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The Simplest Unified Growth Theory

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Abstract. This paper provides a unified growth theory, i.e. a model that explains the very long-run economic and demographic development path of industrialized economies, stretching from the pre-industrial era to present-day and beyond. Making strict use of Malthus' (1798) so-called *preventive check hypothesis* – that fertility rates vary inversely with the price of food – the current study offers a new and straightforward explanation for the demographic transition and the break with the Malthusian era. The current framework lends support to existing unified growth theories and is well in tune with historical evidence about structural transformation.

Keywords: Economic Growth, Population Growth, Structural Change, Industrial Revolution.
JEL: O11, O14, J10, J13.

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1. Introduction

Over the past two centuries, a so-called Malthusian era, represented by stagnant standards of living and a positive relationship between income and fertility, has gradually been replaced by a so-called Modern Growth era, which in turn is marked by sustained economic growth and declining rates of fertility (a demographic transition). Building on seminal work by Galor and Weil (2000), several attempts have been made to try to merge the two eras, and to identify the underlying causes of the demographic transition. An incomplete list of studies includes Boucekkine et al. (2002), Doepke (2004), Galor and Moav (2002), Hansen and Prescott (2002), Jones (2001), Kögel and Prskawetz (2001), Lucas (2002), Strulik (2003) and Tamura (2002).

Common to these so-called unified growth theories is a complex apparatus, offered to motivate the profound shift in human fertility behavior associated with the demographic transition. The story of the fertility drop differs with the specific theory, but the gradual rise in the demand for human capital "has led researchers to argue that the increasing role of human capital in the production process induced households to increase investment in the human capital of their offspring, ultimately leading to the onset of the demographic transition" (Galor, 2005).

The current paper offers a new and straightforward explanation for the demographic transition and the break with the Malthusian era – a theory which nicely complements ideas proposed in existing unified growth theories. More specifically, we provide an endogenous growth model which is consistent with the (stylized) development path of industrialized countries but does not rely on human capital accumulation as the driving force behind the demographic transition.

In the existing literature, a trade-off between child quantity and quality ultimately serves to generate a drop in fertility along with rising incomes. Fertility behavior in the present context, by contrast, relies entirely on Malthus' so-called *preventive check hypothesis* – that the tendency to matrimony, and thus implicitly the desire to give birth to children, is negatively correlated with the price of food (Malthus, 1798). The absence of a quality-quantity trade-off in the current paper eliminates the need of complex theoretical mechanisms hitherto required to explain the demographic transition, and is why we claim to provide the simplest unified growth theory so far proposed.

The current theory draws inspiration from a number of theoretical elements found in the existing literature. The micro foundation is borrowed from Weisdorf (2007), who shows that the Malthusian model conforms nicely to industrial development when the income-effect on the demand for children is removed (i.e. is set to zero). Learning-by-doing mechanisms, invoked in the current framework to trigger growth in productivity, are based on Matsuyama (1992) and Strulik (1997). Yet, the analogy to the R&D-driven growth literature based on Romer (1990) and Jones (1995) is also clearly visible.[1]

Employing a two-sector (dual economy) framework (with agricultural and industry), economic growth in the context of the present model derives from two sources. One is structural transformation in the form of labor being transferred from agriculture to industry. More specifically, agrarian productivity growth in conjunction with *Engel's law* (which states that the proportion of income spent on food falls as income rises) increases the share of labor allocated to industry and therefore raise the industrial sector's output without affecting the performance of the agricultural sector. The other source of growth derives from learning-by-doing in the production processes.

In brief, the development path proposed by the current theory can be described as follows. During early stages of development, the population size is relatively small; the share of labor employed in agriculture is relatively high; and standards of living are relatively close to subsistence. Due to economies-of-scale to population, learning-by-doing effects, to begin with, are relatively small. Hence, the agricultural sector's productivity growth is slow, yet faster than that of industry in which labor resources, and thus learning-by-doing effects, are even smaller.

Productivity growth in agriculture has two effects on development. On the one hand, higher productivity growth in agriculture *relative to industry* makes food, and therefore children, relatively less expensive; this raises fertility and gradually speeds up the rate of population growth. On the other hand, agricultural productivity growth, in combination with Engel's law, increases the share of labor allocated to industrial activities.

With economies-of-scale at work in both agriculture and industry, the transfer of labor out of agriculture gradually shifts the ratio of productivity growth in favor of industry. Sooner or later, therefore, the rate of the industrial sector's productivity growth surpasses that of agriculture. Subsequently, industrial goods are becoming affordable at relatively lower prices, implying that food, and therefore children, become relatively more expensive. Henceforth, fertility declines, and the rate of growth of population slows down, until population growth eventually (endogenously) comes to a halt.

[1]The current model can also be understood as a two-sector extension of Kremer's (1993) model on long-run human economic development, now capturing structural change and micro-founded population growth.

The current theory emphasizes two important features connected to the release of agricultural labor as a source of long-term economic growth. On the one hand, as the analysis below will clarify, a transfer of labor out of agriculture can persist for many (i.e. hundreds of) years before ultimately leading to substantial economic growth, a point often stressed by economic historians (e.g., Clark, 2007). As such, the industrial revolution in the present context is not a sudden break with the past, but a gradual outcome of long-standing processes taking place throughout the Malthusian era. On the other hand, as the analysis below will also establish, a slowdown of economic growth will eventually occur, as the beneficial effects of structural transformation gradually become exhausted.

The paper continues as follows. Section 2 describes the model, Section 3 explores dynamics and stability conditions, and Section 4 performs a quantitative analysis, highlighting the long-run development path predicted by the model. Finally, Section 5 concludes.

2. The Model

2.1. **Introduction.** Consider a closed economy with two sectors: an agricultural sector producing food, and an industrial (i.e. non-agricultural) sector producing manufactured goods. Economic activities extent over infinite (discrete) time. Unless explicitly noted, all variables are considered in period t.

We examine a two-period overlapping generations economy with childhood and adulthood. Productive and reproductive activities take place only during adulthood. For simplicity, reproduction is asexual, meaning that each individual is born to a single parent. Individuals are identical from every aspect, and each adult individual is endowed with one unit of labor which is supplied inelastically to work.

Change in the size of the labor force (i.e. the adult population) between any two periods is given by

$$L_{t+1} = n_t L_t, \tag{1}$$

where n_t is the gross rate of growth of population. As we abstract from mortality, n_t also measures the rate of fertility in period t.

2.2. **Preferences.** Individuals maximize utility derived from the amount of offspring that they have, n_t, and from the consumption of manufactured goods, measured by m_t.

The cause of the demographic transition in the current framework is to be found in the interaction between differential productivity growth and parental preferences. As will become apparent below, a zero income-elasticity on the demand for children implies that fertility responds differently to changes in productivity growth (which enters through the relative price of food), depending on whether they occur in agriculture or industry.

As is demonstrated by Weisdorf (2007), the use of quasi-linear preferences imply that an income-increase in agriculture increases fertility whereas income-increase in industry has the opposite effect. To obtain this result, the utility function of a representative individual is of a quasi-linear type whereby

$$u = \gamma \ln n_t + m_t, \tag{2}$$

with γ being the weight put on children.[2]

To obtain the budget constraint, suppose that over the course of a lifetime an individual consumes a fixed quantity of foods (i.e., calories), measured by η. For simplicity, food is demanded only during childhood and some of it then stored for adulthood.[3] The price of manufactured goods is set to one, and p_t denotes the price of food, measured in terms of manufactured goods (i.e. the relative price of food). By setting $\eta \equiv 1$, it follows that each child consumes one unit of food. This means that the total costs of raising n_t children, measured in terms of manufactured goods, is $p_t n_t$. The individual budget constraint thus writes

$$w_t = p_t n_t + m_t,$$

where w_t is the income of a representative individual, also measured in terms of manufactured goods.

The solution to the optimization problem implies that the demand for children is given by

$$n_t = \frac{\gamma}{p_t}. \tag{3}$$

Consistent with Malthus' (1798) *preventive check hypothesis*, the price-effect on the demand for children is negative. Note, however, that there is no income-effect on fertility. Or to be

[2]Note that the use of quasi-linear preferences is not crucial to results obtained below. In fact, any utility function in which productivity (and therefore income) growth in agriculture and industry (in a general equilibrium) have opposite effects on the demand for children, will provide the same qualitative results as we obtain below. However, the use of the quasi-linear preferences makes the model extremely tractable.

[3]It would not affect the qualitative nature of the results, if, instead, the individual's food demand where to be divided over two periods. However, such a construction would complicate matters severely.

more exact, there is no *direct* income-effect. For, as will become apparent below, an *indirect* income-effect will ultimately enter through the relative price of food.

By comparison, fertility, in the existing literature, responds to a (direct) change in income (e.g. Kremer, 1993, Galor and Weil, 2000, and Jones, 2001). Here, instead, fertility responds to a change in the price of food. By contrast to existing studies, this means – when combining the micro and macro frameworks – that population growth in the current framework reacts to structural changes rather than changes in income per capita. In accordance with stylized historical facts proposed by Crafts (1996) and Voth (2003), therefore, the present work captures the idea that persistent (and rapidly shifting rates of) population growth went on during early phases of industrialization, where standards of living were almost stagnant but when substantial structural transformation took place.

2.3. **Production.** In line with existing unified growth theories, agricultural production is subject to constant returns to labor and land, the latter being measured by X. Land is in fixed supply, and the total amount is normalized to one (i.e. $X \equiv 1$).

Industrial production is subject to constant returns to labor. As is also common in the related literature, this implies that land is not an important factor in industrial production, and that we abstract from the use of capital in production in both sectors.

Inspired by Matsuyama (1992), new technology in each of the two sectors appears as a result of learning-by-doing. More specifically, output and new technology in the respective sectors are produced according to the following production functions:

$$Y_t^A = \mu A_t^{\varepsilon} {L_t^A}^{\alpha} = A_{t+1} - A_t, \quad 0 < \alpha, \varepsilon < 1 \tag{4}$$

$$Y_t^M = \delta M_t^{\phi} L_t^M = M_{t+1} - M_t, \quad 0 < \phi < 1. \tag{5}$$

The variable A_t is total factor productivity in agriculture (superscript A for agricultural goods), and the variable M_t measures total factor productivity in industry (superscript M for manufactured goods). With $0 < \varepsilon, \phi < 1$, there are decreasing returns to learning in both sectors.

Define $g_t^Z \equiv (Z_{t+1} - Z_t)/Z_t$ to be the net rate of growth of a variable Z between any two periods. Then it follows that the net rate of productivity growth in the agricultural sector is $g_t^A \equiv \mu A_t^{\varepsilon-1} \left(L_t^A\right)^{\alpha}$. Similarly, the net rate of productivity growth in industry is $g_t^M = \delta M_t^{\phi-1} L_t^M$.

2.4. **Equilibrium.** The variables L_t^A and L_t^M measure total labor input in agriculture and industry, respectively, and together fulfil the condition that

$$L_t^A + L_t^M = L_t. \tag{6}$$

The share of total labor devoted to agriculture is determined by the food market equilibrium condition. This condition says that total food supply, Y_t^A, equals total food demand, which (when each person child consumes one unit of food) is $n_t L_t$. Using (4), the food market equilibrium condition implies that the share of farmers to the entire labor force is given by

$$\theta_t \equiv \frac{L_t^A}{L_t} = \left(\frac{nL^{1-\alpha}}{\mu A_t^{\varepsilon}} \right)^{\frac{1}{\alpha}}. \tag{7}$$

Note that productivity growth in agriculture releases labor from agriculture, whereas population growth has the opposite effect.

Consistent with existing unified growth theories, we assume that there are no property rights over land, meaning that the land rent is zero. This implies that, in both sectors, labor receives the sector's average product. The labor market equilibrium condition implies that the relative price of food adjusts, so that farmers and manufacturers earn the same income, i.e. $w_t = p_t Y_t^A / L_t^A = Y_t^M / L_t^M$. By the use of (3)-(7), this means that the relative price of food is given by

$$p_t = \frac{\left(\delta M_t^{\phi}\right)^{\alpha} (\gamma L_t)^{1-\alpha}}{\mu A_t^{\varepsilon}}. \tag{8}$$

Note that the relative price of food increases with productivity growth in industry and the size of population. Productivity growth in agriculture, however, has the opposite effect.

Finally, a second use of (3) in (8) provides the rate of fertility in a general equilibrium, and is

$$n_t = \mu \left(\frac{\gamma}{\delta} \right)^{\alpha} \frac{A_t^{\varepsilon}}{M_t^{\alpha\phi} L_t^{1-\alpha}}. \tag{9}$$

It follows that productivity growth in agriculture increases fertility whereas productivity growth in industry and population growth has the opposite effect.

The simplest unified growth model is complete with equations (1) to (9).

3. Balanced and Unbalanced Growth in the Long-Run

Along a balanced growth path, all variables grow at constant rates (possibly zero). Balanced growth rates are identified by a missing time index. According to (4) and (5), a constant rate of productivity growth in each of the two sectors implies that

$$1 = (1 + g^A)^{\epsilon-1}(1 + g^L)^{\alpha} = (1 + g^M)^{\phi-1}(1 + g^L). \tag{10}$$

According to (9), the gross rate of fertility change is

$$\frac{n_{t+1}}{n_t} = \frac{(1 + g_t^A)^{\epsilon}}{(1 + g_t^M)^{\alpha\phi} n_t^{1-\alpha}}.$$

Multiplying by n_t and using the fact that $n_t = (1 + g_t^L)$ for all t we obtain the balanced rate of population growth.

$$(1 + g^L)^{1-\alpha} = \frac{(1 + g^A)^{\epsilon}}{(1 + g^M)^{\alpha\phi}}. \tag{11}$$

Equating (10) and (11), and eliminating g_t^L, it is evident that productivity growth, along a balanced growth path, must fulfill the condition that

$$1 + g^A = (1 + g^M)^{1+\phi\alpha-\phi}. \tag{12}$$

Insert (12) into (11) to get the balanced growth condition between population growth and productivity growth.

$$(1 + g^L)^{1-\alpha} = (1 + g^M)^{\frac{\epsilon(1+\phi\alpha-\phi)}{\alpha\phi}}. \tag{13}$$

Finally insert (13) into (10) to get the condition for population growth along the balanced growth path.

$$(1 + g^L)^{(1-\alpha)(\phi-1)\alpha\phi} = (1 + g^L)^{\epsilon(1+\phi\alpha-\phi)}. \tag{14}$$

For non-zero population growth it has to be true that $(1 - \alpha)(\phi - 1)\alpha\phi = \epsilon(1 + \phi(1 - \alpha))$. But since all involved parameters are larger than zero and smaller than one, the left hand side of the condition is negative and the right hand side is positive. The condition is never fulfilled i.e. there is no population growth along a balanced growth path. The only value that fulfills (14) is $g^L = 0$. Plugging this into (10), we see that $g^A = g^M = 0$ along the balanced path. The economy stagnates along the unique balanced growth path.

This is a noteworthy result. Since positive population growth cannot persist forever (because of a limited space), other long-run growth theories, such as Jones (2001), have to impose the condition that population growth is zero over the long run (which ultimately leads to a zero rate of growth of productivity). In the present context, by contrast, a stationary population level over the long-run occurs endogenously. More specifically, population growth in the current model "digs its own grave" through the feedback-effect of productivity. The reason is that increasing productivity in manufacturing increases the relative costs of children. This slows down the growth of population, thus decelerating productivity growth for the subsequent generation. Eventually, therefore, the economy endogenously converges to a state of zero growth in productivity and population.

While a *balanced* growth path involves stagnant levels of population and income, an *unbalanced* growth path, characterized by imploding or exploding growth, may in principle exist. In the following, we explore the two cases of unbalanced growth, starting with the case of imploding growth. Imploding growth implies perpetually negative population growth, i.e. n_t is smaller than one and L_t is decreasing. It is easy to see that imploding growth is not an option since g_t^A and g_t^M are bound to be non-negative. There is no forgetting-by-doing. With $\lim_{L \to 0} g^A = 0$ and $\lim_{L \to 0} g^M = 0$, we have $\lim_{L \to 0} n = const./L^{1-\alpha}$ from (9). As L_t converges to zero, n_t goes to infinity. A contradiction to the initial assumption of n_t being smaller than one. There is no imploding growth. Intuitively, decreasing marginal returns of labor in agriculture ($\alpha < 1$) prevent implosion. As population size decreases agricultural productivity goes up and prices go down so that fertility and thus next period's population increases.

As for the case of explosive growth, this implies that $g_t^A > g_t^M \geq 0$. In this case, the relative price of food ultimately goes to zero, and fertility to infinity. Since, in this case, productivity growth is faster in agriculture than in industry, the rate of growth of population converges to the rate of growth of productivity in agriculture, so that ultimately $g_t^L = g_t^A$. Hence, for the relative price of food to reach zero, the denominator on the right hand side of (8) must increase faster than the numerator, meaning that $(1+g_t^A)^\epsilon > (1+g_t^M)^{\phi\alpha}(1+g_t^L)^{1-\alpha}$. Using the fact that $g_t^L = g_t^A$, the following condition must therefore hold for unbalanced explosive growth to exist:

$$(1 + g_t^M) < (1 + g_t^A)^{(\epsilon+\alpha-1)/(\alpha\phi)}. \tag{15}$$

With the parameter assumption made so far, this condition might indeed be fulfilled. In order to get rid of empirically unrealistic outcomes, therefore, explosive growth needs to be eliminated by the use of further parameter restrictions. With $g_t^A > g_t^M \geq 0$, the condition stated by (13) is generally not fulfilled when $\alpha + \epsilon - 1 < \alpha\phi$. In fact, it is never fulfilled when $\alpha + \epsilon < 1$, a necessary and sufficient condition, therefore, to rule out unbalanced growth.

The following proposition summarizes the considerations made above about balanced and unbalanced growth.

Proposition 1. *There exists a unique balanced growth path for the simplest unified growth model. It implies zero population growth and zero (exponential) economic growth. If $\alpha + \epsilon < 1$, there exists no unbalanced growth in the long-run.*

Intuitively, the restriction of decreasing returns with respect to knowledge and labor in agriculture (i.e. the assumption that $\alpha + \epsilon < 1$) eradicates the outcome in which population growth is permanently driving hyper-exponential growth in agriculture.

Even under the assumption that $\alpha + \epsilon < 1$, however, learning-by-doing effects may still be sufficiently strong to generate (i) a demographic transition (a fertility transition, strictly speaking); (ii) a structural transformation of the economy; and (iii) a (temporary) take-off of economic growth, i.e. an 'industrial revolution.' In the following, we thus explore the adjustment dynamics predicted by the model.

4. Adjustment Dynamics: Growth in the Middle Ages, Industrialization, and the Productivity Slowdown

In words, the model predicts the following adjustment path. As a starting point, consider a preindustrial, agricultural economy; that is, an economy in which the population level is relatively small; the share of labor employed in agriculture is relatively high; and the level of income per capita is relatively close to subsistence.

Due to economies-of-scale to population, learning-by-doing effects, to begin with, are relatively small. Hence, the agricultural sector's productivity growth is slow, yet faster than that of industry in which labor resources, and thus learning-by-doing effects, are even smaller.

Productivity growth in agriculture has two effects on development. On the one hand, higher productivity growth in agriculture *relative to industry* makes food, and therefore children, relatively less expensive; this raises fertility and gradually speeds up rates of population growth.

On the other hand, agricultural productivity growth, in combination with Engel's law, increases the share of labor allocated to industrial activities.

With economies-of-scale at work in both agriculture and industry, the transfer of labor out of agriculture gradually shifts the ratio of productivity growth in favor of industry. Sooner or later, therefore, the rate of the industrial sector's productivity growth surpasses that of agriculture. Subsequently, food, and therefore children, become relatively more expensive; fertility declines, and population growth slows down until it eventually comes to a halt.

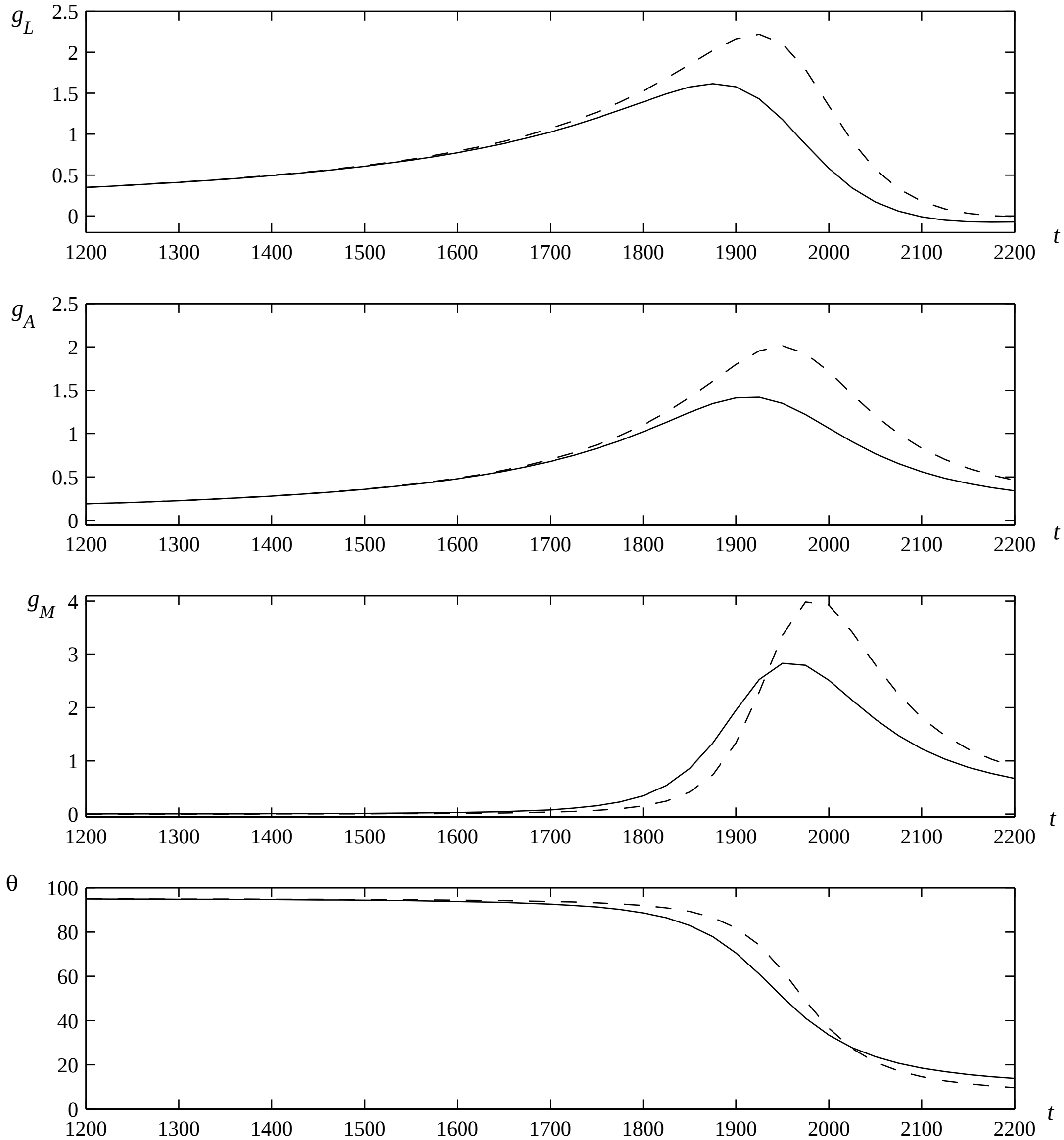
Note, from the inspection of (8), how the size of elasticities (i.e. ϵ and ϕ) shape the inverted u-path of fertility: The higher is ϵ, the stronger is the economic impact on fertility increase during the take-off phase of agriculture (the 'agricultural revolution'). On the other hand, the higher is ϕ, the more pronounced is the fertility decline (the demographic transition) during the take-off phase of industry (the 'industrial revolution').

In order to get a more precise picture of the adjustment dynamics, a calibration of the model is done below.[4] We calibrate the model such that it approximates the peak of the demographic transition in England and the subsequent slowdown of productivity growth. We set parameter values such that population growth reaches its peak of 1.5 percent annually in the year 1875, and such that total factor productivity (TFP) begins to slowdown in the year 1975 when it grows at a rate of 1.5 percent annually.

For the benchmark case, we set the following parameters: $\alpha = 0.8$, $\varepsilon = 0.45$, $\phi = 0.3$, $\mu = 0.5$, $\delta = 1.5$, and $\gamma = 1.5$. Start values are $\theta = L_A/L = 0.95$, $L = 0.1$, $A = 0.1$, $n = 1.08$. For better readability of the results, it is assumed that a generation takes 25 years (approximately the length of the fecundity period), and we then translate generational growth rates into annual ones.

[4] Since we are simulating growth trajectories for a very simple model it can be expected that our results will be inferior against large-scale CGE modelling approaches to the industrial revolution. These studies use multi-factor, multi-sector models taking many important determinants into account that we all ignore (for example, capital accumulation, education, international trade, and energy production). In particular, it can be expected that our model makes large approximation errors at the end of the transition paths when learning-by-doing effects level off and there is neither capital accumulation nor education to foster growth. Our challenge can thus not be to redraw the historical paths of before and after industrialization as accurately as possible. More humbly, our experiment is to explore how much of economic and demographic history can be explained with the simplest conceivable growth model. See Harley and Crafts (2000) and Stokey (2001) for large scale CGE modelling of the industrial revolution.

Figure 1: Long-Run Dynamics According to the Simplest Unified Growth Model



Solid lines: benchmark case, dashed lines: $\gamma = 2$, otherwise benchmark case. From top to bottom the diagrams show population growth, productivity growth in agriculture, productivity growth in manufacturing, and the labor share in agriculture (our measure of structural change).

Figure 1 demonstrates the adjustment path, running from the year 1200 on to the year 2200. The solid lines show the path of the benchmark economy. The dashed lines, by contrast, concerns an alternative economy to be discussed further below.

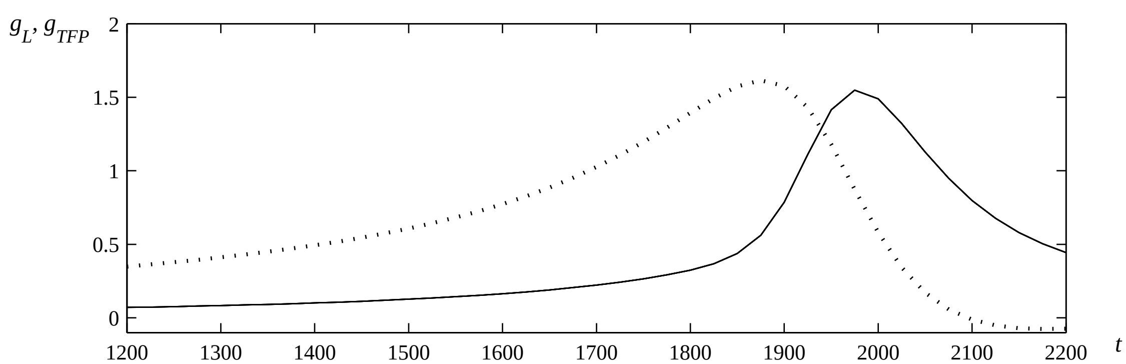
To begin with, nearly the entire labor force is allocated in agriculture. In line with numbers provided by Galor (2005), industrial productivity growth initially is almost completely absent, while productivity growth in agriculture is around 0.15 percent annually.

Agricultural progress, manifesting itself in a slowly decreasing relative price for food, is almost entirely converted into population growth. Due to diminishing returns to labor in agriculture, however, standards of living are hardly affected by the growth of productivity.

With economies-of-scale to population, population growth (in a Boserupian manner) furthers productivity growth in agriculture.[5] Over the subsequent years, therefore, agricultural productivity growth slowly builds up to reach 0.5 percent by the 18th century.

By then, agriculture progress makes possible a substantial transfer of labor into manufacturing, causing an upsurge in industrial productivity growth.[6] By contrast to agriculture, productivity growth in the industrial sector derives from two sources: population growth and a transfer of labor from agriculture. What's more, the fact that the industrial sector (unlike agriculture) is not exposed to diminishing returns to labor positively influences standards of living. From the turn of the 19th century onwards, therefore, economic growth gains momentum.

Figure 2: Long-Run Dynamics: Implied TFP Growth and Population Growth



Solid line: TFP growth, dotted line: population growth

Although productivity growth in industry is now gradually on the rise, its rate of growth during the 19th century is still lower than that in agriculture. Consequently, the relative price of food is still falling, causing further increase in fertility. Strikingly, although substantial structural

[5]See Boserup (1981) for a detailed exploration on how population growth drives agricultural progress.

[6]This outcome of the model is in line with the observation that the direct labor input to produce a ton of grain – while staying almost constant for a long time in history – declined by 70 percent in the 19th century (Johnson, 2002).

and demographic changes takes place in the 19th century, major improvements in standards of living, according to the model, do not appear before the 20th century.

Note that this phase of adjustment dynamics is well in tune with the demo-economic observations made in relation to the industrial revolution. As has been emphasized by economic historians such as Crafts (1996) and Voth (2003), a high rate of structural change and considerable rates of population growth are accompanied by relatively low rates of productivity growth.

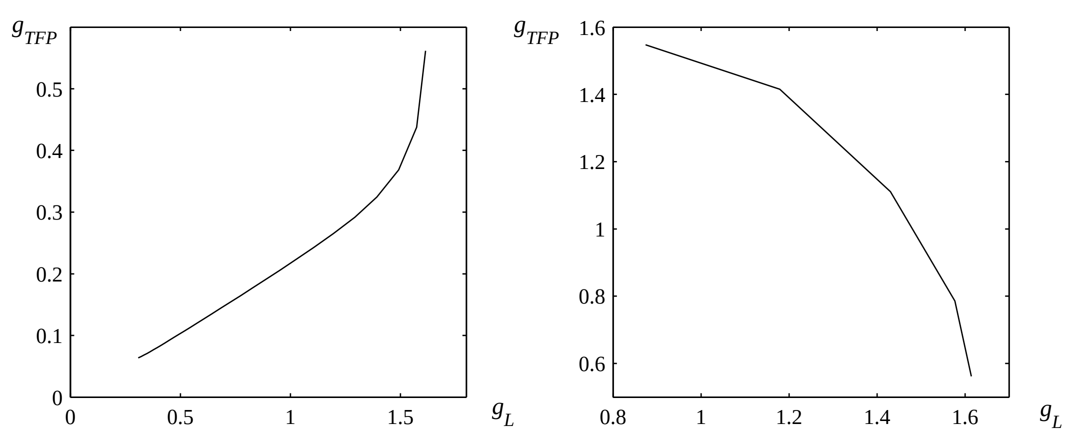
In order to keep track of the relationship between productivity growth on the one hand, and demographic growth on the other, implied total TFP growth is calculated using Domar weights. The implied TFP growth thus writes

$$g_t^{TFP} \equiv p_t \frac{Y_t^A}{Y_t} g_t^A + \frac{Y_t^M}{Y_t} g_t^M = p_t \theta \frac{y_t^A}{y_t} g_t^A + (1-\theta) \frac{y_t^M}{y_t} g_t^M,$$

where $Y = pY^A + Y^M$ measures GDP. Figure 2 combines the adjustment dynamics for implied TFP growth (the solid line) and population growth (the dotted line). As is evident from the illustration, the growth rate of population reaches its peak by the end of the 19th century whereas implied TFP growth peaks about one century later.

This result nicely fits ideas proposed by Galor and Weil (2000). According to their theory, the drop in fertility (the demographic transition) does not emerge before the rate of TFP growth reaches a substantial level.

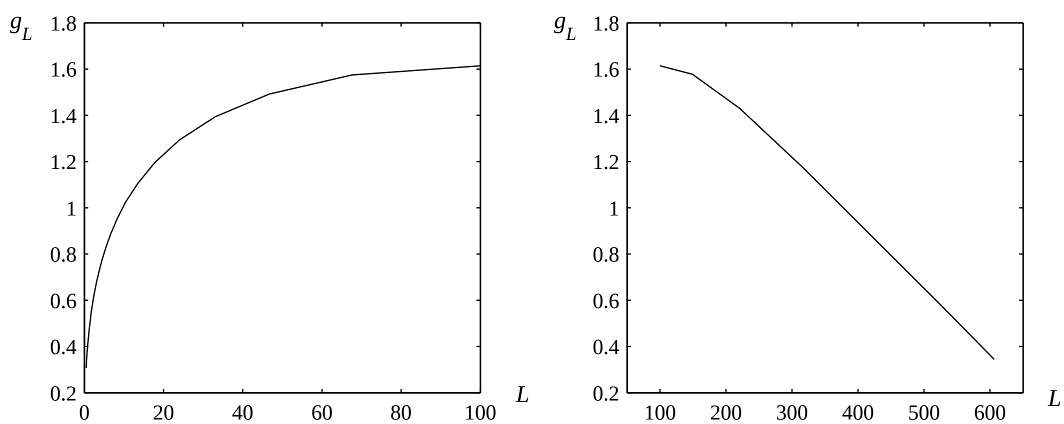
Figure 3: Population-TFP-Nexus:
From 1200 BC to Year 1875 (left) and from 1875 to 1975 (right)



In fact, the current model predicts a structural break for the correlation between TFP growth and population growth. As is evident from Figure 3, the correlation is positive before the year 1875, i.e. during the Malthusian phase, when TFP growth is relatively slow, and the rate of population growth is on the rise. By 1875, the rate of growth of population reaches its peak, and then starts to decline. However, TFP growth, at least for a period of time, continues to rise, indicating a negative correlation between TFP and population growth after 1875.

Empirically, this is not an unfamiliar result. For the past century, Bernanke and Guerkaynak (2002) provide supporting evidence for the negative correlation between TFP and population growth. For the preindustrial period, by contrast, a positive relationship between TFP and population growth is well-documented by Clark (2007).[7]

Figure 4: Population-Size and Population Growth:
From One Million B.C. to Year 1875 (left) and from 1875 to 2000 (right)



The model's prediction concerning the relationship between population size and population growth is also noteworthy. Using a one-sector variant of the current model, but with an exogenously imposed population growth, Kremer (1993) establishes a remarkable positive correlation between the size and growth of population, a phenomenon unseen for any other species than humans. His result is supported empirically, though only weakly for the last data points concerning the 20th century (cf. his Figure 1).

[7]Note, when inspecting the right panel of Figure 3, that the process of development starts at the lower right corner, when population growth is at its highest and TFP growth is relatively weak. It ends in the year 1975, when TFP growth reaches its maximum and population growth is relatively slow. After 1975, a third phase, in which both population growth and TFP growth decline, can be observed, once again suggesting a (modest) positive correlation.

Repeating Kremer's exercise using the present model, a structural break is generated in the year 1875. From one million BC to 1875, the correlation is positive as suggested by Kremer. The slope is non-linear and the best fit of the correlation is obtained for the function $g_L = const \cdot L^{0.2}$. This suggests that the positive correlation was particularly strong during early times, and then became flatter as the population growth rate reaches its peak. After 1875, for industrialized and fully-developed countries, the correlation turns negative, and, indeed, becomes almost linear.

Finally, an analysis of variation in the taste for children provides insight into the understanding the differences in the timing of the industrial revolution and the demographic transition across countries. Compared to the benchmark case (solid lines), the dashed lines in Figure 1 capture the adjustment dynamics of an alternative economy.

In the alternative economy, everything is identical to the benchmark case, except for γ which is set to 2 instead of 1.5. All other things being equal, parents in the alternative economy hence display a stronger taste for children than parents of the benchmark economy.

As has been demonstrated by Strulik (2007), population growth, on average, peaks at higher rates in countries of lower latitude. Following Strulik, therefore, the dashed lines of Figure 1 would capture a country situated closer to the equator than England.

Having a relatively strong taste for children involves parents having a relatively large demand for food. In the alternative economy, therefore, agriculture will dominate for more years than in the benchmark economy. Hence, when the industrial revolution sets off in the benchmark case, industrial productivity growth in the alternative economy is still relatively slow.

In terms of Figure 1, the 'industrial revolution' (i.e. the transfer of labor from agriculture to industry) and the fertility drop in the alternative economy both start about 50 years (or two generations) later than in the benchmark case. However, once structural changes begin to emerge in the alternative economy, industrial productivity growth is faster than in the benchmark case. The reason, of course, is that the alternative economy's population level, and thus its learning-by-doing effects, are larger because of the stronger taste for children.

In terms used by Bloom et al. (2001), the alternative economy ultimately earns its 'demographic dividend,' and it eventually draws nearer to the benchmark economy because of a higher growth rate of productivity and a faster rate of structural change. Effectively, a catching up effect is therefore at play.

5. Conclusion

This paper provides a unified growth theory, i.e. a model consistent with the very long-run economic and demographic development path of industrialized economies, stretching from the Malthusian era to present-day and beyond. Making strict use of Malthus' preventive check hypothesis – that fertility rates vary inversely with the price of food – the current study offers a new and straightforward explanation for the demographic transition and the break with the Malthusian era.

Existing unified growth theories focus on human capital accumulation and a trade-off of child-quantity for child-quality as the driving force behind the demographic transition. The present study offers an alternative explanation, which nicely complements the ideas proposed in the existing literature, especially Galor and Weil (2000) and Kremer (1993), and, in addition, is well in line with historical evidence about structural transformation.

Putting the current theory to the test means testing if ratio of agricultural to industrial productivity growth moves in the directions predicted by the model, keeping in mind that the theory considers a closed economy, and that, during industrialization, countries typically become open up to foreign trade.

References

Allen, R.C., 2000, Economic structure and agricultural productivity in Europe, 1300-1800, *European Review of Economic History* 1: 1-25

Bernanke, B.S. and R.S. Guerkaynak, 2001, Is growth exogenous? Taking Mankiw, Romer, and Weil seriously, *NBER Macroeconomics Annual 16*, 11-57

Bloom, D.E., D. Canning, and J. Sevilla, 2001, Economic Growth and the Demographic Transition, *NBER Working Paper* 8685.

Boserup, E., 1981, *Population and Technological Change. A Study of Long-term Trends*, Chicago University Press, Chicago.

Boucekkine, R., D.de la Croix and O. Licandro, 2002, Vintage Human Capital, Demographic Trends, and Endogenous Growth, *Journal of Economic Theory* 104: 340-375

Clark, G., 2007, *A Farewell to Alms: A Brief Economic History of the World*, Princeton University Press, Princeton.

Crafts, N.F.R., 1996, The first industrial revolution: A guided tour for growth economists, *American Economic Review* 86(2), 197-201.

Doepke, M., 2004, Accounting for Fertility Decline During the Transition to Growth. *Journal of Economic Growth* 9: 347–383

Galor, O., 2005, From stagnation to growth: unified growth theory, in: P. Aghion and S. Durlauf, *Handbook of Economic Growth* Vol 1A, Amsterdam, North-Holland.

Galor, O. and D.N. Weil, 2000, Population,Technology and Growth: From the Malthusian Regime to the Demographic Transition and Beyond, *American Economic Review* 110: 806–828

Harley, C.K. and N.R.R. Crafts, 2000, Simulating the two views of the British industrial revolution, *Journal of Economic History* 60, 819-841.

Johnson, D.G., 2002, Population, food, and knowledge, *American Economic Review* 92, 1-14.

Jones, C.I., 1995, R&D-based model of economic growth, *Journal of Political Economy* 103, 759-84.

Jones, C.I., 2001, Was an industrial revolution inevitable? Economic growth over the very long run, *Advances in Macroeconomics* 1(2), 1-43.

Kögel, T. and Prskawetz, A., 2001, Agricultural productivity growth and escape from the Malthusian trap, *Journal of Economic Growth* 6, 337-357.

Kremer, M, 1993, Population growth and technological change: one million B.C. to 1990, *Quarterly Journal of Economics* 108, 681-716.

Lucas, R.E., 2002, *The Industrial Revolution: Past and Future*, Cambridge: Harvard University Press.

Malthus, T., 1798, *An Essay on the Principle of Population,* London, printed for J. Johnson.

Matsuyama, K., 1992, Agricultural Productivity, Comparative Advantage, and Economic Growth, *Journal of Economic Theory* 58, 317-334.

Mitchell, R.B., 1998, *International Historical Statistics: Europe 1750-199*3, London: MacMillan

Stokey, N., 2001, A quantitative model of the British industrial revolution, 1780-1850, *Carnegie Rochester Conference Series on Public Policy* 55, 55-109.

Strulik, H., 1997, Learning-by-doing, population pressure, and the theory of demographic transition, *Journal of Population Economics* 10, 285-98.

Strulik, H., 2003, Mortality, the Trade-off between Child Quality and Quantity, and Demo-economic Development, *Metroeconomica* 54: 499-520.

Strulik, H., 2007, Geography,Health, and the Pace of Demo-Economic Development, *Journal of Development Economics, 2007*, forthcoming.

Tamura, R., 2002, Human capital and the switch from agriculture to industry, *Journal of Economic Dynamics and Control* 27: 207-242.

Voth, H.-J., 2003, Living standards during the industrial revolution: An economist's guide, *American Economic Review* 93(2), 221-226.

Weisdorf, J.L., 2007, Malthus Revisited: Fertility Decision Making based on Quasi-Linear Preferences, *Economics Letters* (forthcoming)

Wrigley E. A., and R.S. Schofield, 1989, *The Population History of England 1541-1871*, London, Edward Arnold.