < \frac{\delta_\tau \lambda_0 (1 - d)}{\delta_\tau (1 - b) + \delta_\tau d}
\]

Thus, STU is more likely to be observed when the opportunity costs of being unemployed are low (i.e. when labor markets are currently tight (large \( \lambda_0 \)) but little tightness is expected in the future (small \( \lambda_\tau \))), when the direct cost in terms of human capital degradation are low (low \( d \)), when the benefits of STU are high (i.e. unemployment benefits \( b \) are generous and granted for a long period (i.e. \( \delta_\tau \) is large)), and when exiting unemployment after some time is relatively attractive (i.e. welfare assistance \( s \) is low, and taxes on labor income \( \theta \) are not too high). Ceteris paribus, STU without re-education is more prevalent among older workers (low \( \delta_T \)).

3. Comparative Statics

We now discuss how labor market institutions (in particular, the generosity of unemployment benefits \( b \), duration of eligibility for unemployment benefits \( \tau \), labor market taxes \( \theta \), generosity of welfare assistance \( s \)), the characteristics of the unemployed (in particular, human capital \( h \), skill loss \( d \), age \( T - \tau \), retraining costs \( c \)) interact with adverse contemporaneous (\( \lambda_0 \)) or expected (\( \lambda_\tau \)) labor market shocks to explain retraining and the duration of unemployment. By differentiating the functions (7) and (9) with respect to the model’s institutional and individual parameters we obtain how \( h^A \) and \( h^C \) change their position in the career diagram, providing comparative statics of individual career decisions. If we imagine a country’s (or region’s) workforce ordered along the \( h \)-line according to their human capital endowment, we can infer the comparative statics of aggregate short-term and long-term unemployment.

3.1. Unemployment Benefits. In line with empirical evidence, we find that a higher replacement rate \( b \) increases overall unemployment (e.g. Nickell et al., 2005). Our model predicts an increase STU, but not in LTU. In Figure 1, an increase in \( b \) leaves \( h^A \) unaffected and shifts \( h^C \) to the right.

\[
\frac{\partial h^A}{\partial b} = 0, \quad \frac{\partial h^C}{\partial b} = \frac{\delta_\tau h^C}{\delta_\tau (1 - b) + \delta_\tau d} > 0.
\]

Higher unemployment benefits increase STU because they make it worthwhile to let human capital erode for a while or to use an unemployment spell for the accumulation of new skills. Unemployment benefits increase STU at the expense of employment and leave LTU unaffected.
The model predicts that workers with low human capital degradation \((d)\) and old workers (small \(\delta_T\)) are particularly responsive to changes in unemployment benefits.

### 3.2. Duration of Eligibility for Unemployment Benefits

The effects of an increase in the duration of eligibility for unemployment benefits are less straightforward. An increased duration of unemployment benefits increases LTU if it discourages (low-skill) STU to retrain. An increased duration increases total unemployment under some conditions (in particular when labor markets are tight and taxes are high). To simplify the notation, we define \(\delta' \equiv \partial \delta \tau / \partial \tau > 0\) and recall that \(\partial \delta \tau / \partial \tau = -\delta'\). We obtain the following effects of increasing the duration of eligibility for unemployment benefits \((\tau)\).

\[
\frac{\partial h^A}{\partial \tau} = \frac{ce \delta'}{\delta_T(1-\theta)(1-d)} \geq 0, \quad \frac{\partial h^C}{\partial \tau} = \frac{(\delta_r + \delta_T)\lambda_0(1-\theta)d - \phi \cdot e \cdot \delta'}{(1-\theta)\delta_r(1-b) + \delta_Td} \delta'
\]

\[
\phi \equiv [(1-\theta)\delta_T - c](1-b) + (1-\theta)(1-b)\delta_r + cd > 0,
\]

where the positivity of \(\phi\) follows from the retraining condition (5).

Suppose unemployment is not used for retraining \((e = 0)\). In this case, longer eligibility for unemployment benefits does not affect LTU (these workers prefer to stay on the dole anyway). It increases STU because some workers in the higher range of the education spectrum find a short spell in unemployment attractive \((\partial h^C/\partial \tau > 0\) for \(e = 0)\). This will in particular be the case when current labor market frictions are large. Without labor market frictions \((\lambda_0 = 0)\) and without retraining, there is no effect of benefit eligibility on STU because the replacement rate is smaller than one (i.e. \(1-\theta-b > 0)\).

If workers retrain, their option to upgrade skills is the dominant motive for being STU (rather than to collect unemployment benefits). In this case, our simplified model which treats retraining as a one-time event that happens instantly (by paying the retraining cost) may imply a degenerate reaction to longer eligibility of unemployment benefits. Having used the STU spell for retraining, workers actually strive for a new job because the payoff of the newly acquired skills in terms of higher wages occurs only after re-employment.

An (exogenous) extension of the unemployment period \(\tau\) has thus the negative effect of delaying re-employment. The delay devalues retraining such that some workers with intermediate skills who would have retrained with a shorter duration now decide against retraining. Without retraining, they find a permanent stay on the dole attractive and as a consequence LTU increases.
at the expense of STU ($\partial h^A / \partial \tau > 0$ for $e > 0$).

An exogenous extension of the eligibility period $\tau$ also affects workers with relatively high skill levels. These workers would also have chosen to use a STU spell to invest in human capital, but with a higher $\tau$, they find this option less attractive than entering employment. The derivative of $h^C$ shows that devaluation of retraining is the dominant effect on STU if taxes are low and if labor market tightness is low (since both further increase the value of being quickly employed again). While this interpretation of exogenous changes of $\tau$ on human capital are fairly intuitive, the reverse interpretation is perhaps more plausible. In this case, $\tau$ is interpreted as decision variable depending on $h$. Interpreted in this way, the derivative shows that a worker’s preferred duration of STU for retraining purposes decreases with his initial endowment of human capital, $\partial \tau / \partial h^C < 0$.

The “normal” reaction of increasing unemployment for increasing duration of unemployment benefits payments $\partial h^C / \partial \tau > 0$, is obtained when market frictions and taxes are sufficiently high. In this case, longer eligibility makes remaining in STU more attractive and employment decreases. In other words, interaction terms matter (see Belot and van Ours, 2004, and Nickell et al., 2005). In a U.S.–style welfare system where $\lambda_0$ is small and $\phi$ is large (because taxes $\theta$ and benefits $b$ are low), the effect of longer eligibility on STU will be small and possibly ambiguous. In a European–style welfare state where $\lambda_0$ is large and $\phi$ is small the effect of benefit eligibility on STU is positive and possibly large.

3.3. Welfare Assistance. Higher welfare assistance changes the composition but not the overall level of unemployment. Higher welfare assistance increases LTU because some low-skill STU find it attractive to remain on the dole permanently, but it does not increase overall unemployment. In Figure 1, an increase in welfare assistance moves $h^A$ to the right and leaves $h^C$ unaffected.

$$\frac{\partial h^A}{\partial s} = \frac{1}{(1 - \theta)(1 - d)} > 0, \quad \frac{\partial h^C}{\partial s} = 0$$

Workers at the upper end of the education spectrum may consider to use an unemployment spell for some further retraining but staying permanently on the dole is never an option for them. Thus, the decision between employment and STU is unaffected by a change in welfare assistance ($\partial h^C / \partial s = 0$). But some workers who previously planned only a short spell of unemployment
will find LTU attractive when welfare assistance increases. The derivative $\frac{\partial h^A}{\partial s}$ indicates that this effect will be large for those with high rates of human capital degradation.

Some researchers hold generous welfare assistance responsible for continental Europe’s unemployment problem. A single LTU with children, for example, who would live on food stamps in the U.S. receives in Germany Sozialhilfe of 648 Euros per month (including support for housing and heating). Sozialhilfe for other types of households is higher. A couple with two children, for example, receives on average 1601 Euros per month. Several other European countries operate social welfare of comparable generosity (European Commission, 2004). For Germany, Boss (2001) and Sinn et al. (2002) calculate marginal tax rates on (potential) low-wage income of the recipients of social welfare between 80 and 100 percent. Sinn et al. conclude that it does not pay to work for a head of a West German household with two children if his or her productivity is below 50 percent of average productivity.

In the career decision diagram more generous welfare assistance shifts $h^A$ to the right. This moves social benefits above net wage income for a larger fraction of the $h$-ordered population and higher long-term unemployment in the low-productivity segment of the labor market results. The magnitude of this effect is predicted to be large when taxes on (potential) labor income are high.

3.4. Taxation. Higher taxes on labor income increase LTU because they discourage some low-skill STU to exit unemployment. Hence, STU falls. STU may fall for a second reason. Higher taxes may also reduce STU because they make retraining for high-skill persons less profitable. In Figure 1, a higher tax rate on labor income $\theta$ moves $h^A$ to the right and shifts $h^C$ left, if anything.

$$\frac{\partial h^A}{\partial \theta} = \frac{\delta_T s + ce}{\delta_T (1 - \theta)^2 (1 - d)} > 0$$

$$\frac{\partial h^C}{\partial \theta} = -\frac{ce}{(1 - \theta)^2 [\delta_T (1 - b) + \delta_T d]} \leq 0.$$  

Higher taxes reduce the net payoff from working and increase LTU. Note that the effect is particularly strong if retraining is, in principle, worthwhile ($e > 0$). Some low-productivity workers who would have used an STU spell to upgrade their skills when taxes are low are now facing a lower net return of re-education after the tax increase and prefer to stay on the dole.
permanently. Furthermore, the effect is strong if taxes are already high, if welfare assistance is generous, and if skills degrade rapidly during STU.

Yet, lower net returns of retraining also apply to workers of intermediate productivity who previously considered an STU spell and now prefer to exit unemployment immediately. While higher labor taxation increases LTU, it reduces STU for two reasons: an outflow into employment of workers from the higher productivity segment and an outflow into LTU of workers from the lower productivity segment who are permanently discouraged by higher taxes.

We now discuss how personal characteristics of the unemployed determine retraining and the duration of unemployment. As explained below, some variables which are interpreted as reflecting personal characteristics might also be interpreted as due to policy choices. For example, the effect of the variable remaining work-life might be interpreted as a workers age or as (mandatory) retirement age.

3.5. **Human Capital Degradation.** The rate at which a workers human capital erodes can depend on his personal characteristics, i.e. on his skill level, or be a consequence of exogenous factors like the rate at which technological innovations are made. Rapid degradation of human capital increases LTU because retraining is less attractive for the STU, and it reduces STU because the direct cost of degradation while unemployed are too high for high-skill workers. In Figure 1, higher human capital degradation during unemployment moves $h^A$ to the right and $h^C$ to the left.

$$\frac{\partial h^A}{\partial d} = \frac{s + \lambda_T (1 - \theta)}{\delta_T (1 - \theta)(1 - d)^2} - e [(1 - \theta)\delta_T - c] = \frac{h^A}{(1 - d)} > 0, \quad \frac{\partial h^C}{\partial d} = -\frac{\delta_T}{\delta_T (1 - b) + d} < 0.$$

Hence, rapid human capital degradation reduces STU at both ends. Some workers from the upper end of the education spectrum who previously thought about using a spell of STU to upgrade their human capital, now decide against it because the opportunity costs of skill loss while not working are too high. At the lower end of the educational spectrum some workers who previously found a short spell of unemployment attractive, now see their human capital (and potential future wages) erode so fast that staying permanently out of work becomes attractive.

It is worthwhile to point out some similarities and differences of effects of human capital degradation in our model and the model by Ljungqvist and Sargent’s (1998). The main goal of their analysis is to explain the puzzle how labor market institutions can account for high
European unemployment. This is a puzzle because unemployment rates increased massively in Europe relative to the U.S. only after the early 1980s although many of the underlying institutions were already in place before that time. The authors show that the puzzle can be resolved by consideration of turbulence, i.e. human capital degradation ($d$). In their model increased turbulence (as observed in the 1980's) has a strong effect on unemployment in countries with a large welfare state compared to a laisser-faire economy.

Our investigation partly replicates their results but finds also new and different effects. First, while Ljungqvist and Sargent identify unemployment insurance benefits as the main culprit, our analysis suggest that high welfare assistance ($s$), high expected labor market tightness ($\lambda_T$) and high labor income taxes or, more generally, a large welfare state may induce large-scale skill degradation among the LTU. Second, Ljungqvist and Sargent’s study suggests that in particular workers with high initial skill level (high pre-unemployment wage) choose to stay unemployed for a while (i.e. search with low intensity for a new job) in times of turbulence. In contrast, our analysis suggests that employment increases in the high-skill segment when $d$ increases because the opportunity costs of skill-loss are too high in turbulent times. The unemployment pool is filled with workers from the lower end of the skill distribution who find a permanent life on the dole attractive when times are turbulent.

Finally, we can derive a result with respect to working-age. The derivative of $h^C$ is increasing in $\delta_T$ in absolute terms. Thus, the positive effect on employment operates among the youth (with a long remaining work life $T-t$) who exit unemployment earlier under the threat of rapid skill degradation. On the other end of the education spectrum we find that $h^A$ is decreasing in $\delta_T$ implying that the LTU pool is in particular filled by older workers close to retirement age who face a lower threat of skill degradation. Our result is supported by the observation that LTU in Europe increased in particular within the group of older workers (OECD, 2002).

3.6. **Retraining Costs.** An increase in retraining costs discourages retraining among the STU and therefore increases LTU. In Figure 1, higher retraining costs $c$ shift $h^A$ to the right and $h^C$ to the left.

$$\frac{\partial h^A}{\partial c} = \frac{e}{\delta_T(1-\theta)(1-d)} \geq 0,$$

$$\frac{\partial h^C}{\partial c} = -\frac{e}{(1-\theta)[\delta_T(1-b) + \delta_T d]} \leq 0.$$
Obviously, an increase in retraining cost has no effects if retraining does not take place \((e = 0)\) at low \(c\). With retraining, STU is reduced at both ends when \(c\) rises. At the upper end of the education spectrum, some workers who previously considered an STU spell to upgrade their skills now find that they cannot afford it and stay employed. Retraining becomes also too expensive for some at the lower end of the education spectrum. For those persons, however, it is then the best choice to stay out of work permanently.

While retraining costs can be determined by personal characteristics, they may also importantly depend on labor market policy. For example, retraining costs can be reduced by subsidies for (re-) education. Our model suggests a short-run increase of unemployment due to subsidies for retraining. The policy only reduces unemployment in the long run, i.e. when workers re-enter employment who were motivated by the retraining subsidy to shift from LTU to STU (reflected by the move of \(h^A\) to the left). Note that retraining subsidies are particularly effective for persons with rapid human capital degradation since \(\partial h^A / \partial c\) increases and \(\partial h^C / \partial c\) decreases in absolute terms when \(d\) rises.

### 3.7. Age

Comparative statics of increasing working-life \(T\) and, thus, a larger discount factor \(\delta_T\) depend on a number of factors in subtle ways. The derivatives are

\[
\frac{\partial h^A}{\partial \delta_T} = -\frac{ce}{\delta_T^2(1 - \theta)(1 - d)} \leq 0, \quad \frac{\partial h^C}{\partial \delta_T} = \frac{e - dh^C}{\delta_T(1 - b) + \delta_T d}.
\]

The duration of working life can be analyzed in a cross-sectional perspective distinguishing between younger and older workers or in a time-series perspective when investigating exogenous events like an increase in retirement age. In analyzing the effects of a longer work life on STU, we distinguish two cases.

When retraining does not pay off \((e = 0)\), the effects of an increased duration of working life are clear. In this case, our model predicts no effect on LTU because low-productivity workers prefer to stay on the dole permanently anyway. Yet, an increase in the length of working life induces an increase of employment, i.e. a reduction of STU \((\partial h^C / \partial \delta_T < 0\) for \(e = 0)\). The reason is that a longer working life increases the present value of a permanent loss of human capital during an STU spell. Therefore, STU becomes more threatening and people exit unemployment earlier.

With retraining being worthwhile \((e > 0)\), an increase in work-life affects STU because it
extends the time during which the fruits of retraining are reaped and thus makes an STU spell for the purpose of retraining attractive. If the retraining effect dominates (reflected by relatively high $c$), STU increases with the length of working life; if the human capital degradation effect dominates (reflected by relatively high $d$) it decreases.

In any case, however, a prolonged working life reduces LTU when retraining is worthwhile. Some of the workers from the lower end of the productivity spectrum who previously settled on a life on the dole find retraining attractive when the fruits of the educational effort are reaped through a longer working life. This result suggests a strong policy conclusion: Early retirement cannot be a solution of the unemployment problem (as many central European authorities seem to have believed in the 80's and 90's). Quite the opposite is true according to the model. Another way of putting this result is that LTU is particularly attractive for older workers (with small $\delta_T$). Furthermore, the derivative of $h^A$ shows that the incentive to stay on the dole permanently will be strong for older workers when skills erode rapidly and labor income taxes are high.

3.8. Labor Market Tightness. Fluctuations in labor markets are reflected in deviations of actual wage income from wages paid under "normal" circumstances, $w_t = h - \lambda_t$. We model labor market tightness as resulting from exogenous events. These can be temporary, perhaps due to a downturn in the business cycle, or persistent, perhaps due to structural shifts in labor demand. A contemporaneous and temporary recession is captured by an increase in $\lambda_0$, and an expected recession by an increase in $\lambda_\tau$. A permanent structural change is modelled by imposing $\lambda_0 = \lambda_\tau$. The corresponding derivatives are

\[
\frac{\partial h^A}{\partial \lambda_0} = 0, \quad \frac{\partial h^C}{\partial \lambda_0} = \frac{\delta_\tau}{\delta_\tau(1-b) + \delta_T d} > 0
\]

\[
\frac{\partial h^A}{\partial \lambda_\tau} = \frac{1}{(1-d)} > 0, \quad \frac{\partial h^C}{\partial \lambda_\tau} = 0.
\]

In line with standard intuition, a contemporaneous temporary recession increases STU and leaves LTU unaffected. The effect on STU is particularly large if unemployment benefits are high and granted for a long period ($\delta_T$ is large). The effect is particularly pronounced among older workers (with low $\delta_T$). A possibly more surprising prediction of our model is that an expected future downturn of the economy (i.e. higher $\lambda_\tau$) increases LTU. A pessimistic outlook reduces the incentive to retrain during STU, and some workers of the current STU pool decide against
retraining. Without retraining, their productivity falls to a level that makes a permanent life on the dole attractive. This effect is particularly strong if human capital erodes rapidly.

An adverse structural change, perhaps due to a fall in international competitiveness, is modelled by higher $\lambda_0 = \lambda_\tau$ and is obtained as the sum of both effects discussed above. The structural change will thus unambiguously increase LTU and reduce employment. The effect on STU depends again on whether the inflow into STU dominates the outflow into LTU. A comparison of derivatives shows that this is the case if

$$\left(1 + \frac{\delta_\tau}{\delta_\tau}\right) \cdot d < b,$$

i.e. if human capital erosion is relatively small compared to unemployment benefits. In this case, incentives are high enough for skill-upgrading to be the predominant cause of STU.

4. Unemployment Trap and Present-Biased Preferences

This section relaxes the standard assumptions by allowing for present-biased preferences. Such preferences seem to be common and imply time-inconsistent behavior, such as procrastination (Laibson 1997 and O'Donoghue and Rabin 1999). Allowing for such preferences enables us to extend the model to account for the possibility of an unemployment trap in which people end up on the dole permanently, against their best intentions. The unemployment trap we demonstrate is a consequence of mistaken intertemporal optimization. It is therefore an individual-level phenomenon, and not an equilibrium phenomenon as in other accounts of the unemployment trap.

Reconsider the case where retraining costs are sufficiently low and people are sufficiently young such that retraining, in principle, pays off, i.e. condition (5) holds. Diagrammatically, those with human capital below $h^A$ stay long-term unemployed and those with human capital exceeding $h^C$ take up employment immediately in Figure 2. The interesting group consists of those with intermediate human capital endowment, $h^A < h < h^C$, who enter short-term unemployment with the intention to upgrade their skills and re-enter employment later.

The duration of STU can be subdivided in $\tau$ subperiods of eligibility for unemployment benefits, which can conveniently be thought of as weeks or months. Suppose that it takes one subperiod to upgrade the skills, for example, by taking a computer course and suppose that some individuals have present-biased preferences (Laibson, 1997, O'Donoghue and Rabin,
Thus, the present value of life-time income (2') for an STU can be rewritten as

\[ V_t(h, \tau) = b(1 - \theta)h - k_t \cdot c \cdot e \]

(11)

\[
+ \beta \left\{ \sum_{s=1}^{\tau-1-t} \delta^s [b(1 - \theta)h - k_s \cdot c \cdot e] + \sum_{s=\tau-t}^{T-1-t} \delta^s (1 - \theta) \left[ (1 - d)h + \left( \sum_{i=0}^{\tau-1} k_i \cdot e \right) \right] \right\}
\]

with \( \beta \leq 1 \) and where \( k_t \) is an indicator function that takes a value of \( k_t = 0 \) in any subperiod without retraining and \( k_t = 1 \) in the retraining subperiod.\(^5\) If retraining happens, then \( \sum k_t = 1 \) and \( \sum k_t = 0 \) otherwise. The only but crucial difference between (11) and (2') is that now the discount factor from today to the next subperiod is \( \beta \delta \) whereas \( \delta \) remains the discount factor between any two subperiods in the future.

During STU people move from subperiod \( t = 0 \) to subperiod \( \tau - 1 \). We first consider the case in which individuals have perfectly time-consistent preferences (\( \beta = 1 \)) as a benchmark. In this case, people rationally postpone retraining until the ultimate period of STU and then retrain (\( k_{\tau-1} = 1 \)) if retraining is worthwhile, i.e. if

\[ c < (1 - \theta)\delta_T \equiv (1 - \theta) \frac{\delta(1 - \delta^{T-1-\tau})}{1 - \delta}. \]

(12)

Except for the determination of the timing of retraining, this is exactly the result from section 2, and with \( \beta = 1 \) we are back in the standard model.

Now consider agents with present-biased preferences, i.e. the case of \( \beta < 1 \). All future selves of these persons prefer to retrain if condition (12) holds. The analysis below focuses on the case where people are not aware of their time-inconsistency problem, i.e. they do not realize that their tastes change when they get closer to the moment when they originally intended to execute their retraining decision.\(^6\) Like the rational agents they postpone retraining. However, in the last period of eligibility for unemployment benefits (at \( t = \tau - 1 \), when the rational agents

\(^4\) The assumption that it takes one subperiod to re-educate is made for simplicity and could be generalized. The crucial underlying assumption is that retraining has to be taken up once (or never) during unemployment. DellaVigna and Pasermann (2005) integrate present-biased preferences into a search and matching model. They find (theoretically and empirically) a negative effect of impatience on search intensity and exit from unemployment.

\(^5\) Since the argument is independent from labor market tightness we dropped the \( \lambda \)'s in (11) for convenience.

\(^6\) These persons are called naive agents in O’Donoghue and Rabin (1999). Sophisticated agents would realize their present-bias and thus execute retraining inefficiently early during the unemployment spell. Thus, sophisticated agents will not run into the unemployment trap as discussed here.
retrain), the present-biased value of life-time income becomes

\[ V_{\tau-1}(h, \tau) = b(1 - \theta)h - k_{\tau-1} \cdot c \cdot e + \beta \sum_{s=1}^{T-\tau} \delta^s (1 - \theta) \left[ (1 - d)h + \left( \sum_{i=0}^{\tau-1} k_i \cdot e \right) \right] \]  \hspace{1cm} (13)

implying that they will retrain if

\[ c < \beta (1 - \tau) \tilde{\delta}_T. \]  \hspace{1cm} (14)

They will not retrain if the costs of retraining fulfill

\[ \beta \tilde{\delta}_T < \frac{c}{1 - \theta} < \tilde{\delta}_T. \]  \hspace{1cm} (15)

If condition (15) holds, present-biased agents do not retrain when they lose eligibility for unemployment benefits although they originally planned to use the STU spell to upgrade their skills.

Figure 2 illustrates the consequences of a failed retraining episode depending on the worker’s initial skill level. Because retraining was, in principle, worthwhile, failed retraining shifts lifetime income \( V(h, \tau) \) downwards. At the upper end of the education spectrum, between \( h'_C \) and \( h_C \), we find those workers who would have preferred to be employed had they known
that they run into a time-inconsistency problem and do not take up the originally planned retraining. At intermediate education levels, between $h_A'$ and $h_C'$, present-biased preferences have no consequence on unemployment duration (but, of course, on life-time income). At the lower end of the education spectrum, between $h_A$ and $h_A'$, we observe the sad result that workers who originally planned to re-enter employment after a spell of STU used for retraining find themselves lacking the willpower to do so. Yet, with failed retraining and human capital erosion, their productivity is so low to make living on the dole the best available option. Thus, they end up unintentionally in long-term unemployment. Some of the well-educated are lured into STU, but will not remain on the dole permanently. It is the low-educated that fall prey to the unemployment trap and get stuck on the dole permanently.

The preceding discussion indicates the importance of active labor-market policies. If retraining capabilities are equally distributed across initial education levels, a sufficiently high subsidy of retraining costs (or a tax allowance) that drives $c/(1 - \theta)$ below $\tilde{\beta} \tilde{\delta}_T$ would eliminate the time-inconsistency problem according to the model. More generally, any commitment device that enforces retraining would be beneficial for the unemployed (in the sense of increasing his or her long-run utility) and would reduce short-run and long-run unemployment.

The expression "unemployment trap" usually refers to a situation where an unemployed person is unable to increase his income through employment. This applies to all LTU in our model. What we are concerned with here is the possibility that an agent ends up in LTU against his best intentions. Note that the discussion above provides a rationale for an unemployment trap resulting from agents having present-biased preferences but are otherwise rational. Our account of an unemployment trap is not based on strategic interaction. In contrast, an unemployment trap may loom, for example, when multiple equilibria arise due to externalities from hiring restrictions (Saint-Paul 1995), due to discrimination of long-term unemployed (Acemoglu 1995), or from an interplay of social norms and voting on welfare policies (Lindbeck et al. 1995).

A further explanation for why people end up in unemployment they would have chosen to avoid ex ante is based on "forced unemployment", which can easily be integrated into the model of section 2. Forced short-term unemployment occurs if a workers optimal choice (given the prevailing labor market conditions) is to work but the person is for some external reason inhibited from doing so and is therefore in fact unemployed. Forced short-term unemployment increases long-term unemployment. The intuition for this result is as follows: Some workers
with relatively low human capital endowment are only slightly better off by working than by being unemployed. If these workers are forced to enter unemployment, their human capital degrades. If this degradation is pronounced enough and if human capital investments during unemployment are costly enough, their human capital erodes to the point where it does not pay to work again at the time when re-employment is possible.

5. Conclusion

This paper has offered a micro-economic tool for the analysis of short- and long-term unemployment. Still simple enough to be solved analytically and discussed diagrammatically, the model has highlighted the importance of labor market institutions (generosity and duration of unemployment benefits, welfare assistance, taxes, and retraining costs) and how they interact with personal characteristics (age, retraining costs, rate of human capital degradation) and stylized exogenous labor market fluctuations (labor market tightness) to explain unemployment duration and retraining efforts. Our model provides a theoretical foundation for recent empirical studies who find institutional variables at the heart of the unemployment problem.

Despite our mostly intuitive results which are in line with much empirical evidence for several reasons we would like to caution the reader to jump to policy conclusions. First, any quantitative policy assessment would require to embed the model in a macroeconomic context and integrate it into a general equilibrium model of job search and matching. While this would possibly destroy the general solvability and beauty of the model, we expect little value added. This is because the micro-economic model already suggests that “it depends”. Therefore, the second and perhaps more important reason for caution is that the comparative statics have shown that the individual characteristics of the unemployed (education, skill loss, and age) determine the impact of institutional change. This holds in all cases with respect to the magnitude of effects and sometimes even with respect to their sign. As a consequence, the heterogeneity of the workforce must be taken into account when analyzing the implications of policy interventions.

Nevertheless, we believe some general conclusions follow from our model. We identify skill degradation as an important determinant of long-term unemployment as in Ljungqvist and Sargent (1998). In contrast to these authors, we do not find the problem to be aggravated by unemployment benefits. Higher unemployment benefits, while increasing short-term unemployment, can actually be favorable in the long-run because individuals may use a spell of short-term
unemployment for retraining. Our analysis has determined generous welfare assistance, high tax rates, and labor market tightness as the main amplifiers of the impact of skill loss on long-term unemployment and thus as the main culprits of the European unemployment problem. In addition, we identify – in line with the empirical evidence – low-skilled workers and older workers as particularly prone to human capital degradation and long-term unemployment. Finally our analysis has emphasized the role of retraining costs and active labor market policies. These policies are particularly important when the unemployed have present-biased preferences and lack the willpower to retrain. In this case, voluntary short-term unemployment is like a slippery slope and results in a permanent life on the dole, against ones best intentions.
Appendix: Comparative Statics without STU

If life-time incomes intersect in a way that STU is never the best choice as in Figure 3, human capital endowment $h^B$ separates employment and LTU. The relevant derivatives are

\[
\frac{\partial h^B}{\partial b} = \frac{\delta_T h^B}{\delta_T (1-b) + \delta_T} > 0
\]

\[
\frac{\partial h^B}{\partial \tau} = \delta' \left( \lambda_0 - \lambda_r - \frac{s}{1-\theta} + h^B b \right) \frac{1}{\delta_T (1-b) + \delta_T} > 0
\]

\[
\frac{\partial h^B}{\partial s} = \frac{\delta_T}{(1-\theta) \left[ \delta_T (1-b) + \delta_T \right]} > 0
\]

\[
\frac{\partial h^B}{\partial \theta} = \frac{\delta_T s}{(1-\theta)^2 \left[ \delta_T (1-b) + \delta_T \right]} > 0
\]

\[
\frac{\partial h^B}{\partial d} = \frac{\partial h^B}{\partial c} = 0
\]

\[
\frac{\partial h^B}{\partial \delta_T} = \left( \frac{s}{1-\theta} - h^B \right) \frac{1}{(1-\theta) \left[ \delta_T (1-b) + \delta_T \right]^2}
\]

\[
\frac{\partial h^B}{\partial \lambda_0} = \frac{\delta_T}{\delta_T (1-b) + \delta_T} > 0, \quad \frac{\partial h^B}{\partial \lambda_r} = \frac{\delta_T}{\delta_T (1-b) + \delta_T} > 0
\]

In principle, the effects on employment and LTU are the same as for the non-generate case discussed in the text. Sometimes, of course, there are no partial effects observable. Because the LTU will never be at work again they are not affected by retraining costs and do not suffer from (further) increasing skill degradation. This is not the same as saying that there are no effects at all. The parameter variation can be so large that LTU is affected because STU turns out to be worthwhile and the degenerate case ceases to exist.

The sign of $\frac{\partial h^B}{\partial \delta_T}$ is undetermined and only unambiguously positive in the absence of current labor market tightness ($\lambda_0 = 0$). In this case LTU is decreasing with $\delta_T$, i.e. younger workers are less afflicted with unemployment. The sign of $\frac{\partial h^B}{\partial \tau}$ is also ambiguous. It includes a degenerate case where longer eligibility for unemployment benefits leads to lower unemployment. In order to provide an intuition, we assume that labor market frictions are structural
$(\lambda_0 = \lambda_T = \lambda)$, implying that the sign of the derivative equals the sign of

$$b(1 - \theta)\lambda(\delta_r + \delta_T) - s(1 - b).$$

A degenerate case may occur if $s$ is very high and $b$ is low. Then, social benefits are so generously granted compared to unemployment benefits that increasing eligibility for unemployment benefits entails the dominating negative effect of delaying waiting time until eligibility for welfare assistance. Of course, without welfare assistance ($s = 0$) this channel is always closed.

LTU is independent from retraining costs and human capital degradation because the choice is between being never and always unemployed ($\partial h^B / \partial d = \partial h^B / \partial c = 0$). The other derivatives are in line with the case discussed in the main text: LTU increases for higher unemployment benefits, higher welfare assistance, higher taxes rates, and higher labor market frictions.
References


