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An Equilibrium Analysis of the Gender Wage Gap

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Abstract

This paper develops a theory of the gender wage gap. In a general equilibrium model, spouses devide their labor between a formal sector and a home sector. Due to indivisibility effects, productivity of labor in the formal sector is negatively related to labor used in the home; at the same time labor inputs are complementary in home production. We show that initial beliefs about the gender wage gap are self-fulfilling, and a central result is multiplicity of equilibria. Spouses allocate their labor equally, if they expect to earn the same wage rates, which ex post reinforces equal wage rates; whereas they allocate their labor differently, if they expect to earn different wage rates. The latter situation manifests itself in a gender wage gap. By use of numerical examples, we show that welfare is highest when spouses allocate labor equally. We relate this finding to policy recommendations.

Key words: Gender Wage Gap, Household Models, Household Production, Labor Markets.

JEL Classification codes: D13, J16, J22, J30

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1 Introduction

The gender wage gap has been explained by an inherent source of difference between men and women, which causes women to earn lower wages. For example, Elul et al. (2002) suggest that gender-differences in wages can be attributed to demographic reasons. Men marry younger women, and men therefore, before getting married, have the opportunity to settle where they receive maximal compensation. Women, on the other hand, marry at younger age, and are accordingly more likely to settle where their compensation is not at maximum. Siow (1998) attribute differences in earnings to a biological factor: differences in fecundity. Women are only able to have children in a limited period of their lives, whereas men are not subject to this restriction. Men therefore need extra income to have children when old, and thus have an incentive to work more than women. This leads to higher human capital accumulation and, consequently, higher male wage rates relative to female wage rates.

The starting point for our analysis of the gender wage gap, on the contrary, is an economy in which, male and female individuals, with one of each represented in a household, are ex ante generically *identical* except for gender. Within this symmetric framework, we explain the possibility that the gender wage gap is lasting rather than disappearing. Our analysis demonstrates that gender equality in terms of equal education and equal opportunity are necessary, but not sufficient, conditions to eliminate the gender wage gap. Beliefs about the gender wage ratio play a crucial role in this result in that they are self-fulfilling. As such beliefs, even in modern society, are likely to be shaped by historical facts, today's notion of relative wages is arguably biased towards a gender wage gap. Due to the self-fulfilling nature of such a notion, our model suggests that a gender wage gap will persist unless actions are taken to change society's beliefs.

We develop a general equilibrium model in which we link two sectors; a home sector and a formal sector. Our model is based on two main assumptions: indivisibility effects of labor in the formal sector, the workplace; and complementarity of husband's and wife's labor input in home production.

Home production takes place within households. Households are described by a standard unitary household model and labor is allocated as to achieve intra-household efficiency. This means that labor of each spouse is allocated so the marginal value product of labor in home production equals the wage rate. If spouses expect to earn different wage rates, they allocate their labor differently. On the other hand, if spouses expect to earn the same wage rate, they allocate their labor identically. Complementary in home production means that one household member cannot produce without labor input from her spouse. This assumption is certainly justifiable for household activities which concern reproduction. More generally, one can argue that without mutual affection and attention, there will be no household production by either family member.

The indivisibility effects of labor in the workplace imply that one employee working $2T$ hours produces more than two employees each working T hours. Arguments in favor of this relationship include sunk costs such as start-up costs. Moreover, if more workers are assigned on the same project, they may have to exchange information and update one another, which is likely to be costly in terms of decreasing productivity. The assumption also reflects profitability of availability. The more time an employee spends at the job, the more likely the employee is able to act immediately in case of emergencies and urgent requests. Arguments can furthermore be made in favor of learning by doing effects in the workplace; the more time a worker spends producing, the more productive the worker becomes.[1] A consequence of indivisibility of labor in employment is that the less labor a worker puts into home production, the more productive the worker is in employment.

Firms are non-discriminating,[2] competitive, and hire workers taking as given the supply of labor (in terms of hours)[3] of each worker. They hire workers until the marginal productivity of a worker equals her marginal costs (her salary). Firms are therefore willing to hire both low and high productivity workers, if the workers' salaries vary accordingly. In equilibrium, a worker who works long hours earns a high wage, and a worker who works short hours earns a low wage.

A key result of our model is the existence of multiple equilibria. In particular, there exist: both an equilibrium in which spouses differ in their labor allocation, and earn different wages, and an equilibrium in which all workers have identical labor allocations, and earn the same wage. In this way, our model explains gender-differences in earnings by gender-differences in labor allocation. In turn, gender-

[1] It can be argued that the gap in hourly wages between part-time and full-time jobs to some extent reflects differences in effectiveness between short and long hours. Such a gap has been reported repeatedly, e.g., by the U.S. Department of Labor (2005). In 2004, a full-time worker in the US earned about $19 per hour, whereas part-time workers earned only $10. Yet a portion of the difference may be accounted for by occupational differences. Hirsch (2000) finds that the part-time wage gap diminishes considerably by controlling for age, gender, skill level, and other variables. Also Bonke et al. (2005) discuss how increasing hours in household production are correlated with wages. They find that household work has a negative effect on female wages. The same is not completely true for men, however; low-end male wages are positively correlated with household work.

[2] Meyersson Milgrom et al. (2001) find that, within Sweden, men and women doing the same work for the employer are paid the same salary. In academic labor markets, however, evidence of discrimination has consistently appeared (Blackaby et al. 2005).

[3] This assumption at first may seem to contradict the conventional assumption that labor demand is decreasing in wages. This would be true if we did not distinguish between the number of workers and number of hours worked. Indeed, we postulate that workers and hours are not perfectly substitutable (see, e.g., Cahuc and Zylberberg (2004) for a discussion).

differences in labor allocation occur if the initial beliefs about wages are stereotype.[4] If indeed beliefs are stereotype, the labor market dictates a wage rate for women and a wage rate for men.[5]

Another question we address is whether the gender wage gap equilibrium is Pareto efficient. By use of numerical examples, it turns out that a gender wage gap situation is inferior, as it brings less welfare to society than the situation in which spouses' labor allocation and wage rates are the same. Household members share identical factor shares in household production. Accordingly, in the equilibrium in which labor allocation is identical across spouses, total labor input in home production may be less than in the unequal case, and yet produce more household service. This implies welfare gains, since the extra freed labor is used in the firm.

In a closely related, but independent paper, Albanesi and Olivetti (2006) also argues that the gender wage gap may be explained as a result of the interaction between a family, who allocates labor between the home and the firm, and the firm which hires labor. Unlike our paper, however, which emphasizes interaction between household production and effectiveness of labor use in the workplace, Albanesi and Olivetti focus on labor market attachment; household members choose both effort and labor market hours. They examine two situations: the situation with an initial difference in productivities of men and women, and the situation without. In the latter, which resembles the assumptions in our model, the authors find two types of equilibria which they refer to as gendered and ungendered. In contrast to our analysis, their focus is not on complementarity in home production and indivisibility

[4] By stereotype, we mean traditional patterns of sex roles.

[5] We stress that this mechanism is conceptually different from discrimination in that in our model the wage rate for men and women coincides when initial beliefs are unisex.

effects of labor in the workplace, but like our analysis, the self-fulfilling nature of initial beliefs determines the outcome.

Our model is also related to the framework outlined in Becker's seminal work on sexual division of labor published in 1985. Becker argues that even if husband and wife are intrinsically identical, they gain from a division of labor between employment and household work; one specializes in employment and the other specializes at home. Such a division raises the productivity of both persons. Becker's result is driven partly by Becker's assumption about the production function in home production; the household service is produced according to a production function where inputs of each family member are perfect substitutes in production. As we consider a household production function in which labor from both household members is complementary in production, our results are different in that, despite the specialization gains in the workplace which arises from the indivisibility assumption, household members do not specialize completely in different sectors.

Chichilnisky and Eisenberger (2005) study production functions, learning curves, and the division of labor within families. Using a logistic learning curve which changes from convexity to concavity through an inflection point, they find that for the concave part, which represents highly skilled labor, equal division of tasks is efficient whereas for unskilled labor efficiency involves specialization. The latter result replicates the Becker (1985) outcome. Their model implies that within advanced economies, where labor is highly skilled, differences in labor allocation between men and women can be ascribed to market failures. Chichilnisky (2005), who also uses a logistic learning curve, argues that missing contracts between the home and the workplace, and missing property rights to household services, lead to an outcome with an unequal division of labor between husband and wife. Firms and families play a game similar to the Prisoner's Dilemma, and the outcome is a Pareto inefficient gender wage gap situation.

There is also a large body of empirical literature which attempts to explain the gender wage gap. Explanations include the so-called family gap: women who marry and have children experience a higher wage gap than unmarried women with no children (Ginther 2004; Waldfogel 1998; Winder 2004), job segmentation, i.e., men and women are allocated differently to occupations that differ in the wages they pay (Meyersson Milgrom et al. 2001), and self-selection of women into sectors that have experienced a relatively lower wage growth (Rosholm and Smith 1996). Other explanations suggest that family-friendly policies may have adverse effects on female wages (Gupta et al. 2006), or that evidence of a glass ceiling effect[6] prevents women from being paid the same as men (Meyersson Milgrom and Petersen 2006). Finally, Blackaby et al. (2005) suggest that discrimination causes women to be underpaid. Still, however, a large fraction of the gender wage gap seems to remain unaccounted for (Blau and Kahn forthcoming).

The paper proceeds as follows. In the next section we provide some data to motivate our model. Section 3 develops a model in full generality by describing the representative two-person family, the representative firm and the equilibrium. In section 4, we solve our model and present the results. In section 5, we discuss the welfare aspects of equilibria, and in section 6, we discuss policy implications. The final section provides some concluding remarks.

[6] The glass ceiling effect refers to a situation within firms in which there is a rank or level beyond which women are rarely promoted.

2 Background

Female participation in the labor force has increased substantially during the last half century in advanced economies. As an illustration, fig. 1 shows how men and women's labor force participation rates have evolved in the US since the 1950s.

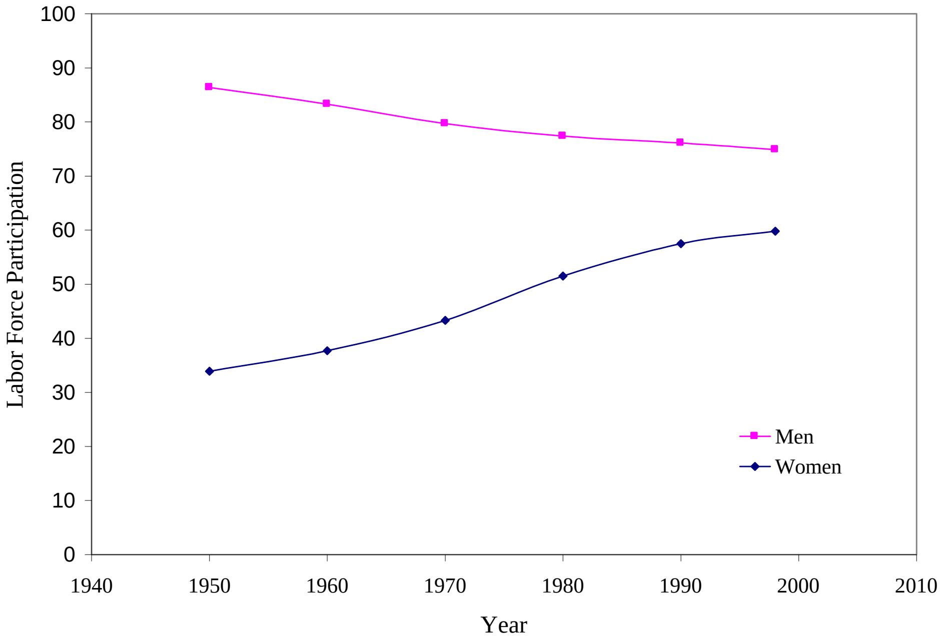

Figure 1: US labor force participation for men and women, 1950-98. (Data from Fullerton 1999, table 1.)

The female labor force participation rates rose from 33.9 percent in 1950 to 59.8 percent in 1998. In the same period, the male labor force participation rates declined from 86.4 percent to 74.9 percent. As a result, the difference in labor force participation rates went down from 52.5 percent in 1950 to 15 percent in 1998 (Fullerton 1999).

In addition, women's educational achievements are rising. In the US, women have overtaken the role as the most educated sex since the mid-90s (Freeman 2004).

Yet despite these advancements of women's position in the labor force, and despite "equal work equal pay" regulations in many countries,[7] women do not seem to be making the same salaries as men.

After women entered the labor force, the gender wage gap[8] has been closing. In the US, the gap converged in the 1980s after a stable period in the 1960s and 1970s (Blau and Kahn 2000). Since then the convergence has slowed. Fig. 2 shows how the gender wage gap has evolved. The slope in the first part of the period is significantly[9] different from the slope of the second period.

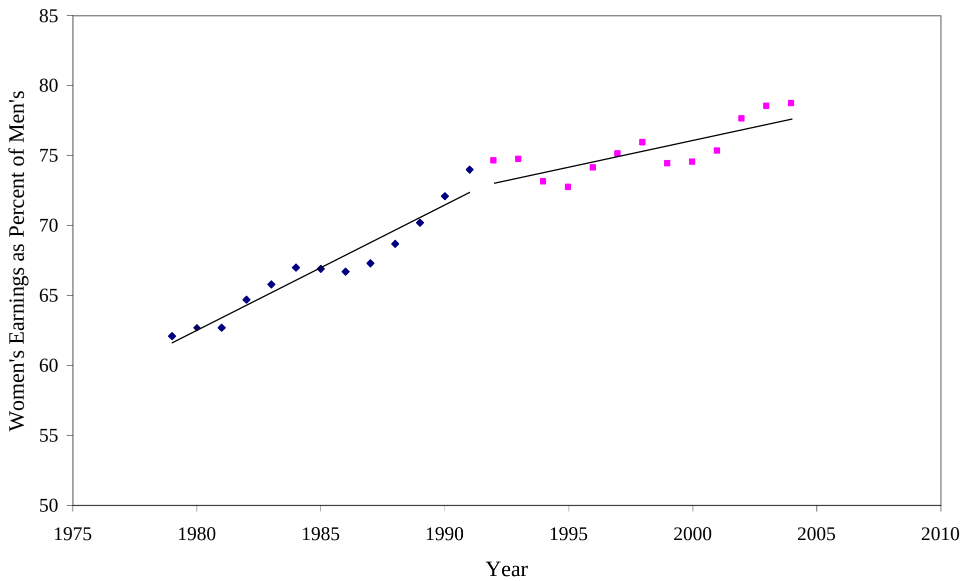

Figure 2: The US gender wage gap, 1979-2004. (Data from U.S. Department of Labor 2005, table 16.)

[7] ILO's Equal Remuneration Convention no. C100 has, since its enactment in 1951, been ratified by 162 countries, http://www.ilo.org/ilolex/english/convdisp2.htm.

[8] In general, the gender wage gap is a rough estimate that includes both differences in earnings across "male and female occupations" as well as differences in male and female earnings within the same occupation. One should therefore be careful when comparing wage gap estimates.

[9] The null hypothesis of a common slope is rejected at the 1 percent level by use of a f-test.

Indeed, Blau and Kahn (forthcoming) find that in the US, the gender wage gap has remained almost constant since the early 1990s. Similar findings are presented for other advanced economies, such as those of Sweden (Edin and Richardson 2002) and Denmark (Gupta et al. 2006). In the OECD countries, on average, women earn 84 percent of male hourly earnings (OECD 2002). There is some evidence, however, that each new cohort of women is faring better than previous ones (Blau and Kahn 2000).

The aim of this paper is to develop a theory that may explain this persistency of the gender wage gap. We base our theory on another empirical regularity; namely, that today's division of labor between spouses within the household looks surprisingly traditional. Numerous time-use studies show that wives spend relatively more time in home production than husbands, and that husbands spend relatively more time in the workplace than wives. Table 1 shows the results from different surveys of which most are sampled in the early 1990s.[10] It presents hours spent on household work and labor market work on a working weekday.

Table 1 Allocation of Daily Work Hours between the Home and the Workplace

	Market		Home		Total	
	Women	Men	Women	Men	Women	Men
Canada	8.5	9.6	2.8	1.7	11.3	11.3
Netherlands	4.1	6.7	4.0	2.0	8.1	8.7
Norway	7.2	8.7	3.4	2.1	10.6	10.8
UK	6.9	8.8	3.3	1.3	10.2	10.1
US	8.4	9.3	2.5	1.5	10.9	10.8
Italy	6.5	7.9	4.0	0.9	10.5	8.8
Austria	7.9	9.8	3.7	1.3	11.6	11.1

Source: Data from Freeman and Schettkat 2005, table 7.

[10] Freeman and Schettkat (2005) contains a full specification of the sampling years.

The table shows that for a range of developed economies, women do relatively less market work and men do relatively less work in the home. Yet in total (with the exception of Italy), men and women roughly spend the same amount of time on the two activities. Short (2000) reports that the recent situation in UK is similar to the pattern in table 1. In 1999, British men still use less time in household production than do British women, and British men still use more time on paid work than do British women. Bonke et al. (2005) find similar results in Danish data.

This pattern in labor allocation is in accordance with the fact that women occupy 68 percent of all part-time jobs. About half of those women are married, whereas the share of part-time workers, who are married men, is as low as 9 percent (U.S. Department of Labor 2005, table 4).

In summary, it seems that despite the fact that women and men roughly share same initial educational levels and opportunities, ex post, their labor patterns diverge. In particular, this divergence manifests itself as unequal labor allocation between the home and the labor market, and as gender-differences in wages. We proceed to suggest how this pattern can be rationalized.

3 The Model

The economy consists of two sectors, a formal sector and a home sector. Each sector is constituted by a number of identical firms and families. The home sector produces household services and the formal sector produces a market commodity.

The constant $\overline{N}$ denotes the number of families. Families consist of a husband and a wife, who are identified by an index $i \in \{1, 2\}$. Family members are ex ante identical except for gender. They supply labor to the firm, l_i, and to the family, t_i, and have constant labor endowments, $\overline{T}$. We think of the labor endowment as the daily number of hours used for work activities (cf. table 1); thus,

$$l_i + t_i = \overline{T},^{11}$$

and henceforth, l_i and t_i are in the following expressed in number of hours as a share of total daily labor endowment. Family members do not derive utility from leisure and personal time.

3.1 Families

The representative family consumes the market commodity, x, and household services, z, which we think of as including activities such as food preparation, dish washing, household up-keeping, care for clothes, child care, shopping, do-it-yourself work, gardening, etc. The market commodity is purchased from the market. The household service, on the other hand, is produced and consumed entirely within the home.[12]

We assume strict essentiality and complementarity in home production in the sense that one family member cannot produce without labor input from her spouse. Specifically, home production is given as

$$z = (t_1 t_2)^{\frac{\beta}{2}}, \tag{1}$$

where, if $z > 0$, then $t_1 > 0$ and $t_2 > 0$. Moreover, $0 < \beta \leq 1$. We assume there are constant or decreasing returns to male and female labor input taken together. The literature shows no strong prior on this point, but the constant returns formulation

[11] We shall refer to any combination $(t_i, l_i) = (t_i, \overline{T} - t_i)$ as the family member's labor (or time) allocation between the home sector and the firm sector.

[12] One could argue that household services are to a certain extent available on a formal market. Time-use studies, however, show that families produce (at least part of) the service themselves (Bonke et al. 2005; Freeman and Schekatt 2005; Short 2000).

is often used for its convenience in empirical analysis (Apps and Rees 1997; Aronsson et al. 2001). Note that the factor shares of female and male labor input are taken to be identical. This reflects the idea that husband and wife are equally productive if they allocate their labor equally.

Each family member has identical preferences and an equal weight in the family welfare function in conformity with a conventional unitary household model[13] with household production. The family utility function, u, is for convenience taken to be linearly additive:

$$u(x, z) = ax + bz, \tag{2}$$

where $a > 0$ and $b > 0$ are parameters. For given *hourly* wage rates, w_1 and w_2, the family maximizes its utility

$$\max_{t_1, t_2} u(x, z) \tag{3}$$

subject to its budget, production, and labor constraints:

$$\begin{aligned} p_x x &= w_1 l_1 + w_2 l_2, && (4) \\ z &= (t_1 t_2)^{\frac{\beta}{2}}, && (5) \\ l_i + t_i &= \overline{T}, \qquad i \in \{1, 2\}, && (6) \\ l_i &\geq 0, \qquad t_i \geq 0, && (7) \end{aligned}$$

by efficiently allocating labor to home production and to earning market wages. The price, p_x, of the market commodity is our numeraire.

The household service is not traded, and therefore it has no market price. However, a price for the household service, p_z, can be defined as a shadow price at an

[13] This aspect of our model could be made more general by using a collective household model (Chiappori 1988) which allows household members to have different preferences and to have different weights in the family welfare function.

optimum. Using the wage rate as the shadow price of labor input to home production, we can, as intra-household efficiency in the family consumption allocation requires that the marginal rate of substitution between the two goods equals their price ratio, $\frac{\partial u/\partial x}{\partial u/\partial z} = \frac{p_x}{p_z}$, derive p_z as the ratio $\frac{b}{a}$.

In a solution where $l_i > 0$ and $t_i > 0$, an efficient allocation of labor endowments is reached when the marginal value product of labor in home production equals its opportunity cost (the hourly wage rate). Specifically, the first order conditions to the family utility maximization problem in such a solution can be expressed as

$$\frac{b}{a}\frac{\partial(t_1t_2)^{\frac{\beta}{2}}}{\partial t_1} = w_1, \tag{8}$$

$$\frac{b}{a}\frac{\partial(t_1t_2)^{\frac{\beta}{2}}}{\partial t_2} = w_2, \tag{9}$$

and dividing (8) by (9), we obtain an expression for the gender wage ratio (or gap):

$$\frac{w_1}{w_2} = \frac{t_2}{t_1}. \tag{10}$$

By (10), the wage ratio equals the inverse ratio of labor input into household production; if a family member earns relatively higher wage rates than her spouse, she allocates relatively less time in home production than her spouse, and vice versa.[14] If family members earn the same wage rate, they allocate labor in the same way.

The first order conditions, (8) and (9), are in general not satisfied in case of boundary solutions, i.e., when $l_i = 0$ or $t_i = 0$. Such situations occur if the marginal cost of the household service is different from its price for any allocation of labor, namely, if $w_i / \frac{\partial(t_1t_2)^{\beta/2}}{\partial t_i} \neq p_z$.

[14] This prediction is tested by Albanesi and Olivetti (2006) on American data. They find a significant negative correlation between the husband-wife ratio of earnings and their home hours ratio.

3.2 Firms

The representative firm operates in a competitive market and produces the market commodity taking labor as input. It decides how many male and female workers, N_1 and N_2, to employ taking the hours of labor supplied by each worker and the hourly wage rates as given.

Each worker produces an output, and, thus, labor inputs from each gender are perfect substitutes. Let e denote effectiveness of each unit of labor input at the firm. We assume that effectiveness is not gender-specific, but only a function of labor allocation. In particular, we assume that due to indivisibility, effectiveness is (linearly) increasing in hours per day in employment:

$$e = e(l_i) = l_i, \tag{11}$$

so $\frac{\partial e(l_i)}{\partial l_i} > 0$.

Total firm output per day, q, is the sum of output produced by each employee per day given as

$$q = A\left[e(l_1)l_1N_1 + e(l_2)l_2N_2\right], \tag{12}$$

where A is a positive productivity term, and $\frac{\partial q}{\partial N_i} = Ae(l_i)l_i$ is the marginal productivity of a worker i, which depends on the number of hours the worker puts into production. As l_i is taken as given, from the standpoint of the firm, the firm has constant returns to scale in employment. An implication of (11) in (12) is that longer hours worked at the firm lead to higher marginal productivity of labor as well as of workers.

The firm decides how many workers to recruit in order to maximize its profits, π, which are the firm's revenues minus its costs. As the price of the market commodity is the numeraire, the profit maximization problem is to

$$\max_{N_1,N_2} \pi = \{A\left[e(l_1)l_1N_1 + e(l_2)l_2N_2\right] - w_1l_1N_1 - w_2l_2N_2\}. \tag{13}$$

Taking w_1, w_2, l_1, and l_2 as given, in a competitive market, the firm employs workers until their marginal daily productivity, $Ae(l_i)l_i$, equalizes their marginal daily costs, w_il_i. Hence,

$$\frac{\partial \pi}{\partial N_1} = Ae(l_1)l_1 - w_1l_1 = 0 \Leftrightarrow Ae(l_1) - w_1 = 0, \tag{14}$$

$$\frac{\partial \pi}{\partial N_2} = Ae(l_2)l_2 - w_2l_2 = 0 \Leftrightarrow Ae(l_2) - w_2 = 0. \tag{15}$$

Since labor, l_1 and l_2, is measured in hours, the solution to the firm's problem depends on the relationship between hourly wages and effectiveness of an hour of labor at the firm adjusted by the productivity term A. In the following, we refer to $Ae(l_i)$ as the average productivity of labor per hour, $i = \{1, 2\}$. We have three different situations describing the firm's employment demand:

$$N_i = \begin{cases} \infty \text{ if } Ae(l_i) > w_i, \\ [0, \infty[\text{ if } Ae(l_i) = w_i, \\ 0 \text{ if } Ae(l_i) < w_i. \end{cases} \tag{16}$$

If $Ae(l_i) > w_i$ the firm would want to hire an infinite amount of type i workers, if $Ae(l_i) < w_i$ the firm would not want to hire any type i workers, and if $Ae(l_i) = w_i$ the firm is indifferent about the number of type i workers.

3.3 Equilibrium

The conditions for existence of a competitive[15] equilibrium in the economy involve: (i) the labor market, (ii) the market commodity, and (iii) the household service.

[15] The economy is competitive although there is an inherent externality: the spillover effect from household service production to labor market productivity which the family does not internalize.

There are two types of equilibria. An *interior* equilibrium, in which the production levels of both x and z are strictly positive,[16] and a *specialized* equilibrium, in which only one sector is producing.

An *interior* equilibrium involves a positive price vector, (w_1, w_2), at which markets for male and female labor, as well as the market commodity, and the household service, clear; and for which the marginal conditions for an optimum given by the firm's and the family's first order conditions are satisfied.

There is a market clearing condition for each of the two goods. For every family, maximization of utility ((3)-(7)) yields a labor allocation which satisfies

$$p_z z = \frac{b}{a} (t_1 t_2)^{\frac{\beta}{2}}, \tag{17}$$

so that household services consumed equal household services produced. Also the market commodity production must equal the market commodity demand. As the firm's production technology is linear homogenous in employment, we can normalize the number of firms to unity. In this case,

$$q = \overline{N}x \tag{18}$$

holds, where

$$q = A\left[e(l_1)l_1N_1 + e(l_2)l_2N_2\right] \text{ and } \overline{N}x = \overline{N}(w_1l_1 + w_2l_2). \tag{19}$$

Finally, the employment clearing conditions are as follows. If there is a solution with a finite market commodity production, then from (16) we have that

$$Ae(l_i) = w_i \tag{20}$$

[16]Due to complementarity of male and female labor input in home production it follows that when home production is operative then $t_1 > 0$ and $t_2 > 0$. Moreover, as we prove in proposition 2 below, an equilibrium where only one spouse spends all time in home production does not exist. If instead, the household service is produced separately by each adult, at least one individual will completely specialize labor resources in this sector. This result resembles Theorem 2.3 in Becker (1991, 34).

holds.[17] Hence, (20) is a necessary condition for an interior competitive equilibrium. Together with the constant returns assumption on N_i (not on l_i), (20) implies that the competitive firm is indifferent about the number of workers it employs. In equilibrium, the number of female and male workers, N_1 and N_2, equals the number of families $\overline{N}$; i.e., $N_1 = N_2 = \overline{N}$.

Substituting (11) in (20) gives the following employment clearing conditions

$$Al_1 = w_1, \tag{21}$$

and

$$Al_2 = w_2. \tag{22}$$

Productivity of an hour of labor equals the hourly wage. In the interior equilibrium, female and male labor supply equals female and male labor demand when (8) equals (21), and (9) equals (22). Using $l_i = \overline{T} - t_i$ we can derive two equations in t_1 and t_2 :

$$\begin{aligned} A(\overline{T} - t_1) &= \frac{b}{a}\frac{\beta}{2} t_1^{\frac{\beta-2}{2}} t_2^{\frac{\beta}{2}}, & (23) \\ A(\overline{T} - t_2) &= \frac{b}{a}\frac{\beta}{2} t_1^{\frac{\beta}{2}} t_2^{\frac{\beta-2}{2}}. & (24) \end{aligned}$$

Eq. (23) and (24) states that in equilibrium, average productivity of one hour of labor in the workplace has to equal the marginal value product of labor in home production.

We can now characterize an *interior* equilibrium as any combination of t_1 and t_2 which solves (23) and (24). Such a combination clears markets for male and female labor, and supports a price vector, (w_1, w_2), for which also the market for x clears, and firms earn zero profits.

[17]If $Ae(l_i) < w_i$ holds, production of the consumption good is zero, and if $Ae(l_i) > w_i$ holds, the firm earns positive profits.

4 Results

In solving the model, it this useful to define a labor allocation for which $t_1 = t_2$ as *symmetric,* and one for which $t_1 \neq t_2$ as *asymmetric.* Both cases can occur, but by (21) and (22), only an asymmetric situation leads to a gender wage gap. Specifically, we have

Proposition 1 (*Symmetric equilibrium.*) *If* $0 < \beta < 1$ *and* $\frac{b}{a}\frac{\beta}{2A} < \left(\frac{\overline{T}}{2-\beta}\right)^{2-\beta}(1-\beta)^{1-\beta}$, *then there exist two interior symmetric equilibria. If* $\beta = 1$ *and* $\frac{b}{a}\frac{1}{2A} < \overline{T}$, *then there exists one interior symmetric equilibrium.*

Proof. See Appendix.

When $t_1 = t_2 \equiv t$, equations (23) and (24) collapse into

$$A(\overline{T} - t) = \frac{b}{a}\frac{\beta}{2}t^{\beta-1}. \tag{25}$$

In equilibrium, the average productivity of an hour of labor in the firm equals the marginal value product of labor in home production as illustrated in fig. 3.

The dashed line in fig. 3 illustrates the hourly wage rate as a function of labor used in home production (i.e. t) which satisfies the zero profit condition in (16). Due to the negative spillover from household production to productivity at the firm, the hourly wage decreases. The solid lines illustrate the marginal value product of labor in home production. The horizontal line illustrates the case where $\beta = 1$. The curved lines represent examples for $0 < \beta < 1$. The innermost curve illustrates $\frac{b}{a}\frac{\beta}{2A} < \left(\frac{\overline{T}}{2-\beta}\right)^{2-\beta}(1-\beta)^{1-\beta}$ and the two intersections with the dashed line illustrate the two equilibria. The uttermost curve illustrates a situation where $\frac{b}{a}\frac{\beta}{2A} > \left(\frac{\overline{T}}{2-\beta}\right)^{2-\beta}(1-\beta)^{1-\beta}$, which is an economy without an interior symmetric equilibrium, as the marginal value product of labor in home production exceeds the hourly wage rate for all allocations of labor resources. In this situation, only the home sector is operative.

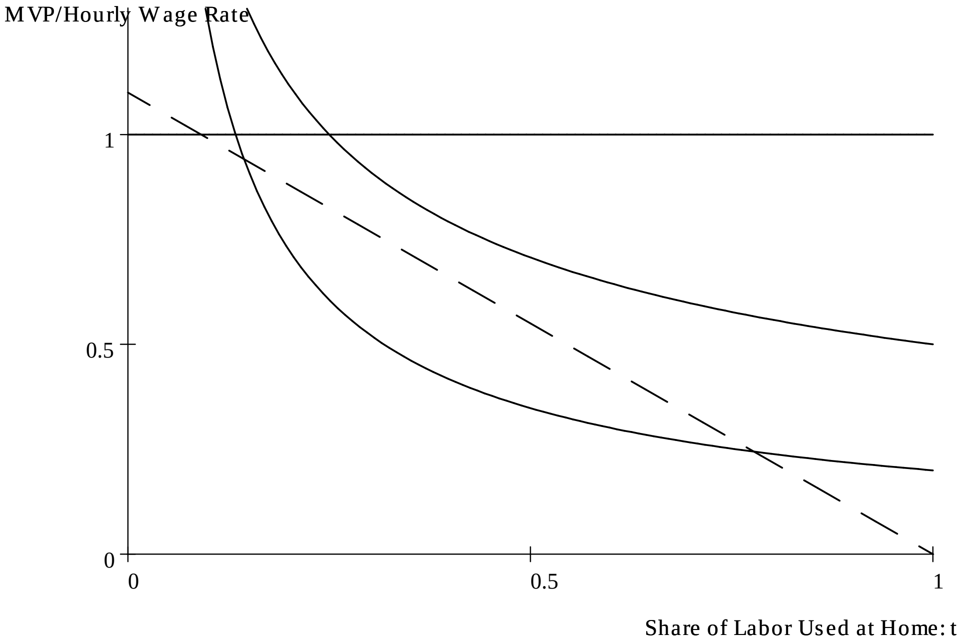

Figure 3: An illustration of symmetric equilibria. Symmetric equilibria exist in points where the hourly wage (illustrated by the dashed line) equals the marginal value product of labor in home production (illustrated by the solid line). Spouses are in the same intersection point.

Assume the hourly wage is such that it corresponds to one of the intersection points between the dashed and the solid line in fig. 3. In this case, the firms are willing to hire the workers, since marginal productivity just equals the marginal costs. Moreover, workers do not want to supply neither more nor less labor to the firm. If they supply more (i.e., decrease their labor input into home production), the marginal value product of labor in home production exceeds the given hourly wage rate. If they supply less (i.e., increase their labor input into home production), the marginal value product of labor in home production is less than the given hourly wage rate.

The intuition behind the existence of two symmetric equilibria can be explained as follows. When the household production function is concave in total labor input ($0 < \beta < 1$), for small t's, the marginal value product of labor in home production is high and larger than the corresponding average productivity of an hour of labor in the firm, or equivalently, the hourly wage. As t increases, marginal productivity of labor in home production decreases to a point where it is exceeded by the hourly wage rate. As t becomes even larger, however, the negative spillover from home production onto average productivity at the firm increases further. Eventually, the spillover damages productivity to an extent that marginal value product of labor in home production again exceeds the hourly wage rate.

Likewise, in the situation where $\beta = 1$, if the hourly wage rate coincides with the marginal value product of labor used in home production, the family is indifferent as to how much labor they supply to the firm. The firm, however, is only willing to hire labor when labor productivity is equal to, or higher than, the wage they must pay. To the left of the intersection point in fig. 3, however, firms would demand an infinite number of workers; therefore this cannot be an equilibrium. On the other hand, when $\frac{b}{a}\frac{1}{2A} > \overline{T}$, the marginal value product of labor in home production exceeds the hourly wage for all allocations of labor resources, and all labor is used in the home.

Similarly, we analyze the asymmetric equilibrium:

PROPOSITION 2 (*Asymmetric equilibrium.*) *If* $\frac{b}{a}\frac{\beta}{2A} < \left(\frac{\overline{T}}{2}\right)^{2-\beta}$, *then there exist two interior asymmetric equilibria.*

PROOF. See Appendix.

The asymmetric solution is illustrated by fig. 4. In fig. 4, again the dashed line illustrates decreasing hourly wages as a function of labor used in home production. The uttermost curve illustrates the situation where $\frac{b}{a}\frac{\beta}{2A} > \left(\frac{\overline{T}}{2}\right)^{2-\beta}$; the situation without an interior solution. The innermost curve intersects the dashed line twice, and illustrates the situation where husband and wife, despite being completely identical ex ante, allocate their labor endowments differently between the home and the workplace.

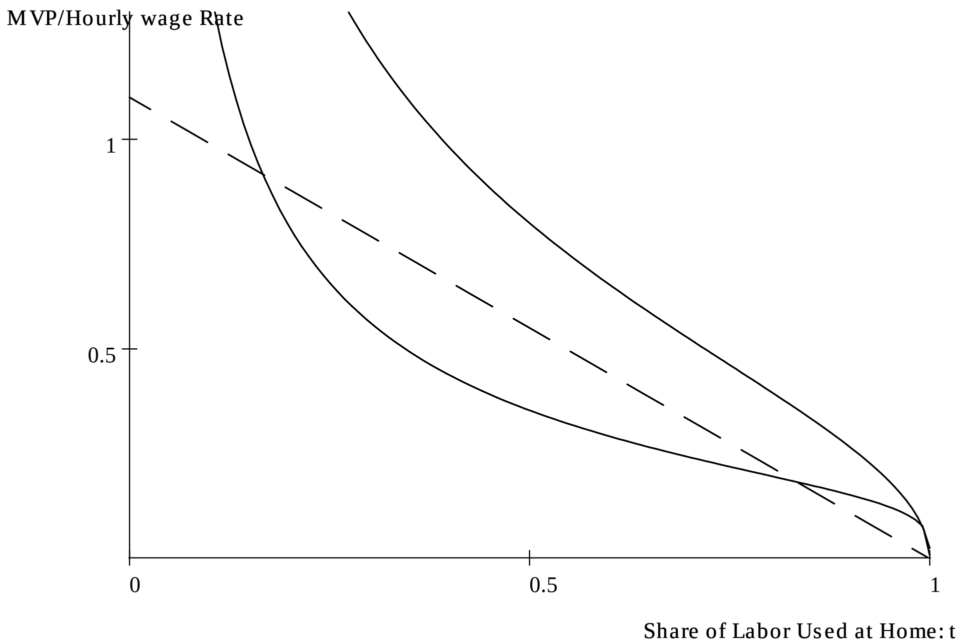

Figure 4: Asymmetric equilibria. Asymmetric equilibria are given by the pair of points where the hourly wage rate (illustrated by the dashed line) equals the marginal value product of labor in home production (illustrated by the solid line). Household members are in *separate* intersection points.

Whereas in the symmetric equilibrium, each spouse allocates the same labor to home production, and therefore an equilibrium is a situation where household members are "located" in the same intersection point of the two curves in fig. 3, the asymmetric equilibrium is an equilibrium, in which one family member is "located" in one intersection point and, simultaneously, the spouse is "located" in the other intersection point. In general, we have two possible pairings of gender and "location." Household members may allocate labor according to traditional gender roles, or inversely.

Proposition 3 (*Multiple interior equilibria.*) *If an interior asymmetric equilibrium is supported by a positive price vector* (w_1, w_2), *there exists another price vector* $(\widetilde{w}_1, \widetilde{w}_2) \neq (w_1, w_2)$ *which supports an interior symmetric equilibrium.*

Proof. See Appendix.

By proposition 3, we establish that for some sub-interval of the parameters A, a, and b, the model has multiple interior equilibria which results in either gender-similarities or gender-differences in labor allocation.

This result is important. It mirrors a self-fulfilling nature of expectations about gender roles. The family's efficient response to traditional beliefs about earnings is to actually allocate labor as stereotypical workers. On the other hand, the family's efficient response to unisex beliefs is to allocate labor identically.

Proposition 3 suggests that the persistency in the gender wage gap relates to persistency in the perception of the patterns of sexual roles. We explore this issue further in detail in section 5, but first we notice that the model has interesting comparative statics for the asymmetric equilibrium.

Proposition 4. *Assume the economy is in an asymmetric equilibrium. A higher productivity level* A *is associated with a larger gender wage gap.*

Proof. See Appendix.

For a given initial asymmetric labor allocation, we consider a situation where A

increases and, consequently, labor productivity in the firm goes up. The person with the lower t (the most labor allocated to the firm) experiences the highest increase in hourly wage as average productivity of an hour of labor increases in A at the rate $(\overline{T} - t)$.

As is clear from (10), the family's efficient labor allocation response to such a change in the relative hourly wage rates is that the person, who works most at the firm, allocates more labor to the firm, and the person, who works most at the home, allocates more labor to the home. Hence, the person who works most in the home ends up earning a lower wage than in the original equilibrium. In this way, increases in A magnify any existing differences productivity, and the model predicts higher wage differentials for higher paid formal sectors.

Proposition 4 can be taken to the evidence. It predicts that the gender wage gap is larger within families that work in sectors with higher wage rates. Fig. 5 is a scatter plot of female/male earnings ratio against male earnings.

Each dot represents an occupation like civil engineers, lawyers, photographers, etc. If couples predominantly exist within similar occupations,[18] fig. 5 confirms proposition 4. Indeed, the corresponding regression[19] reveals a statistically significant negative relationship between male median weekly earnings and the gender wage gap.

[18]Some empirical evidence for educational homogamy, i.e., individuals marry individuals with similar characteristics such as occupation, education, and religion, is presented in Blossfeld and Timm (2003).

[19]The intercept estimate is 92.99 (38.66) and the slope estimate is -0.02 (-5.26), where the numbers in the parenthesis are the t-statistics. The fraction of the variation in the wage gap explained by the regression is above 20 percent (R^2 is 0.21).

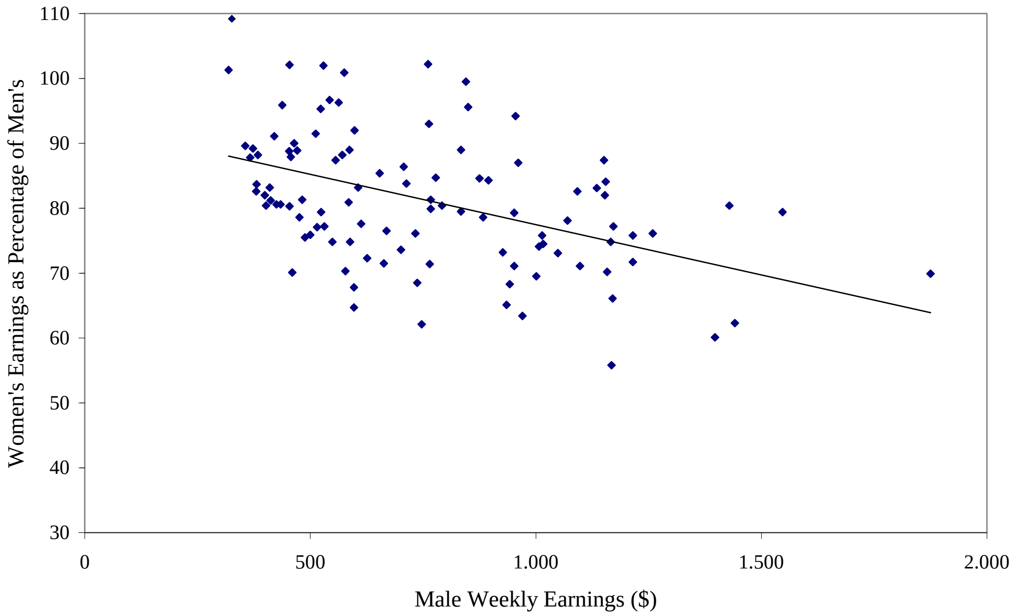

Figure 5: An illustration of the gender wage gap across occupations in the US. The figure shows that higher male earnings within an occupation are correlated with a larger gender wage gap, i.e., lower women's earnings relative to those of men's. (Data from U.S. Department of Labor 2005, table 2.)

Proposition 5. *Assume the economy is in an asymmetric equilibrium. A higher β reduces the gender wage gap if $\frac{b}{Aa} > \frac{2}{\beta}\exp\left(\frac{\beta-2}{\beta}\right)$.*

Proof. See Appendix.

First, we analyze the situation where $\frac{b}{Aa} > \frac{2}{\beta}\exp\left(\frac{\beta-2}{\beta}\right)$ is satisfied. In this case, increases in β increase the marginal value product of labor in home production for both household members at given labor allocations. The increase is largest, however, for the person, who works less in the home. Therefore, there is room for "redistribution" of the marginal value product. The benefit of letting the "outside working" work more in home production more than compensates for the loss of home production that the "home working" person sacrifices to enable the reallocation of labor.

Due to changes in the spillover effect from this reallocation of labor, also the equilibrium wage rates are affected. Since, in response to a high β, the "outside working" person works less out, and "home working" person works less at home, the differences in productivities at the firm diminish, and in an equilibrium, wage rates are more equal.

The explanation for the opposite case, the situation where $\frac{b}{Aa} < \frac{2}{\beta} \exp\left(\frac{\beta-2}{\beta}\right)$, follows the same logic. However, in this situation the marginal value product declines instead of increases in response to increased β. As it declines most for the "outside working" person, it pays for the family to let this person work even less in the home, at the expense of letting the "home working" person work more at home. This leads to a higher wage gap.

When the weight on the household service in the utility function, b, is low relative to the weight on the market commodity, a, the wage gap is more likely to increase in response to higher β. In this case, the initial labor allocation across spouses is already relatively specialized in an asymmetric equilibrium. In the limit, when t of one spouse approaches zero, increases in β diminish the marginal value product of labor in home production.

5 Welfare Analysis by Numerical Examples

The public debate about gender roles often favors equality on the labor markets and in the home (Hakim 2004), but less often are the welfare properties of the gender wage gap discussed. According to Becker (1985), welfare increases with specialization, whereas in Chichilnisky (2005), in a society with high skill levels, equal labor division across family members generates the highest welfare.

This section provides some numerical examples, which are reported in table 2 and table 3, to illustrate the welfare properties of the current model. Table 2 shows simulated symmetric equilibria, and table 3 shows simulated asymmetric equilibria.

Table 2 Simulated Symmetric Equilibria ($\overline{T} = 10$)

		t[a]		x		z		$u(x,z)$	
$\frac{b}{Aa}$	β	t^l :	t^h	l[b] :	h[c]	l :	h	l :	h
10	0.3	0.07	9.69	197.21	0.19	0.45	1.98	201.71	19.96
10	0.6	0.05	8.74	198.01	3.18	0.17	3.67	199.66	39.90
10	0.9	0.00	6.25	200.00	28.13	0.00	5.20	200.00	80.16
15	0.3	0.12	9.54	195.23	0.42	0.53	1.97	203.17	29.93
15	0.6	0.14	8.05	194.44	7.61	0.31	3.50	199.05	60.03
15	0.9	0.02	4.15	199.20	68.45	0.03	3.60	299.65	122.44
20	0.3	0.18	9.37	192.86	0.79	0.60	1.96	204.82	39.93
20	0.6	0.30	7.26	188.18	15.02	0.49	3.29	197.89	80.72
20	0.9	0.90	1.00	165.62	162.00	0.91	1.00	183.81	182.00
13	0.9	0.01	5.02	199.60	49.60	0.02	4.27	199.81	105.14

[a] Across the symmetric equilibria, t^l and t^h are the "low" and "high" equilibrium values, ($t^l < t^h$), of labor spent in home production.

[b] The label "l" indicates values which corresponds to t^l.

[c] The label "h" indicates values which corresponds to t^h.

Let overall welfare in society, W, be given by the sum of household utilities and firm profits

$$W = \overline{N}u(x, z) + \pi = \overline{N}u(x, z), \tag{26}$$

where the last equality follows from firms earning zero profits in equilibrium.

The family consumes different ratios of the household service and the market commodity across equilibria. In table 2, we notice that utility is highest for the equilibrium in which spouses allocate most labor to the workplace, i.e., in $t = t^l$. This is due to the negative spillover effect of home production onto labor productivity at the firm. The extra production of the market commodity more than compensates for the decline in home production.

In table 3, we report the equilibrium in which women spend most time in the household.[20] Hence, the gender wage gap, ρ, is the wife-husband wage ratio.

Table 3 Simulated Asymmetric Equilibria ($\overline{T} = 10$)

$\frac{b}{Aa}$	β	$t_1^{a} : t_2$	x	z	$u(x, z)$	ρ
10	0.3	9.84 : 0.16	96.85	1.07	107.56	0.00
10	0.6	9.49 : 0.51	90.32	1.60	106.37	0.00
10	0.9	8.10 : 1.90	69.22	3.42	103.43	0.06
15	0.3	9.73 : 0.27	94.75	1.16	112.08	0.00
15	0.6	9.05 : 0.95	82.81	1.91	111.41	0.01
15	0.9	- : -	-	-	-	-
20	0.3	9.62 : 0.38	92.69	1.21	116.98	0.00
20	0.6	8.47 : 1.53	74.08	2.16	117.21	0.03
20	0.9	- : -	-	-	-	-
13	0.9	5.42 : 4.58	50.35	4.24	105.51	0.71

[a] In the asymmetric equilibrium, t_1 and t_2 denote the labor allocated to home production by the woman and the man respectively.

In the asymmetric equilibrium, the gender-difference in labor allocation is smaller when β is high. This is what we expect, since by proposition 5, when $\frac{b}{Aa} > \frac{2}{\beta}\exp\left(\frac{2-\beta}{\beta}\right)$ is fulfilled (which is the case when $\frac{b}{Aa} \in (10, 20)$), the wage gap is increasing in β. Table 3 also confirms proposition 4. A higher A increases the gender wage gap, i.e., decrease the wife-husband wage ratio.

We compare the asymmetric equilibrium with the symmetric equilibrium which yields the higher welfare. We notice that production of the market commodity is higher everywhere in the symmetric equilibrium than in the asymmetric equilibrium,

[20] The results equally apply to the reversed situation in which men are spending most time in the household.

but production of the household service is higher in the asymmetric equilibrium than in the symmetric equilibrium. Yet the extra production in the formal sector makes up for the loss of household services and welfare is higher in the symmetric equilibrium. The simulations thereby suggest that, for the model presented in the current paper, a gender wage gap is Pareto inferior in that utility in the symmetric equilibrium (with least labor used in home production) is higher everywhere than utility in the asymmetric equilibrium.

The explanation for this result is that when the economy is in an interior asymmetric equilibrium, the home production suffers a productivity loss as family members are not allocating identical amounts of labor input. Since labor input of each spouse has identical factor shares, and labor input is complementary in production, clearly the cost minimizing labor allocation in household production is when spouses allocate identical amounts of labor. Accordingly, in the symmetric equilibrium, total labor input in home production may be less and the asymmetric case (which is always equal to $\overline{T}$ as $t_1 = \overline{T} - t_2$ cf. the proof of proposition 2) and yet produce more household service.

The last simulation in both tables offers a parameterization which gives a gender wage ratio that lies in the range of typical gender wage ratio estimates, cf. OECD (2002).

6 Discussion

Albeit the model does not provide a priori insight as to the specific equilibrium outcome among the possibilities of interior equilibria, a key prediction is that if families believe that wages are stereotype, the economy will experience a gender wage gap with women earning less than men. In this sense, the gender wage gap is explained as a self-fulfilling prophecy.

A natural way for today's families to decide on labor allocation would be to use information on "yesterday's" wages. If, for what could be historical and cultural reasons, women used to be less educated than men and to participate less in the labor force, cf. fig. 1, it would be rational that women historically (and yesterday) earned less than men. In this way, even though the premises, which determined the historical labor market outcome have changed, in that today, women and men share the same starting point to become equally productive in both the home and in the workplace, current beliefs about earnings may be "historically biased" in favor of stereotype beliefs. Hence, one may argue that the persistence of the gender wage gap in developed societies is possibly due to a self-fulfilling "history bias" on beliefs.

The literature offers explanations of why families appear not to "choose" a symmetric labor allocation. Table 4 demonstrates how gender roles are viewed within British families.

Table 4 Couples Aiming for Symmetric Roles

	Dual-earner[a]		Full-time Workers[b]	
	Men	Women	Men	Women
Percentage choosing symmetric roles	44	41	44	56

Source: Hakim (2004).

[a]Dual-earner couples refer to households in which either spouse reports being in employment.

[b]Full-time workers couples consist of full-time working husbands and wives.

The numbers suggest that the majority of couples aim for traditional roles.

Hakim (2004) argues that one explanation is that women regard themselves as secondary earners, and that employment does not provide them with their central identity.[21] This would be in conformity with our analysis. An implication of our model is that family reality and family beliefs about earnings have to change simultaneously for the economy to move from the stereotype asymmetric equilibrium to a unisex symmetric equilibrium. Hence, effective policies are policies that can change norms of society. Without such policies the gender wage gap is likely to persist as a rational reaction to stereotype family beliefs about gender roles, even when there is no actual gender discrimination or other initial differences between the two sexes.

Of course, there may be many reasons as to why families do not change their traditional beliefs. One explanation is that both men and women view equity as a relevant concept in the workplace, but neither view the home as a workplace. Roughly speaking, if housework is a "woman's labor of love," equity does not come into question. Moreover, men and women may define certain jobs as feminine and others as masculine. A woman is less of a woman if she does not keep the house, and the man is less of a man if he does. If men compare themselves to other men, and women to other women, and since the majority of households have unequal division of labor, both the woman or the man are likely to perceive traditional gender roles as normal and desirable (Hakim 2003; Valian 1999).

[21] She also finds that families without children have a traditional division of labor.

7 Concluding Remarks

This paper investigates how the persistence of the gender wage gap in an economy where male and female workers are ex ante identical with respect to education, ability, and skill levels can be explained.

We show that complementarity of spousal labor in home production, and indivisibility of labor input in the workplace may lead to multiplicity of equilibria in which families' beliefs about the gender wage gap are self-fulfilling. This means that if family members believe that women earn less than men, ex post, intra-household labor allocation justifies such beliefs. Therefore, women's history on the labor market may have severe implications for the labor market outcome today, which are not easily overcome. Yet the welfare analysis reveals potential welfare gains to closing the gender wage gap.

Naturally, the approach to explaining the gender wage gap offered by the present model hopes just to offer a small piece of the gender wage gap puzzle. Besides the large literature that concerns differences in human capital accumulation, a literature largely initiated by Becker (1985), others have suggested that differences in wages can be attributed to a theory of "male-dominated institutions," or preference theory suggesting that women prefer to prioritize household tasks (Hakim 2004). Gender differences in networking (Montgomery 1991), or statistical discrimination (Moro and Norman 2003, 2004), may also lead to differences in wage rates.

8 Appendix

8.1 Proof of Proposition 1

When $t_1 = t_2 \equiv t$, the equations (23) and (24) collapse into

$$(\overline{T} - t)t^{1-\beta} = \frac{b}{Aa}\frac{\beta}{2}. \tag{27}$$

(This is eq. (25) in the main text.)

In general, a solution to (27) exists when the right hand side, which is parametrically given, is smaller than, or equal to, the maximum value of the left hand side.

When $\beta < 1$, the left hand side is an inversely "U-shaped" polynomial with a unique maximum that is positively skewed. The maximum is found by first differentiating the left hand side wrt t, then setting this expression equal to zero, and finally isolate for t : $\frac{\partial(\overline{T}-t)t^{1-\beta}}{\partial t} = 0 \Rightarrow t^{-\beta}\left[\overline{T} - 2t + (t - \overline{T})\beta\right] = 0 \Leftrightarrow \overline{T}\left(\frac{1-\beta}{2-\beta}\right) = t$. Substituting this expression back into (27) determines the maximum value of the left hand side as of this equation a function of β; $\arg\max_t(\overline{T} - t)t^{1-\beta} = \left(\frac{\overline{T}}{2-\beta}\right)^{2-\beta}(1-\beta)^{1-\beta}$. Hence, in an interior equilibrium $\frac{b}{Aa}\frac{\beta}{2} \leq \left(\frac{\overline{T}}{2-\beta}\right)^{2-\beta}(1-\beta)^{1-\beta}$ must be satisfied. When the equation holds with equality, there is exactly one solution, otherwise there are two solutions.

When $\beta = 1$ the left hand side of (27) is linear and equal to $(\overline{T} - t)$. Hence, the maximum is given when $t = 0$, so $\arg\max_t(\overline{T} - t) = \overline{T}$. For an interior equilibrium to exist, $\frac{b}{a}\frac{1}{2A} < \overline{T}$. $\square$

8.2 Proof of Proposition 2

An interior equilibrium is given when (23) and (24) are satisfied simultaneously. Dividing (23) and (24) means $\frac{\overline{T}-t_1}{\overline{T}-t_2} = \frac{t_2}{t_1}$ must hold. Rewriting this expression yields

$\overline{T}t_1 - t_1^2 = \overline{T}t_2 - t_2^2 \Leftrightarrow \overline{T}(t_1 - t_2) = t_1^2 - t_2^2 \Leftrightarrow \overline{T}(t_1 - t_2) = (t_1 + t_2)(t_1 - t_2) \Rightarrow t_1 + t_2 = \overline{T}$ for $t_1 \neq t_2$.

Substituting $t_1 = \overline{T} - t_2$ back into either (23) or (24), rearrange and solve for t_2 gives

$$(\overline{T} - t_2)t_2 = \left(\frac{b}{Aa}\frac{\beta}{2}\right)^{\frac{2}{2-\beta}}. \tag{28}$$

Eq. (28) is a second-order polynomial. By inspection we find that the shape of the left hand side is a symmetric parabola for which $\arg\max_{t_2}(\overline{T} - t_2)t_2 = \left(\frac{\overline{T}}{2}\right)^2$. The left hand side is a constant larger than zero. If $\frac{b}{Aa}\frac{\beta}{2} > \left(\frac{\overline{T}}{2}\right)^{2-\beta}$, then there is no solution to (28), and if $\frac{b}{Aa}\frac{\beta}{2} = \left(\frac{\overline{T}}{2}\right)^{2-\beta}$, then there is one solution ($t_1 = t_2 = \frac{\overline{T}}{2}$), and if $\frac{b}{Aa}\frac{\beta}{2} < \left(\frac{\overline{T}}{2}\right)^{2-\beta}$, then there are exactly two solutions satisfying $t_1 \neq t_2$. $\square$

8.3 Proof of Proposition 3

By proposition 1, $\frac{b}{Aa}\frac{\beta}{2} \leq \left(\frac{\overline{T}}{2-\beta}\right)^{2-\beta}(1-\beta)^{1-\beta}$ and $\frac{b}{Aa}\frac{1}{2} < \overline{T}$ are necessary conditions for an interior symmetric equilibrium when $0 < \beta < 1$ and when $\beta = 1$ respectively. The interior symmetric equilibrium is supported by a positive price vector which we denote $(\widetilde{w}_1, \widetilde{w}_2)$. By proposition 2, $\frac{b}{Aa}\frac{\beta}{2} < \left(\frac{\overline{T}}{2}\right)^{2-\beta}$ is a necessary condition for an interior asymmetric equilibrium, which is supported by another price vector which we denote (w_1, w_2).

We want to prove that when there exists an asymmetric equilibrium, then there also exists a symmetric equilibrium, i.e., that

$$\left(\frac{\overline{T}}{2}\right)^{2-\beta} \leq \left(\frac{\overline{T}}{2-\beta}\right)^{2-\beta}(1-\beta)^{1-\beta} \quad \forall \quad 0 < \beta < 1, \tag{29}$$

and

$$\left(\frac{\overline{T}}{2}\right) \leq \overline{T} \quad \forall \quad \beta = 1. \tag{30}$$

We prove each in turn. First, simplify (29) to get

$$\left(\frac{1}{2}\right)^{2-\beta} \leq \left(\frac{1}{2-\beta}\right)^{2-\beta} (1-\beta)^{1-\beta}.$$

Let $LHS \equiv \left(\frac{1}{2}\right)^{2-\beta}$ and $RHS \equiv \left(\frac{1}{2-\beta}\right)^{2-\beta} (1-\beta)^{1-\beta}$. We examine LHS and RHS for $\beta \to 0$ and $\beta \to 1$ respectively.

$$\begin{aligned} \underset{\beta\to 0}{LHS} &= \frac{1}{4} \text{ and } \underset{\beta\to 1}{LHS} = \frac{1}{2}, \\ \underset{\beta\to 0}{RHS} &= \frac{1}{4} \text{ and } \underset{\beta\to 1}{RHS} = 1. \end{aligned}$$

Hence, in the limits $RHS \geq LHS$. In order to study monotonicity, we first take logs:

$$\begin{aligned} \ln(LHS) &= (2-\beta)\ln\left(\frac{1}{2}\right), \\ \ln(RHS) &= (1-\beta)\ln(1-\beta) + (\beta-2)\ln(2-\beta), \end{aligned}$$

and then we take the derivative wrt β:

$$\begin{aligned} \frac{\partial[\ln(LHS)]}{\partial\beta} &= \ln(2) > 0, \\ \frac{\partial[\ln(RHS)]}{\partial\beta} &= -\ln(1-\beta) + \ln(2-\beta) > 0. \end{aligned}$$

Hence, both sides of (29) are monotonically increasing. Furthermore,

$$\begin{aligned} \frac{\partial^2[\ln(LHS)]}{\partial\beta^2} &= 0, \\ \frac{\partial^2[\ln(RHS)]}{\partial\beta^2} &= \frac{1}{(1-\beta)(2-\beta)} > 0. \end{aligned}$$

We can thus conclude, that if an interior asymmetric equilibrium exists, then also an interior symmetric equilibrium exists for $0 < \beta < 1$.

Second, simplify (30) to get

$$\frac{1}{2} \leq 1,$$

which is true. $\square$

8.4 Proof of Proposition 4

From the proof of proposition 2 we have that an interior asymmetric equilibrium must satisfy

$$(\overline{T} - t_2)t_2 = \left(\frac{b}{Aa}\frac{\beta}{2}\right)^{\frac{2}{2-\beta}}, \tag{31}$$

where $\overline{T} - t_2 = t_1$.

Differentiate the right hand side wrt A to get

$$\frac{\partial \left(\frac{b}{Aa}\frac{\beta}{2}\right)^{\frac{2}{2-\beta}}}{\partial A} = \left(\frac{b}{a}\frac{\beta}{2}\right)^{\frac{2}{2-\beta}} (\frac{2}{\beta - 2})A^{\frac{2}{\beta-2}-1} < 0,$$

which means that an increase in A shifts down the right hand side of (31).

The left hand side of (31) is an inverted "U-shaped" parabola, and therefore the distance between the values of $(\overline{T} - t_2)$ and t_2 which solves the system increases as A increases.

The wage gap is given as a function of t_2 and t_1 by (10): $\frac{w_1}{w_2} = \frac{t_2}{t_1}$. The more t_1 and t_2 differs, the higher the gender wage gap. $\square$

8.5 Proof of Proposition 5

From the proof of proposition 2 we have that an interior asymmetric equilibrium must satisfy

$$(\overline{T} - t_2)t_2 = \left(\frac{b}{Aa}\frac{\beta}{2}\right)^{\frac{2}{2-\beta}} \tag{32}$$

where $\overline{T} - t_2 = t_1$.

In order to analyze the effect of a change in β we take logs on both sides of this equation:

$$\ln(\overline{T} - t_2) + \ln(t_2) = \frac{2}{2-\beta}\left[\ln\left(\frac{b}{2Aa}\right) + \ln(\beta)\right].$$

The left hand side does not vary with β. For the right hand side we find that

$$\frac{\partial\left\{\frac{2}{2-\beta}\left[\ln\left(\frac{b}{2Aa}\right)+\ln(\beta)\right]\right\}}{\partial\beta}=\frac{2}{2-\beta}\left\{\frac{1}{2-\beta}\left[\ln\left(\frac{b}{2Aa}\right)+\ln(\beta)\right]+\frac{1}{\beta}\right\}.$$

As $0<\beta\leq 1$, $\ln(\beta)$ is non-positive, and $\ln\left(\frac{b}{2Aa}\right)<0$ if $\frac{b}{2Aa}<1$, we can conclude that $\frac{\partial\left\{\frac{2}{2-\beta}\left[\ln\left(\frac{b}{2Aa}\right)+\ln(\beta)\right]\right\}}{\partial\beta}>0$ only if $\frac{b}{Aa}>\frac{2}{\beta}\exp\left(\frac{\beta-2}{\beta}\right)$. Again, the left hand side of (32) is an inverted "U-shaped" parabola, and therefore the distance between the values of $(\overline{T}-t_2)$ and t_2 which solves the system increases as β increases when $\frac{b}{Aa}>\frac{2}{\beta}\exp\left(\frac{\beta-2}{\beta}\right)$.

The wage gap is given as a function of t_2 and t_1 by (10): $\frac{w_1}{w_2}=\frac{t_2}{t_1}$. The more t_1 and t_2 differs, the higher the gender wage gap. □

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