AUDITING COST OVERRUN CLAIMS^{*}

David Pérez-Castrillo[#]

University of Copenhagen & Universitat Autònoma de Barcelona

Nicolas Riedinger

ENSAE, Paris

Abstract: We consider a cost-reimbursement or a cost-sharing procurement contract between the administration and a firm. The firm privately learns the true cost overrun once the project has started and it can manipulate this information. We characterize the optimal auditing policy of cost overrun claims as a function of the initial contractual payment, the share of the cost overrun paid by the administration, the cost and the accuracy of the auditing technology, and the penalty rate that can be imposed on fraudulent firms. We also show that this possibility of misreporting reduces the set of projects carried out and biases the choice of the quality level of those projects that the administration carries out.

Keywords: cost overruns, auditing, procurement.

JEL codes: H57, L50, D82.

^{*} We thank Inés Macho-Stadler, Juan-Enrique Martínez-Legaz, and Pau Olivella for helpful comments. D. Pérez-Castrillo acknowledges the financial support from projects DGES PB 97-0181 and SGR 98-62.

[#] Centre for Industrial Economics
Institute of Economics; University of Copenhagen
Studiestraede 6; DK-1455 Copenhagen K; Denmark
phone: +45 35323013 fax: +45 35323000
e-mail: David.Perez@uab.es

1. Introduction

In the contracts that rule the relationships between government agencies and private firms, the final cost of the project is a primary ingredient. In particular, target-cost pricing is a widely applied formula in procurement contracts (see, for example, Cummins (1977) for an analysis of the use of this type of contract in defense procurement). Target-cost-pricing contracts are based on two elements: an initial payment made by the administration, which is related to the "estimated costs," and the payment of a share of the cost overruns, that is, of the difference between the final cost and the agreed-upon cost estimate.¹ This second element is very important since the weight of the payment for cost overruns on the total project cost can actually be very large. For example, Peck and Scherer (1962) estimated that, for U.S. defense programs, development costs exceed original predictions by 220 percent on average.

The modern theory of procurement looks for the characterization of the form of the optimal contracts. It has emphasized the importance of the ex-ante asymmetric information between sponsor and contractor with respect to the cost function of the contractor, or the unobservability of its cost-reducing effort. In particular, Laffont and Tirole (1986) analyze a relationship in which both the sponsor and the contractor are risk neutral and where the two previous asymmetric information problems are present. They show that the optimal contract is linear in final costs.²

Implementing contracts that depend on the actual cost overruns requires the administration to be able to assess the true final cost of the project. However, a firm can inflate its costs in several ways. For example, it can shift costs from one project to another, if it is working on several projects with different sponsors. It can also claim that

¹ For example, the Spanish Code of Public Markets specifies that, for the large public markets, the administration shall attribute the project with a fixed price and that this price can only be revised if there appear unexpected new costs or constraints when the project is been carried out. Similarly, the French Code of Public Markets states that, whenever the extra-contractual costs arrive at a level of a fifteenth of the initial amount, the cost overruns must be paid by the administration (10% of the cost overruns can be left to the account of the firm, depending on the reasons of the extra cost). Finally, cost-reimbursement type contracts are the most frequently used on NASA programs. For a description of contract types available for use in Government contracting, see NASA (1997).

² McAfee and McMillan (1986) show that a cost-sharing-plus-fee contract perform better than a cost-reimbursement contract when the final cost is observable and it depends upon the effort of the firm and some exogenous variables.

good (an expensive) staff has been working on the project, although they were doing something else. Avoiding to pay for false cost overruns is a central concern in procurement contracts.

In this paper, we consider that the final cost, and hence the true cost overrun, is private information of the firm once the project has started. We assume that the contract between the administration and the contractor is linear in cost overruns; that is, we consider the several variants of cost-reimbursement and cost-sharing contracts. Once it knows the true cost, the firm makes a cost overrun claim, not necessarily the true one.³ The administration can possibly learn the true cost overrun if it decides to audit the firm. But auditing is costly and imperfect: it sometimes discovers the fraud, other times it does not. We assume that the administration is able to commit (for example, by law) to a certain auditing policy. We characterize the optimal auditing strategy as a function of the initial contractual payment, the share of the cost overrun paid by the administration, the cost and the accuracy of the auditing technology, and the penalty rate that can be imposed on fraudulent firms. We also analyze the effects that this potential misbehavior of firms has on the quality of the project chosen by the administration. The analysis is made under the assumption that both the administration and the firm are risk neutral.⁴

We find that the optimal auditing policy is very simple. It sets a cut-off value for cost overrun claims. No claim below this value is audited. Claims above it are audited, either randomly (when the audit technology is very precise and/or the penalty rate is high), or systematically. We show that it is optimal for the administration to be very tolerant with cost overrun claims if the audit technology is expensive, it is not very

³ Casual evidence about strategic behavior by firms is provided by Ganuza-Fernández (1996). In 1993, 77% of the projects developed in Spain had a cost overrun and 33% of them had a cost overrun between 19 and 20%, which shows that cost overruns seem rather the rule than the exception. It also shows that firms use the fact that if they stay under the 20%, they are only subject to a limited control (under the Spanish Code, projects with cost overrun of more than 20% of the contracted price are subject to strict control system).

⁴ The analysis that we develop in this paper not only applies to public procurement, but also to the procurement to private firms. We have chosen to refer to the "sponsor" as the "administration" and to the "contractor" as the "firm". The main reason for our choice is that the assumption of *ex ante* commitment to the auditing policy is crucial for our results and it is usually easier for the public administration to commit "by law" to a given auditing strategy (especially when the strategy is very simple, hence bureaucratic) than for a private firm.

accurate in finding out frauds, and/or the penalty rate is low. In this case, the expected profits of the firm are high.

The behavior of the firm facing the optimal auditing policy is qualitatively very different depending on the ability of the administration to discover and punish the offenders. If the means to discover and punish misbehaviors are effective enough, a random audit for cost overruns claims above some cut-off value is optimal. The firm will claim the true cost overruns if it lies above the cut-off level, and it will claim the cut-off level otherwise. In particular, under the optimal policy, only truthful claims are audited in this case. On the other hand, if the audit technology is poor and the penalty rate is low, then the firm will always commit fraud. If the true cost overrun is high enough, the firm will claim the highest possible cost overrun. Otherwise, it will claim the cut-off cost overrun that allows it to avoid the audit. The very high claims are audited and, maybe, the misbehavior is discovered and punished.

We also analyze in the paper the effects of the possibility of firm's misbehavior on the choice of the quality of the project made by the administration. We model a situation in which the administration chooses both the (verifiable) quality of the project and the auditing policy. When choosing the quality of the project, the administration not only takes into account the expected cost for the completion of the project but also the expenses due to the non-observability of the final cost (auditing costs and extra profits for the firm). First, we show that if the project is carried out, the quality chosen is in general different from the optimal quality were the final cost verifiable. The administration biases the quality towards those levels that, because of the form of their cost distribution, make auditing easier. That is, the administration trades off between the inefficiency in the decision on quality and the inefficiency due to the extra costs due to the auditing activity. Second, we prove that projects that are profitable for the administration are sometimes discarded because of the possibility of firm's misbehavior. Hence, the possibility of fraud reduces the set of projects carried out by the administration.

In our model, auditing is a means to verify the true cost (overrun) of the firm. Auditing can also be a way to alleviate the adverse selection problem that appears, for example, when a regulated firm is better informed than the regulator about a (*ex ante*)

parameter that affects its cost function. This is the framework analyzed by Baron and Besanko (1984). They assume that the regulator is able to observe the (*ex post*) realized cost by auditing at a cost. They show that the optimal contract menu involves auditing the firm that reports a high cost parameter and the imposition of a penalty when the final cost is low. In their model, the relevant private information of the firm is not the final cost, as it is in our model, but the cost function. At equilibrium, the firm will always report truthfully but, if the cost parameter is high and the final cost low, it is audited and penalized.

The analysis of optimal auditing strategies has been the subject of research in other economic problems. In particular, our basic model shares characteristics with the models developed by Sánchez and Sobel (1993) and Macho-Stadler and Pérez-Castrillo (1997), which study optimal auditing in tax evasion frameworks. Also, Souam (1999) analyzes the optimal auditing strategy for the authority in charge of competition policy enforcement when it cannot perfectly observe the characteristics and behavior of the firms.⁵

Finally, let us notice that some authors use a different definition of cost overrun than the one we have taken. Lewis (1986) considers a situation where the project requires a number of tasks to be completed. He shows that the cost distribution for a task at the end of the project dominates the cost distribution for a task at the beginning of the project, in the sense of first order stochastic dominance. He refers to this effect as cost overruns. In a related model, Arvan and Leite (1990) analyze the endogenous compensation scheme. They also show the presence of cost overruns, thought of as a combination of the stochastic dominance property in cost per task, a lack of cost minimization by the contractor, and an excessive variability in contractor remuneration.

The remainder of this paper is organized as follows. In the next section we present the basic model in which, for convenience, we assume that the administration fully compensates the cost overruns. In Section 3, we characterize the optimal auditing policy for a given project. In Section 4, we analyze the effects of the possibility of fraud and the

⁵ See also Martin (1998) for a model of resource allocation by a competition authority that audits several industries, when final prices not only depend on firms' behavior.

application of the optimal auditing policy on the choice of project quality by the administration, when it can freely set the initial contractual payment. In Section 5, we show how our analysis can be applied to the situations in which the fines are not financial. Also, we prove that the results generalize easily if the administration only pays a share of the cost overruns. Finally, in Section 6 we conclude. All the proofs are presented in the Appendix.

2. The Basic Model

We consider a cost-reimbursement contract between the administration and a firm. The contract concerns the completion by the firm of a project of observable quality. It engages the administration to make an initial payment of c_0 in exchange for the project.⁶ In addition to this initial payment, the administration must pay the cost overrun encountered by the firm.⁷

At the time the contract is signed, both the firm and the administration have only imperfect knowledge of the true cost of the project. Moreover, they cannot affect the final cost. The true cost c (we will also call it the final cost) is distributed along the interval $[\underline{c}, \bar{c}]$ according to the distribution function $F(c)$. We suppose that $F(c)$ is continuously differentiable and that $f(c) = F'(c) > 0$ for $c \in]\underline{c}, \bar{c}[$. We denote the hazard rate associated with $F(c)$ by $\phi(c) \equiv F(c)/f(c)$. The function $\phi(c)$ is assumed to be increasing in c .⁸ We denote by c_m the expected cost of the project.

The firm (but not the administration) learns c before the work ends. We denote by e the true cost overrun (or the true extra cost); that is, e is the difference between the true cost c and the payment agreed upon in the contract c_0 if this difference is positive, and it is zero otherwise, i.e., $e = \max\{c - c_0, 0\}$. After the firm has observed the final cost of the project, it sends a cost overrun claim $s \geq 0$ (not necessarily the true one!) to the

⁶ It is often the case that the firm receives at least a positive payment c_0 even if the final cost is lower than this level. In those cases in which the administration just covers the cost, without a minimal level, then the initial payment is zero.

⁷ We assume that the administration fully covers the extra cost for notational simplicity. In Section 5, we will generalize our results to the situations where the reimbursement does not cover the total cost overrun, but only a fraction of it (cost-sharing contracts).

administration. The firm is risk neutral and it announces the cost overrun $s(e)$ in order to maximize its expected profit, given the true extra cost e . We take the convention that a firm indifferent between declaring s and $s' > s$ will declare s .

If the administration audits the firm, the audit is successful, and the announced cost overrun is larger than the true cost overrun, then the firm has to pay a fine. We assume that the penalty is proportional to the difference between the announced cost overrun and the true one. A firm that announces a cost overrun $s > e$, if discovered, suffers a penalty of $\pi(s-e)$. Consequently, it receives a total payment (taking also into account the initial payment c_0) of $c_0 + e - \pi(s-e)$. When the firm has not been audited, the audit was not successful, or the audit revealed an honest attitude (or it discovered a cost overrun larger than the claim), the firm receives $c_0 + s$. That is, there is no compensation for a firm that declares a cost overrun that is inferior to the real one.

Before the firm announces its declared cost overrun s , the administration chooses the audit policy, i.e., the function $p(s)$, for $s \in [0, \bar{c} - c_0]$. The amount $p(s)$ is the probability that the administration audits the firm if it announces a cost overrun s . We assume that the administration can commit to this probability-of-auditing function. It only has interest in auditing if the cost overrun is strictly positive, hence $p(0) = 0$. We denote by $k > 0$ the cost of auditing one project. We suppose that the audit succeeds with probability $\theta \in]0, 1]$. In particular, $\theta = 1$ corresponds to a situation where the audit is perfect, in the sense that it uncovers any possible fraud with certainty. A low value of θ corresponds to those cases in which the technology of audit, or the auditor's skills, make it difficult for the administration to find out whether the cost overrun claimed by the firm is justified.

The administration is also assumed to be risk neutral. It maximizes a weighted difference of the expected profit of the firm Π and the expected total cost supported by the administration C_a . This total cost is the expected cost of the audit plus the total price paid to the firm, net of the expected penalty. Indeed, we suppose for the moment that the

⁸

We make this assumption for simplicity. We will comment on it later.

penalty is monetary and it is totally perceived by the administration.⁹ Therefore, the maximization problem for the administration is:

$$\text{Max}_{p(.)} \{ \alpha \Pi(p(.)) - C_a(p(.)) \},$$

where $\alpha \in [0, 1]$ indicates the weight that the administration gives to the profit of the firm. In particular, $\alpha = 0$ corresponds to a situation where the administration minimizes its expected cost and $\alpha = 1$ to a situation where the administration maximizes the expected social surplus.

We could introduce an opportunity cost of the public funds λ (as it is often done in procurement models) and maximize the function $\alpha \Pi - (1+\lambda) C_a$. Introducing this type of opportunity cost does not qualitatively influence the results; we would solve the same program substituting α by $\alpha/(1+\lambda)$. Hence, we choose to omit this term to ease the notation.

3. The Optimal Inspection Policy

We look for the optimal auditing policy for the administration, taking into account that the firm, once the audit policy is announced and having observed the real cost, will declare a cost overrun in order to maximize its expected profits. Denote by $E(s, e, p(.))$ the expected payment net of the penalty received by a firm whose true cost overrun is e and that declares a cost overrun s , when the audit policy is $p(.)$. That is,

$$E(s, e, p(.)) = c_0 + s - p(s) \theta (1+\pi) (s - e) \text{ if } s \geq e.$$

(Notice that we can discard any $s < e$, since this behavior is dominated by the declaration of $s = e$. See also Lemma 1 in the Appendix.) Denote by $s(e)$ the optimal reporting of a firm that faces a true cost overrun e (when the audit policy is $p(.)$) and let $E^*(e, p(.)) = E(s(e), e, p(.))$. We can now express the expected profit of the firm Π at the time the contract is signed and the expected total cost supported by the administration C_a , as a function of the audit function $p(.)$:

⁹ In Section 5, we will analyze the situations in which, because of legal problems concerning

$$\begin{aligned}\Pi(p(.)) &= \int_0^{\bar{c}-c_0} E^*(e, p(.)) dG(e) - c_m \\ C_a(p(.)) &= \int_0^{\bar{c}-c_0} [E^*(e, p(.)) + kp(s(e))] dG(e),\end{aligned}$$

where $G(\cdot)$ is the distribution function of the true cost overruns, that is, $G(e) = F(c_0 + e)$, for all $e \in [0, \bar{c} - c_0]$.

The characteristics of the optimal inspection policy and of the firm's announcement strategy depend crucially on the relative position of θ and $1/(1+\pi)$, that is, on the accuracy of the auditing and on the level of the fine.

To provide the optimal policy for the case in which $\theta(1+\pi) \geq 1$, let us denote:

$$\begin{aligned}a &= \phi^{-1}\left(\frac{k}{(1+\pi)(1-\alpha)\theta}\right) \text{ if } \phi(\bar{c}) > \frac{k}{(1+\pi)(1-\alpha)\theta} \\ a &= \bar{c} \text{ otherwise} \\ b &= \max\{0, a - c_0\}\end{aligned}$$

The parameter a is defined in the space of possible costs, while the parameter b takes value in the space of possible cost overruns, i.e., $b \in [0, \bar{c} - c_0]$. Notice, in particular, that the value of a is independent of the initial contractual payment c_0 . The following proposition characterizes the optimal audit policy in the case where $\theta(1+\pi) \geq 1$.

Proposition 1. If $\theta(1+\pi) \geq 1$, the following audit probability function is an optimal solution of the audit problem:

$$\begin{aligned}p^*(s) &= 0 \text{ if } s \leq b, \\ p^*(s) &= 1/[(1+\pi)\theta] \text{ otherwise.}\end{aligned}$$

Facing this policy, the optimal firm's cost-overrun claim is the following:

$$\begin{aligned}s(e) &= b \text{ if } e \leq b, \\ s(e) &= e \text{ if } e > b.\end{aligned}$$

monetary penalties or because of firm's liquidity constraints, fines are non-monetary.

Before we comment on this proposition, let us remark that the qualitative characteristics of the optimal auditing policy do not depend on our assumption that the hazard rate $\phi(c)$ is an increasing function of c . If $\phi(c)$ is not increasing, then the optimal policy has the same form as the one presented in the proposition. The only difference is that the cut-off value b is more difficult to characterize.¹⁰ Also, the qualitative properties of the optimal policy remain if the administration has a fixed budget to devote to auditing. In this case, the threshold value is characterized as the minimum b compatible with the budget.

[Insert Figure 1 about here]

According to the optimal auditing policy (see Figure 1), in those situations in which the contractual prize c_0 is high enough, so that $c_0 \geq a$, the administration audits all the projects claiming cost overruns with the same probability. Facing this policy, the firm only declares cost overruns when they are real. In the opposite case, i.e., when $c_0 < a$, the administration admits the cost overrun as long as it is not too high, and else audits with a fixed probability. The firm always declares a cost overrun of at least $s = b = a - c_0$. It declares a larger one only if it is real.

Under the optimal auditing policy, the precise value of the contractual prize c_0 is not relevant as long as it is inferior to the cut-off value a , since the administration will end up paying at least a independently on the final cost. The proposition provides a rationale for the use of an initial payment that the firm keeps even if the final cost happens to be lower than this level. If the administration has all the bargaining power in the contractual relationship (as we will suppose in the next section) it is optimal for it to propose a contract with an initial payment $c_0 = a$ and to implement an audit policy consisting in auditing every cost overrun with probability $1/[(1+\pi)\theta]$.

The optimal auditing policy described in Proposition 1 is random for cost overrun claims higher than the cut-off value b (except in the limit case in which $\theta(1+\pi) = 1$). Indeed, if a firm claims a cost overrun $s > b$, then it is audited with some probability lower than 1. This randomness is somehow in contrast with the bureaucratic procedures

¹⁰ See also the proof of the proposition in the Appendix for a clarification of this statement.

that are most often used in practice. Note, however, that besides the randomness, the rule to be applied in case of a claim of cost overrun is quite simple, very easy to implement and, hence, well adapted to bureaucratic procedures.

Another characteristic of the optimal policy is that the administration never audits a fraudulent firm.¹¹ Facing this policy, the firm declares its true cost overrun unless it is lower than b , in which case it claims b (see Figure 1). Given that the administration only audits claims higher than b , ex post, only firms declaring the true cost overrun are audited. Hence, under the optimal audit policy, penalties are never paid. This property of the optimal policy seems a bit unpleasant. Indeed, the administration only audits firms that it "knows" they will turn out to be honest. The optimal audit policy is of course not optimal ex post. Precisely, the existence of the "bureaucratic" rules makes it possible for the policy not to be ex post optimal, but ex ante optimal.

Given the expression of the cut-off value b , we can assess that the administration is more tolerant with cost overrun claims the more expensive the audit technology (k) is, the more difficult it is to find out a fraud (lower θ), the lower the rate of penalty (π) is, and the more weight (α) it confers to the profits of the firm. In particular, if $\alpha = 1$, then $b = \bar{c} - c_0$, hence every claim of cost overrun will be covered by the administration without audit. (This last property would be false if we introduced an opportunity cost of public funds.) All of the effects are in accordance with intuition.

Notice also that if $f(\bar{c}) = 0$ and $\alpha \neq 1$, then it is always the case that $b < \bar{c} - c_0$, whatever the cost of the audit k , that is, it is optimal to audit high enough cost overrun claims. On the other hand, when $f(\bar{c}) \neq 0$ and $\alpha \neq 1$, then there exists a bound for the unitary cost of audit such that the administration never audits (i.e., $b = \bar{c} - c_0$) if the cost is higher than this bound.

Finally, we write down the expected profit of the firm. It is equal to $(c_0 + b)F(c_0 + b) + \int_{c_0+b}^{\bar{c}} c dF(c) - c_m = \int_{\bar{c}}^{c_0+b} F(c) dc$, which is strictly positive as long as k

¹¹ Notice that this is also the case in Baron and Besanko (1984). In their model, facing the optimal contract menu, the firms declare honestly. However, a firm reporting a high cost parameter is audited if "bad luck" makes the final cost to be low.

> 0 . Moreover, the expected profit has a lower bound equal to $\int_{\underline{c}}^a F(c)dc$, whatever the initial contractual payment.

We now analyze the case in which it is difficult for the administration to ascertain whether the cost overrun claimed by the firm is justified (i.e., θ is low), and/or the fine that can be imposed to the firm is not very high (π is low). The following proposition characterizes the optimal policy when $\theta(1+\pi) < 1$. We will use the following notation:

$$\beta = \bar{c} - c_0 - (1+\pi)\theta(\bar{c}-a) \text{ if } a \geq c_0$$

$$\beta = 0 \text{ if } a < c_0.^{12}$$

The parameter β takes value in the space of possible cost overruns, i.e., $\beta \in [0, \bar{c}-c_0]$. It will play a similar role as the parameter b in Proposition 1, although there are important differences between the two cases.

Proposition 2. If $\theta(1+\pi) < 1$, then the following audit probability function is an optimal solution of the audit problem:

$$p^*(s) = 0 \text{ if } s \leq \beta,$$

$$p^*(s) = 1 \text{ otherwise.}$$

Facing this policy, the firm's cost-overrun claim is the following:

$$s(e) = \beta \text{ if } e \leq a - c_0,^{13}$$

$$s(e) = \bar{c} - c_0 \text{ if } e > a - c_0.$$

Figure 2 summarizes the results of this proposition.

[Insert Figure 2 about here]

¹² Any $\beta \in [0, (1 - (1+\pi)\theta)(\bar{c}-c_0)[$ is equivalent to $\beta = 0$, for $a < c_0$. For all those values, the optimal firm's announcement is a cost overrun equal to $\bar{c}-c_0$. Hence, the administration is indifferent between any β in the interval. Notice also that, if $a \geq c_0$, then the cut-off value can also be written as $\beta = a - c_0 + [1 - (1+\pi)\theta](\bar{c}-a)$, which is more similar to b in Proposition 1.

¹³ Notice that $a - c_0 < \beta$.

The optimal audit policy in this case is quite similar to that described in Proposition 1. The difference with the previous case does not concern the form of the audit policy, but the behavior of the firm. In particular, in this case, the probability θ that the audit succeeds in finding evidence of fraud is so small (for the given level of penalty π) that the threat of the audit is never strong enough. There is no audit policy that makes the firm declare its true cost overrun. Fraud always exists.

An interesting characteristic of the policy is that it is very bureaucratic: the probability of an audit is always equal to either 0 or 1. Therefore, it is very easy to verify that the rule has been followed. Moreover, in this case, the policy is more appealing ex post than the policy in the case where $\theta(1+\pi) \geq 1$, since the audit is always directed towards dishonest firms (they are all dishonest!). Sometimes the fraud made by the audited firms is discovered. Therefore, the expected collected fine is not zero.

Similarly to what happened after Proposition 1, if $\theta(1+\pi) < 1$ the cut-off value a , and hence also β , is non-decreasing in k and α and non-increasing in θ and π . Finally, notice that we can check that the expected profit is now strictly greater than $\int_{\underline{c}}^a F(c)dc$, whatever the contract cost.

4. Optimal quality of the project

In this section, we look for the optimal decision of the administration concerning the quality of the project. That is, we are interested in the analysis of the effects of the possibility of fraud and the application of the optimal auditing policy on the quality chosen for the project.

To develop our analysis, we suppose, first, that the administration not only decides on the contractual payment and the audit policy, but also on the quality of the project. This quality is observable and the value (denoted by Q) that the consumers attribute to a level of quality is known. The information of the administration concerning the cost of a project of quality Q is represented by a distribution function $F_Q(c)$ continuously differentiable on c with support $[\underline{c}(Q), \bar{c}(Q)]$. As in the previous sections,

we assume that, for every $Q > 0$, $f_Q(c) \equiv F'_Q(c) > 0$, for all $c \in]\underline{c}(Q), \bar{c}(Q)[$ and, for simplicity, that $\phi_Q(c) \equiv F_Q(c)/f_Q(c)$ is an increasing function of c .

Concerning the behavior of the distribution function with respect to the quality, we assume, first, that the cost of producing $Q = 0$ is zero, that is, $\underline{c}(0) = \bar{c}(0) = 0$. Second, for all c , $F_Q(c)$ and $f_Q(c)$ are twice-continuously differentiable functions of Q on $]0, \infty[$. We do not assume continuity on 0 and hence, we take into account the possibility of fixed costs. Finally, the expected cost $c_m(Q)$ is an increasing, convex, and twice-continuously differentiable function.

The administration chooses the initial contractual payment to the firm freely. That is, we assume that the administration has all the bargaining power. In order to contemplate the choice of the quality of the project, the worth of the project for the administration is now included in its objective function. Hence, the program solved by the administration is the following:

$$\begin{aligned} \text{Max} \quad & \{Q + \alpha \Pi(p(.)) - C_a(p(.))\} \\ (c_0, p(.), Q) \end{aligned}$$

We denote $Q_{opt}(\alpha, k, \theta, \pi)$ the solution of this program. We will usually only write the relevant arguments in each case, to simplify notation.

Since we are interested in those situations in which it is reasonable for the administration to carry the work out, we suppose that $\text{Max}_Q \{Q - c_m(Q)\} > 0$. We denote by Q^* the solution of this maximization. Remark that $Q_{opt}(\alpha=1) = Q^*$ whatever the values of the other parameters. Indeed, if $\alpha = 1$, the administration does not care about transferring profits to the firm, so it does not audit at all. Moreover, we also have $Q_{opt}(k=0) = Q^*$ if $\theta(1+\pi) \geq 1$.

As previously, we could introduce an opportunity cost of public funds λ . In this case, we would use the hypothesis $\text{Max}_Q \{Q - (1+\lambda) c_m(Q)\} > 0$. However, with such an opportunity cost, it is not true anymore that $Q_{opt}(\alpha=1) = Q^*$. The reason is that the administration may find it worthwhile to audit the firm, even if $\alpha = 1$. But the following results are not substantially modified. We will indicate when the introduction of this term leads to a qualitative difference.

Following the results of the preceding sections, we can calculate the optimal audit policy $p_Q^*(.)$, for a given Q . The optimal contractual payment c_0 is not relevant as long as it lies in the interval $[0, a(Q)]$, where $a(Q)$ is the optimal cut-off value corresponding to quality Q . Since the administration chooses c_0 , it will decide any c_0 in the previous interval, so that $b(Q) + c_0 = a(Q)$ (or $\beta(Q) + c_0 = \bar{c}(Q) - (1+\pi)\theta(\bar{c}(Q) - a(Q))$, if $\theta(1+\pi) < 1$). The optimal quality then solves:

$$\text{Max}_Q \{Q + \alpha\Pi(p_Q^*(.)) - C_a(p_Q^*(.))\}.$$

Next result allows us to discuss the effect on the optimal quality of a change in the auditing cost (due, for example, to some gains in efficiency by the administration). We analyze whether the administration will then ask for a project of higher or lower quality. We can also interpret the analysis as a comparison of the levels of quality of public works in different fields, when the auditing cost varies among fields.

Proposition 3. If $Q_{opt}(.) > 0$ in a neighborhood of k and the optimal threshold cost (either b or β) for $Q_{opt}(k)$ is also interior (i.e., $\phi(\bar{c}(Q_{opt}(k))) > k/[(1+\pi)(1-\alpha)\theta]$), then:

$$\frac{dQ_{opt}(k)}{dk} \text{ has the same sign as } \frac{\partial \left(F_Q \circ \phi_Q^{-1} \left(\frac{k}{(1+\pi)(1-\alpha)\theta} \right) \right)}{\partial Q} \Big|_{Q=Q_{opt}(k)}.$$

When looking for the optimal quality, the administration takes into account the expenses related to the completion of the project and the expenses due to the non-observability of its final cost (auditing costs and extra profits for the firm). If the cost of an audit increases, the administration cares more about the expenses related to the auditing activity. Therefore, an increase in the unit cost k should lead to a policy in which fewer audits take place. This is precisely the meaning of Proposition 3. The expression $(F_Q \circ \phi_Q^{-1})(k/[(1+\pi)(1-\alpha)\theta])$ is equal to $F_Q(a(Q))$, that is, the probability that the administration does not carry out the audit when it follows the optimal auditing strategy. When k increases, the quality chosen must be such that $F_Q(a(Q))$ increases, that is, the audit takes place less often. Note that an increase in $F_Q(a(Q))$ does not necessarily imply

an increase in $a(Q)$ nor a decrease in Q since the distribution function of the true cost changes with the quality chosen.

For a given c , the value of $(F_Q \circ \phi_Q^{-1})(c)$ is small when the density function f_Q is “flat”. It may seem reasonable that $(F_Q \circ \phi_Q^{-1})(k / [(1 + \pi)(1 - \alpha)\theta])$ is a decreasing function of Q . But it can also be an increasing function. Hence, Proposition 3 points out that an increase in audit costs leads to a worse situation from a social point of view, but not necessarily to a lower quality. The following examples clarify this message.

Example 1. Suppose that the distribution of the absolute difference between the cost and the average cost, i.e., $c(Q) - c_m(Q)$, does not depend on Q for values of Q . It is then easy to check that $(F_Q \circ \phi_Q^{-1})(c)$ is independent of Q . Hence, $Q_{opt}(k)$ is independent of k and it is equal to Q^* , as long as it is positive (and it becomes eventually zero for k large enough, as we will see below).

Example 2. Suppose that the distribution of the relative difference between the cost and the average cost, i.e., $[c(Q) - c_m(Q)] / c_m(Q)$, follows an independent distribution of Q . That means that there exists a function F such that $F_Q(c) = F(c / c_m(Q))$. In this situation, the expression $(F_Q \circ \phi_Q^{-1})(c) = (F \circ \phi^{-1})(c / c_m(Q))$ is a decreasing function of Q . Therefore, $Q_{opt}(k)$ is a decreasing function of k , eventually zero for k large enough.

Example 3. Suppose that, for a given Q , the distribution F_Q is uniform. Therefore, the distribution is defined in a non ambiguous way by the expected cost $c_m(Q)$ and the length of the support $L(Q)$. An easy calculation shows then that $(F_Q \circ \phi_Q^{-1})(c) = c / L(Q)$, it is then inversely proportional to $L(Q)$. When k increases, the optimal quality changes in the direction where the support is smaller, this is to say, where the knowledge of the administration is more precise.

From the point of view of the administration, auditing costs add to the costs of the public work. Sometimes, this extra cost is so high that it makes it not worthwhile to carry the project out, even if the project would have been profitable under symmetric

information. The following proposition characterizes the behaviour of $Q_{opt}(k)$, for a fixed α .¹⁴

Proposition 4. If $\text{Max}_{Q>0} \{Q - \alpha c_m(Q) - (1 - \alpha)\bar{c}(Q)\} \geq 0$, then $Q_{opt}(k) > 0$ for all $k \geq 0$. Otherwise, there exists $k_{lim}(\alpha)$ such that if $k > k_{lim}(\alpha)$, then $Q_{opt}(k) = 0$.

When the cost of an audit is very large, the optimal audit policy is to never audit, which leads to a payment of $\bar{c}(Q)$ to the firm. The preceding proposition establishes the conditions under which carrying the work out and paying the highest possible prize is socially preferable to not producing at all.

It can be interesting to compare Q^* , the value of the work when the audit cost is zero, and $Q_\infty(\alpha)$, the value of the work when the audit cost is infinity. If $\bar{c}(Q) - c_m(Q)$ is an increasing function, then $Q_\infty(\alpha) \leq Q^*$. The interpretation is similar to the preceding proposition. If the precision of the information received by the administration lowers with the quality of the project, then the optimal quality when the audit cost is very high is inferior to the optimal quality when the cost is low. Similarly, a sufficient condition for $Q_\infty(\alpha) \geq Q^*$ is that $\bar{c}(Q) - c_m(Q)$ is a decreasing function.

Notice that, when $\theta(1 + \pi) \geq 1$, the result of the maximization problem of the administration does only depend on k , θ , and π through the expression $k/[(1 + \pi)\theta]$. A reduction of the audit cost k is therefore equivalent to an increase of the penalty coefficient π , or to an increase of the probability of the audit success θ . In this case, we can alternatively interpret propositions 3 and 4 as a study on the variation of the optimal quality in function of π or θ .

Finally, we can also establish some properties about the optimal quality as a function of α .

Proposition 5. If $\text{Max}_{Q>0} \{Q - \bar{c}(Q)\} \geq 0$, then $Q_{opt}(k, \alpha) > 0$ for all $\alpha \in [0, 1]$ and $k \geq 0$. Otherwise, there exists $\alpha_{lim} \in [0, 1]$ such that:

¹⁴ We take the convention that if the administration is indifferent between a project of zero quality and a project of positive quality, it will chose the last one.

(a) if $\alpha \geq \alpha_{lim}$, then $Q_{opt}(k, \alpha) > 0$ for all $k \in]0, \infty[$, and

(b) if $\alpha < \alpha_{lim}$, then there exists $k_{lim}(\alpha)$ such that $Q_{opt}(k, \alpha) = 0$ for all $k > k_{lim}(\alpha)$.

Moreover, $k_{lim}(\alpha)$ is increasing in α .

If the administration values the profit of the firm enough, it will let the project be realized (i.e., $Q_{opt} > 0$) whatever the cost of the audit. We notice, however, that this property is not longer true if we introduce an opportunity cost for the public funds. Indeed, under the hypothesis that $Q - (1+\lambda) c_m(Q) > 0$, the administration could decide not to start the project if the audit cost is too high, even when $\alpha = 1$.

5. Extensions

5.1 Non-financial Penalty

We have supposed up to now that the administration is able to impose a financial penalty to the firm. However, in many cases, the administration cannot impose such a fine because of legal matters, or just because the firm does not have the financial means to pay such a fine. After all, the financial equilibrium of the firm is a main reason behind the reimbursement of the cost overruns. In this case, the term $\pi(s - e)$ represents a non-financial penalty, for example, a lost of reputation for the firm, or a certain period where the administration does not accept any projects from that firm. In our model, the difference with the case with monetary fines is that, now, although the firm pays it, the administration does not receive any penalty.

How does the non-financial nature of the fine influence the optimal audit policy? As before, the results do depend in a crucial way on the relative position of $1/(1+\pi)$ and θ .

Proposition 6. If $\theta(1+\pi) \geq 1$, the optimal audit policy described in Proposition 1 is still optimal when the penalty is non-financial.

Therefore, both the optimal auditing policy and firms' behavior are independent of the nature of the penalty, as long as $\theta(1+\pi) \geq 1$. There is no welfare loss in this case

with respect to a situation where the penalty is financial. Similarly, we can easily verify that the results of Section 4 also hold with non-financial penalties.

The main reason for the optimal policy not to depend on the nature of the penalty in the case where $\theta(1+\pi) \geq 1$ is that, facing the optimal auditing policy, the firm actually never pays the fine. However, this is not longer true when $\theta(1+\pi) < 1$, which makes the analysis more difficult. We can not solve for the optimal auditing policy in this case. However, we can state the following property:

Proposition 7. If $\theta(1+\pi) < 1$ and the penalty is non-financial, then the policy described in Proposition 2 is not optimal anymore when $\beta \in]0, \bar{c} - c_0[$. Indeed, in this case, the administration can improve on this policy by raising the cut-off value β .

In the case $\theta(1+\pi) < 1$, there is a welfare loss due to the non-financial nature of the penalty. In the class of auditing policies consisting in auditing every claim of cost overrun higher than a certain cut-off value, the administration finds it optimal to be more lenient when the penalty is non-financial. Given that the administration does not receive the worth of the fine, although the firm "pays" it, an audit policy that leads the firm to pay a fine creates more inefficiencies than when the fine was financial. Therefore, it is optimal to reduce the expected penalties paid, that is, to increase the cut-off value β .

5.2. Partial Repayment of the Cost Overrun

Our results also apply to cost-sharing contracts, that is, to situations where the administration only pays a share δ of the cost overruns, with $\delta \in]0, 1[$, leaving the fraction $1-\delta$ to the firm. This hypothesis corresponds, for example, to the French Code of Public Markets, where $\delta = .90$ for certain types of cost overruns. Proposition 8 shows that the optimal auditing policy in this case is the same as for the case with full reimbursement. The only quantitative difference with respect to our basic framework concerns the precise expression for the cut-off value a .

Proposition 8. Propositions 1 and 2 still hold when the administration only pays a share $\delta \in]0, 1[$ of the cost overruns, with the only difference that the parameter a is now defined by:

$$a = \begin{cases} \phi^{-1}\left(\frac{k}{\delta(1+\pi)(1-\alpha)\theta}\right) & \text{if } \phi(\bar{c}) > \frac{k}{\delta(1+\pi)(1-\alpha)\theta}, \\ \bar{c} & \text{otherwise.} \end{cases}$$

Notice that the cut-off value a is decreasing with the rate of reimbursement δ . The higher this percentage, the larger the proportion of audited projects.

The results of Section 4 on the optimal quality also generalize to cost-sharing contracts. The main difference is that, if both the initial payment c_0 and the rate δ are low enough, then the expected profits of the firm are negative. Hence, it is not necessarily true, as it was in Section 4, that the contractual payment c_0 is not relevant as long as it lies in $[0, a(Q)]$. Consider the case with $\theta(1+\pi) \geq 1$ (the other case is similar). There are two possibilities. If the expected profit with the contractual payment in $[0, a(Q)]$ is non-negative, that is, $a(Q) + \int_{a(Q)}^{\bar{c}(Q)} \delta(c - a(Q)) dF_Q(c) - c_m(Q) \geq 0$ (where $a(Q)$ is determined as in Proposition 8 for the level of quality Q), then we are in the same situation as in the previous section. However, if the previous expected profits are negative, then the administration will choose the minimum c_0 compatible with non-negative profits, that is, $c_0 + \int_{c_0}^{\bar{c}(Q)} \delta(c - c_0) dF_Q(c) - c_m(Q) = 0$, and it will apply the optimal auditing policy given this level of payment.¹⁵

6. Conclusion

We have characterized the optimal policy of auditing cost overrun claims for an administration that has signed a cost-reimbursement or a cost-sharing contract and that is unable to directly observe the realization of the true cost overruns of a project. The optimal auditing policy is very simple. It can be implemented as a bureaucratic procedure

of the same type of the procedures that govern many of the particulars of the contractual arrangements between the administration and firms. This fact is important, since our results depend on the assumption that the administration is able to commit to an audit strategy that is optimal *ex ante*, but that is not optimal *ex post*. The rigidity of the bureaucratic procedures can help to implement the optimal policy.

Moreover, we have shown that the possibility of misbehavior has two additional consequences. On the one hand, the extra costs due to the auditing activity reduce the set of projects carried out. On the other hand, if the project is carried out, the administration biases the chosen quality towards levels that make auditing easier.

In contrast with our paper, the modern theory of procurement looks for the optimal contract when the government does not know some firm's characteristics and/or it wants to give the firm incentives to decrease the cost of the project. In this literature, cost-based contracts allow to alleviate the asymmetric information problems. In particular, Laffont and Tirole (1986) show that cost-sharing-plus-fee contracts are optimal when both government and firm are risk neutral (and the adverse selection and moral hazard problems appear in an additive way). Our approach is somewhat different. We have assumed that, at the time the contract is signed, government and firm have the same information about the cost function. Moreover, they cannot influence the realization of the final cost. It is only once the firm starts working that it learns (more than the government) about the true cost of the project. It can manipulate its report to increase the payment received as cost reimbursement. This framework seems to adapt well to research, design, or study efforts. (In these projects, for example, NASA uses cost-plus-fixed-fee contracts.) In any case, we think that both approaches are complementary. The question of the joint endogenous determination of both the contract and the auditing policy stays open.

¹⁵ In this case, for Proposition 3 to hold, the requirement that b or β be interior also imposes the cut-off value a to be higher than the minimum c_0 compatible with non-negative profits.

Appendix

Proof of Proposition 1.¹⁶ We use the following two lemmas, whose proof is not difficult:

Lemma 1. (a) A firm never claims a cost overrun lower than the real one, i.e., $s(e) \geq e$.
 (b) If $p(s) \geq 1/[(1+\pi)\theta]$ for $s > e$, then the firm will claim a cost overrun different from s .
 (c) The firm declares truthfully if and only if $p(s) \geq 1/[(1+\pi)\theta]$, for all $s \in]e, \bar{c}-c_0]$.

Property (a) of Lemma 1 comes from the fact that there is no reward for a firm that declares a cost overrun lower than the real one. Property (b) holds because $E(s, e, p(.)) \geq c_0 + e$ if and only if $p(s) \theta (1+\pi) \leq 1$, for $s \in]e, \bar{c}-c_0]$. This property implies, in particular, that the administration will never choose an audit probability strictly higher than $1/[(1+\pi)\theta]$. Finally, property (c) easily follows from property (b) and the fact that if $p(s) \theta (1+\pi) < 1$ for some $s \in]e, \bar{c}-c_0]$, then $E(s, e, p(.)) > c_0 + e$.

Lemma 2. The function $p(s(e))$ is non-decreasing in e . Moreover:

$$E^*(e, p(.)) = \bar{c} - \int_e^{\bar{c}-c_0} (1+\pi)\theta p(s(v)) dv.$$

We sketch the proof of Lemma 2. Take $e, v \in [0, \bar{c}-c_0]$. Then,

$$E^*(e, p(.)) = c_0 + s(e) - p(s(e)) \theta (1+\pi) (s(e)-e) \geq c_0 + s(v) - p(s(v)) \theta (1+\pi) (s(v)-e).$$

Note that the inequality comes from the optimality of $s(e)$ if $s(v) \geq e$, and it also holds if $s(v) < e$ given that $p(s(v)) \theta (1+\pi) \leq 1$ by Lemma 1. Combining the previous equations with similar equations for $E^*(v, p(.))$ and $s(e)$, we obtain that, for every $e, v \in [0, \bar{c}-c_0]$:

$$p(s(v)) \theta (1+\pi) (e-v) \leq E^*(e, p(.)) - E^*(v, p(.)) \leq p(s(e)) \theta (1+\pi) (e-v).$$

Finally, the two properties stated in Lemma 2 come easily from the previous inequalities.

Remember that the problem faced by the administration is the following:

¹⁶ This proof of the proposition (including the two lemmas) is related to proofs in Sánchez and Sobel (1993) and Macho-Stadler and Pérez-Castrillo (1997). The main difference is that, in our paper, the administration must pay at least the price c_0 independently on the real cost of the project.

$$\begin{aligned} & \text{Max} \left\{ - (1-\alpha) \int_0^{\bar{c}-c_0} E^*(e, p(s(e))) dG(e) - \int_0^{\bar{c}-c_0} kp(s(e)) dG(e) - \alpha c_m \right\} \\ & \text{s.t. } s(e) \in \arg\max_s E(s, e, p(s)). \end{aligned}$$

We can use lemmas 1 and 2 to state the problem under the following equivalent form, after integration by parts:

$$\begin{aligned} & \text{Max} \int_0^{\bar{c}-c_0} H(e) p(s(e)) dG(e) & \text{Max} \int_0^{\bar{c}-c_0} H(e) q(e) dG(e) \\ & \text{s.t. : } p(s(e)) \text{ non-decreasing in } e & \Leftrightarrow \text{s.t. : } q(e) \text{ non-decreasing in } e \\ & p(s(e)) \in \left[0, \frac{1}{(1+\pi)\theta} \right] & q(e) \in \left[0, \frac{1}{(1+\pi)\theta} \right] \end{aligned}$$

where $H(e) = (1-\alpha)(1+\pi)\theta\chi(e) - k$, and $\chi(e) \equiv G(e)/G'(e) = F(c_0+e)/f(c_0+e) = \phi(c_0+e)$, for all $e \in [0, \bar{c}-c_0]$, is the hazard rate associated to $G(e)$. Notice that the function $q(\cdot)$ takes values on the true cost overruns, while $p(\cdot)$ takes values on the claims. Sánchez and Sobel (1993) have analyzed the results of this type of maximization problem. They shown that, if $H(e)$ is continuous, there always exists a solution of the form:

$$\begin{aligned} q^*(e) &= 0 \text{ if } e \leq b^* \\ q^*(e) &= 1/[(1+\pi)\theta] \text{ otherwise.} \end{aligned}$$

Given the behavior of the firm, the function $q^*(\cdot)$ translates into the following audit probability function:

$$\begin{aligned} p^*(s) &= 0 \text{ if } s \leq b^* \\ p^*(s) &= 1/[(1+\pi)\theta] \text{ otherwise.} \end{aligned}$$

We now characterize the value of b^* . It is clear that the behavior of the firm facing this auditing strategy is to declare a cost overrun equal to b^* when the true cost overrun is lower than this cut-off value, and to be honest otherwise. Also, the function $H(e)$ is increasing, given that $\chi(e)$ is increasing (because $\phi(c)$ is increasing). Then, there are three possible cases. First, if $H(e)$ is always negative (this case corresponds to $a = \bar{c}$), then $b^* = \bar{c} - c_0$. Second, if $H(e)$ is always positive, then $b^* = 0$. Finally, if $H(e)$ is zero at some value in $]0, \bar{c}-c_0[$, then the optimum is given by $(1-\alpha)(1+\pi)\theta\chi(b^*) - k = 0$, i.e., $b^* = a$

$-c_0$. Notice that if $\phi(c)$ is not increasing, the characteristics of the policy are the same, but the precise calculation of b^* is more difficult.

Proof of Proposition 2. The first part of the proof is the same as in Proposition 1. We can state the maximization program of the administration in the same form as before, the only difference is that the function $q(e)$ must lie in the interval $[0, 1]$. This does not change the form of the solution, which is that $q(e) = 1$ for $e > b^*$, and $q(e) = 0$ otherwise, for some b^* . Finding the expression for the optimal b^* is similar as before, but translating this into a cut-off cost overrun is more complex.

The difference with the previous case lies in the behavior of the firm. This is so because an audit probability of 1 does not avoid the fraud, given that $(1+\pi)\theta < 1$. If the administration sets a cut-off value β° , then a firm with cost overrun $e \geq 0$ will choose between reporting β° and not being audited, or claiming a cost overrun of $\bar{c} - c_0$ and being audited for sure (it is easy to check that the other possibilities are dominated by these two strategies). It will choose to claim β° if and only if $c_0 + \beta^\circ \geq \bar{c} - (1+\pi)\theta(\bar{c} - c_0 - e)$, i.e., $e \leq \bar{c} - c_0 - [\bar{c} - c_0 - \beta^\circ]/[(1+\pi)\theta]$. In particular, the firm will always claim the highest cost overrun possible, $\bar{c} - c_0$, if $\beta^\circ < [1 - (1+\pi)\theta](\bar{c} - c_0)$.

We now characterize the optimal cut-off values. If $(1-\alpha)(1+\pi)\theta\chi(0) - k \leq 0$, i.e., if $a \geq c_0$, then $b^* = a - c_0$. To implement a policy that makes the firm to be audited if and only if its true cost overrun is above $b^* = a - c_0$, the cut-off value must be $\beta = \bar{c} - c_0 - (1+\pi)\theta(\bar{c} - a)$. On the other hand, if $a < c_0$, then $b^* = 0$ (the firm will always be audited). Any policy with $\beta \in [0, (1 - (1+\pi)\theta)(\bar{c} - c_0)[$ will induce a firm's strategy that will make it to be audited.

Proof of Proposition 3. Denote $J(k, Q) \equiv Q + \alpha \Pi(Q, p_Q^*(.)) - C_a(k, Q, p_Q^*(.))$. Then, the optimal level of quality is the argument that maximizes: $\text{Max}_Q J(k, Q)$. In an interior solution, it is the case that $\partial J / \partial Q(k, Q_{opt}(k)) = 0$ and $\partial^2 J / \partial Q^2(k, Q_{opt}(k)) \leq 0$. Therefore, according to the implicit function theorem, the ratio $dQ_{opt}(k)/dk$ has the same sign as the ratio $\partial^2 J / \partial Q \partial k(k, Q_{opt}(k)) = \partial^2 J / \partial k \partial Q(k, Q_{opt}(k))$.

We now prove the proposition when $\theta(1+\pi) \geq 1$. In this case, we also have that $J(k, Q) = \text{Max}_{b^\circ} \Gamma(b^\circ, k, Q)$, where:

$$\begin{aligned} & \Gamma(b^\circ, k, Q) \\ &= Q - c_m(Q) - (1-\alpha) \left[(c_0 + b^\circ) F_Q(c_0 + b^\circ) + \int_{c_0 + b^\circ}^{\bar{c}(Q)} c dF_Q(c) - c_m(Q) \right] - \frac{k}{(1+\pi)\theta} (1 - F_Q(c_0 + b^\circ)) \\ &= Q - c_m(Q) - (1-\alpha) \int_{\underline{c}(Q)}^{c_0 + b^\circ} F_Q(c) dc - \frac{k}{(1+\pi)\theta} (1 - F_Q(c_0 + b^\circ)) \end{aligned}$$

That is, $\Gamma(b^\circ, k, Q)$ is the value of the function that the administration maximizes when it audits with probability $1/[(1+\pi)\theta]$ above the cut-off value b° , and with zero probability below b° .

Using the envelop theorem, we have:

$$\frac{\partial J}{\partial k}(k, Q) = \frac{\partial \Gamma}{\partial k}(b, k, Q) = -\frac{1}{(1+\pi)\theta} [1 - F_Q(c_0 + b)] = -\frac{1}{(1+\pi)\theta} \left[1 - F_Q \circ \phi_Q^{-1} \left(\frac{k}{(1+\pi)(1-\alpha)\theta} \right) \right].$$

Therefore, $\left. \frac{\partial (F_Q \circ \phi_Q^{-1} \left(\frac{k}{(1+\pi)(1-\alpha)\theta} \right))}{\partial Q} \right|_{Q=Q_{opt}(k)}$ has the same sign as

$\partial^2 J / \partial k \partial Q(k, Q_{opt}(k))$, and hence it has also the same sign as $dQ_{opt}(k)/dk$.

The proof for the case with $\theta(1+\pi) < 1$ is completely similar. The only quantitative difference is that, in this case, $\frac{\partial J}{\partial k}(k, Q) = -\left[1 - F_Q \circ \phi_Q^{-1} \left(\frac{k}{(1+\pi)(1-\alpha)\theta} \right) \right]$, but this difference does not modify the proposition.

Proof of Proposition 4. The objective function of the administration is decreasing in k . If the value function is positive when the cost k tends toward infinity, i.e., $\text{Max}_{Q>0} \{Q - \alpha c_m(Q) - (1-\alpha)\bar{c}(Q)\} \geq 0$, then the value function is positive for all $k \geq 0$. This implies that $Q_{opt}(k) > 0$ for all $k \geq 0$. Otherwise, the value for the administration is negative for k very high and positive for $k = 0$. The second part of the proposition then derives easily from the fact that value function is decreasing in k .

Proof of Proposition 5. The value function $\text{Max}_{Q>0} \{Q - \alpha c_m(Q) - (1-\alpha)\bar{c}(Q)\}$ is a continuous and increasing function of α . Also, we have made the hypothesis that it is

positive when $\alpha = 1$. Therefore, if the value function is also positive when $\alpha = 0$, then it is positive everywhere, which proves the first part of the proposition. If the function is negative when $\alpha = 0$, then there exist a limit threshold for α such that the value function is positive above this threshold and negative below it. As to the property that $k_{lim}(\alpha)$ is increasing in α , notice that, in $k_{lim}(\alpha)$, the value of the objective function of the administration is zero. Given that the objective function is increasing in α , and it is decreasing in k , the implicit function theorem allows us to make sure that $k_{lim}(\alpha)$ is also increasing in α .

Proof of Proposition 6. Notice, first, that the behavior of the firm only depends on the audit policy and it is independent on the nature of the penalty. Also, for a given audit policy, the objective function of the administration in case of financial fine is equal to the sum of the objective function in case of non-financial fine plus the amount paid as penalty. But, as we have shown, the optimal auditing policy in case of financial fine induces a behavior for the firm such that it never pays the fine. Therefore, the policy is also optimal when the penalty is non-financial.

Proof of Proposition 7. We analyze to audit policies that audit with zero probability cost overruns below a certain cut-off level β° and with probability 1 below it. We look for the optimal audit policy within this class, when the fine is non-financial. We assume that β is interior, and we start by assuming that β° is also interior. Characterizing β° is equivalent to characterizing the threshold a° such that the firm announces a cost overrun equal to β° if the true cost overrun is smaller than a° (see Proposition 2). We characterize the optimal threshold a° . Under this type of policy, we have:

$$\begin{aligned}\Pi &= (\beta^\circ(a^\circ) + c_0)F(a^\circ) + \int_{a^\circ}^{\bar{c}} [\bar{c} - (1 + \pi)\theta(\bar{c} - c)]dF(c) - c_m \\ \text{where } \beta^\circ(a^\circ) + c_0 &= \bar{c} - (1 + \pi)\theta(\bar{c} - a^\circ), \text{ and} \\ C_a &= (\beta^\circ(a^\circ) + c_0)F(a^\circ) + \int_{a^\circ}^{\bar{c}} [\bar{c} - \theta(\bar{c} - c)]dF(c) + k(1 - F(a^\circ)).\end{aligned}$$

The first order condition of the program $\text{Max}_{a^\circ} \{ \alpha\Pi - C_a \}$ is:

$$-(1 - \alpha)(1 + \pi)\theta F(a^\circ) + kf(a^\circ) + \pi\theta(\bar{c} - a^\circ)f(a^\circ) = 0.$$

That is, $\phi(a^\circ) = \frac{k}{(1+\pi)(1-\alpha)\theta} + \frac{\pi}{(1+\pi)(1-\alpha)}(\bar{c} - a^\circ)$.

Therefore, $\phi(a^\circ) > \phi(a) = \frac{k}{(1+\pi)(1-\alpha)\theta}$. Hence, $a^\circ > a$. Of course, this proof is only correct if both a and a° are interior solutions. If a is interior, a similar argument works. If $a = \bar{c}$, then $a^\circ = \bar{c}$ as well. If $a = c_0$, then it can be the case that we still have $a^\circ = c_0$, or that $a^\circ > a$.

Proof of Proposition 8. The proof of this proposition is qualitatively similar to that of propositions 1 and 2.

References

- Arvan, L. and A.P.N. Leite, 1990, Cost overruns in long term projects, International Journal of Industrial Organization 8, 443-47.
- Baron, D. and D. Besanko, 1984, Regulation, asymmetric information, and auditing, Rand Journal of Economics 15 (4), 447-470.
- Cummins, J.M., 1977, Incentive contracting for national defense: a problem of optimal risk sharing, Bell Journal of Economics 6, 168-185.
- Ganuza-Fernández, J.J., 1996, El comportamiento estratégico en la contratación pública, Ph.D. thesis, Universidad Carlos III de Madrid, Spain.
- Laffont, J.-J. and J. Tirole, 1986, Using cost observation to regulate firms, Journal of Political Economy 94, 614-641.
- Lewis, T.R., 1986, Reputation and contractual performance in long-term projects, Rand Journal of Economics 141,157.
- Martin, S., 1998, Resources allocation by a competition authority, W.P., Center for Industrial Economics, University of Copenhagen.
- McAffee, R.P. and J. McMillan, 1986, Bidding for contracts: a principal agent analysis, Rand Journal of Economics 17 (3), 326-338.
- Macho-Stadler, I. and D. Pérez-Castrillo, 1997, Optimal auditing with heterogeneous income sources, International Economic Review 38 (4), 951-968.
- NASA, 1997, Award fee contracting guide, www.hq.nasa.gov/office/procurement.
- Peck, M.S. and F.M. Scherer, 1962, The weapons acquisition process: an economic analysis, Boston: Harvard Univ., Graduate School of Business Administration.
- Sánchez, I. and J. Sobel, 1993, Hierarchical design and enforcement of income tax policies, Journal of Public Economics 50, 345-369.
- Souam, S., 1999, Optimal antitrust policy under different regimes of fines, International Journal of Industrial Organization, forthcoming.

Figure 1.a: Optimal auditing policy and firm's behavior
when $(1+\pi)\theta \geq 1$ and $a > c_0$.

Figure 1.b: Optimal auditing policy and firm's behavior
when $(1+\pi)\theta \geq 1$ and $a \leq c_0$.

Figure 2.a: Optimal auditing policy and firm's behavior
when $(1+\pi)\theta < 1$ and $a > c_0$.

Figure 2.b: Optimal auditing policy and firm's behavior
when $(1+\pi)\theta < 1$ and $a \leq c_0$.