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Ordinal comparison of multidimensional deprivation: theory and application

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Abstract

This paper develops an ordinal method of comparison of multidimensional inequality. In our model, population distribution g is more unequal than f when the distributions have common median and g can be obtained from f by one or more shifts in population density that increase inequality. For our benchmark 2x2 case (i.e. the case of two binary outcome variables), we derive an empirical method for making inequality comparisons. As an illustration, we apply the model to childhood poverty in Mozambique.

JEL classification: D63, I32, O15

Keywords: Qualitative data, multidimensional first order dominance, multidimensional inequality, ordinal comparison

1 Introduction

The primary aim of this paper is to contribute to the development of ordinal concepts of multidimensional inequality. We extend the Allison and

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Foster (2004) framework for assessing inequality with one-dimensional categorical ordinal data (such as self-assessed health status). We assume that well-being is measured in several dimensions (attributes) where outcomes are categorical and can be ranked according to their desirability along each dimension. There is no numerical scale with cardinal properties associated with the categories. Only ordinal information about the desirability of outcomes is available. Briefly, we say that population distribution f is more unequal than population distribution g, if the distributions have common median and f can be obtained from g by one or more shifts in population density that all increase inequality.

We highlight our model is ordinal and differs in this respect from existing models of multidimensional inequality. Take the well-known concentration index.[1] It is a measure of two-dimensional inequality, and is a variant of the Gini index. Data on well-being along one dimension (say income) is used to rank individuals, so in this dimension only ordinal information is required. At the same time, data on well-being along the second dimension (say individual health status) is measured on a cardinal scale. Gravel and Moyes (2006) and Gravel, Moyes, and Tarroux (2008) develop another notion of two-dimensional inequality where one of these is cardinally measurable. For discussions of other (cardinal) multidimensional inequality measures, such as the various multidimensional generalizations of the Gini index and the Atkinson-Kolm-Sen approach, we refer to the surveys by Maasoumi (1999) and Weymark (2006).

In Section 2 we motivate, illustrate and provide intuition, while Section 3 contains general definitions and a comparison of our approach with that of Allison and Foster (2004). Section 4 addresses the empirically important 2x2 case (i.e., the case of two binary outcome variables), and we develop a procedure for detecting inequality in practice. The test for inequality boils down to comparisons of medians plus verification of a system of inequalities (the exact system depending on the location of the median). The test requires straightforward computations and can be carried out using a spreadsheet.

[1] See for example Wagstaff et al. (1989, 1991) and Kakwani et al. (1997) for studies of socioeconomic health inequality using this method.

In Section 5 we show how our model can be applied to analyze deterioration and dispersion in multidimensional deprivation. Micro data on childhood poverty in Mozambique is used, and we rely on a bootstrapping method for statistical analysis of sample data. Section 6 concludes and identifies future research needs.

2 An ordinal approach to multidimensional inequality: illustration and intuition

Suppose a person's well-being can be measured using two 0-1 binary variables, so there are four possible outcomes. Let $(0,0)$ denote the outcome where both variables take the value 0, $(1,0)$ the outcome where the first variable takes the value 1 and the second variable the value 0, and so on. In the figure below arrows point to better adjacent outcomes.

$$\begin{array}{ccc} (0,0) & \rightarrow & (1,0) \\ \downarrow & & \downarrow \\ (0,1) & \rightarrow & (1,1) \end{array}$$

Outcome $(0,0)$ is the worst and $(1,1)$ is the best. We assume it is unknown which of the two intermediate outcomes $(0,1)$ and $(1,0)$ is better. A population is characterized by how people are distributed among the four outcomes. This can be illustrated as follows:

$$f: \quad \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & \frac{2}{16} & \frac{4}{16} \\ 1 & \frac{4}{16} & \frac{6}{16} \end{array}$$

where $\frac{2}{16}$ of the population has $(0,0)$, $\frac{4}{16}$ has $(0,1)$ and $(1,0)$ respectively, and $\frac{6}{16}$ $(1,1)$. Call this distribution f, and compare with distribution g:

$$g: \quad \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & \frac{4}{16} & \frac{2}{16} \\ 1 & \frac{2}{16} & \frac{8}{16} \end{array}$$

Note that both f and g have a median value of 1 in each of the two dimensions, and that g can be obtained from f by moving density of an amount of $\frac{1}{8}$ from outcome $(0,1)$ to outcome $(0,0)$ and by moving a similar amount of density from $(1,0)$ to $(1,1)$. In other words, g can be obtained from f by a *correlation-increasing switch* (Epstein and Tanny 1980, Boland and Proschan 1988).[2] Intuitively, g is obtained from f by a balanced movement of density from the two intermediate outcomes to the two extremes. It is reasonable to say that g is more unequal than f. The marginal distributions are the same for both distributions, and if a person experiences a bad outcome in one of the dimensions at g, the conditional probability that the other outcome is also bad is higher for g than for f.

In practice, it is very unlikely to find two population distributions where one can be obtained from the other by a correlation-increasing switch. This would require that the difference in density between the two distributions for the outcome $(0,0)$ is exactly equal to the corresponding difference for the outcome $(1,1)$. Unless the populations (or number of observations) underlying the two distributions are extremely small this is only going to happen in exceptional cases.

However, let us consider distribution h:

$$h: \quad \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & \frac{4}{16} & \frac{2}{16} \\ 1 & \frac{3}{16} & \frac{7}{16} \end{array}$$

where $\frac{4}{16}$ of the population has $(0,0)$, $\frac{3}{16}$ $(1,0)$, $\frac{2}{16}$ $(0,1)$, and $\frac{7}{16}$ $(1,1)$. Note that f, g and h have the same median outcome $(1,1)$. Obviously, h cannot be obtained from g or f by a correlation-increasing switch. Distribution h can be obtained from g by moving population density amounting to $\frac{1}{16}$ from outcome $(1,1)$ to $(1,0)$. We regard the common two-dimensional median as reference outcome,[3] and say that h was obtained from g by a *median-*

[2] The notion of a *correlation-increasing switch* has been discussed by Atkinson and Bourguignon (1982), Tsui (1999), Atkinson (2003), Bourguignon and Chakravarty (2003), and others, in the context of multidimensional inequality and poverty measurement.

[3] The multidimensional median is the vector of coordinate-wise medians.

preserving spread (Allison and Foster, 2004). Intuitively, h is more unequal than g. It is obtained from g by moving density away from their common median. Note that a median-preserving spread is a switch of density from one outcome to another, which may or may not involve movement of density away from an intermediate outcome, depending on the location of the median.

Accordingly, we will say that a distribution is *ordinally more unequal* than another if it is possible to obtain the first distribution from the second through a sequence of median-preserving spreads and/or correlation-increasing switches. For instance, in our example, h is ordinally more unequal than f since there exists a distribution g, such that g can be obtained from f through a correlation-increasing switch and h can be obtained from g through a median-preserving spread.

In Section 3, we proceed to a formal definition of our ordinal inequality concept in a general N attribute context. An obstacle in empirical implementation is that it is difficult to check if a given distribution is more unequal than another. Thus, in subsequent sections we focus on the empirically important 2x2 case.

3 General formulation

An *outcome* is a vector $x = (x_1, ..., x_N)$ described by N attributes, x_j, $j = 1, ..., N$, where each attribute is defined on an attribute set $X_j = \{1, ..., n_j\}$. The set of outcomes to be considered is the product set $X = X_1 \times \cdots \times X_N$ of the attribute sets X_j.

The statement $x \leq y$ will mean that $x_j \leq y_j$ for all j, and $x < y$ will mean that $x_j \leq y_j$ for all j and $x \neq y$.

A *pseudo-distribution* is a real-valued function f on X with $\sum_{x \in X} f(x) = 1$. A pseudo-distribution f is a *distribution* if $f(x) \geq 0$ for all $x \in X$. Let f_j denote the marginal distribution on X_j.

Let $m_j(f_j)$ denote the median of f_j on X_j. The (multidimensional) median of f is the vector $m(f) = (m_1(f_1), ..., m_N(f_N))$, of N coordinate-wise medians.

We say that g can be derived from f by a *bilateral transfer* (of density), if there are outcomes x, y such that $g(z) = f(z)$ for $z \neq x, y$. It is *diminishing* if $y \leq x$ and $g(x) \leq f(x)$. It is *median-preserving* if $m(f) = m(g)$, and for an outcome z it is z-*directed* if $f(x) \leq g(x)$ and $x < y \leq z$ or $z \leq y < x$. It is *inequality-increasing* if it is both median-preserving and median-directed.

Distribution g *first order dominates* distribution f if g can be derived from f by a finite sequence of diminishing bilateral transfers, i.e., if there are distributions $f = f_1, f_2, ..., f_k = g$, where f_{i+1} is obtained from f_i by a diminishing bilateral transfer $i = 1, ..., k-1$.[4]

For a pair of outcomes x, y let $\max(x, y)$ denote the outcome where the ith attribute is $\max\{x_i, y_i\}$, and let $\min(x, y)$ be the outcome where the ith attribute is $\min\{x_i, y_i\}$. Let f and g be two distributions for which there are outcomes x, y, v, w such that $g(z) = f(z)$ for $z \neq x, y, v, w$. We say that g is derived from f by a *correlation-increasing switch* if we can choose x, y, v, w such that $v = \min(x, y)$ and $w = \max(x, y)$, $f(x) - g(x) = f(y) - g(y) > 0, f(v) - g(v) = f(w) - g(w) < 0$.

If g can be derived from f by a finite sequence of median-preserving inequality-increasing bilateral transfers and correlation-increasing switches, we say that g is *ordinally more unequal* than f, or, as an equivalent statement, f is *ordinally more equal* than g. Formally, g is ordinally more unequal than f if there are distributions $f = f_1, f_2, ..., f_k = g$, where f_{i+1} is obtained from f_i by a correlation-increasing switch or a median-preserving inequality-increasing bilateral transfer, $i = 1, ..., k-1$.

Before proceeding, compare these definitions and concepts with the one-dimensional case pursued by Allison and Foster (2004). For $N = 1$, $X = X_1 = \{1, ..., n_1\}$, $f = f_1$ and $g = g_1$, define $F(k) = \sum_{j=1,..,k} f(j)$ and G in a similar way. Allison and Foster (2004) say g has a *greater spread* than f whenever $m(g) = m(f)$ and $G(k) \geq F(k)$ for all $k < m(f)$ and $G(k) \leq F(k)$ for all $k \geq m(f)$. For $N = 1$, g has greater spread than f precisely if g is more unequal than f (as defined here). Thus, the present model is a

[4] For general references on stochastic dominance, see, e.g., Shaked and Shanthikumar (1994) or Müller and Stoyan (2002).

generalization of Allison and Foster's one-dimensional case.

Our 'more unequal' relation is based on the concept of median-preserving inequality-increasing spreads. A central question is how to test if one distribution is 'more unequal' than another. For the one-dimensional case, empirical implementation is straightforward: For two one-dimensional distributions f and g, testing that g is more unequal (i.e. has greater spread) than f is a matter of checking whether n_1 inequalities hold.[5] For the multi-dimensional case, empirical implementation is non-trivial. The next section solves the 2x2 case.

4 The 2x2 case

In this section, we assume that an outcome is a vector $x = (x_1, x_2)$ described by two attributes, where each attribute x_j is defined on an attribute set $X_j = \{0, 1\}$, $j = 1, 2$. Thus, the outcome set is the product set $X = \{0, 1\} \times \{0, 1\}$. For an outcome $x = (x_1, x_2)$ we use the notation $f(x_1, x_2)$ for $f(x)$.

4.1 Finding first order dominance relations

Let f and g denote distributions on X. By a general result on multidimensional first order dominance,[6] we get that f first order dominates g if and only if the following four inequalities are satisfied: $g(0,0) \geq f(0,0), g(0,0) + g(0,1) \geq f(0,0) + f(0,1), g(0,0) + g(1,0) \geq f(0,0) + f(1,0)$, and $g(0,0) + g(1,0) + g(0,1) \geq f(0,0) + f(1,0) + f(0,1)$.

Alternatively, we can define the function $D(f, g) = \min\{g(0,0) - f(0,0)$, $g(0,0) + g(0,1) - f(0,0) - f(0,1)$, $g(0,0) + g(1,0) - f(0,0) - f(1,0)$, $g(0,0) + g(1,0) + g(0,1) - f(0,0) - f(1,0) - f(0,1)\}$; then $D(f, g) \geq 0$ if and only if f first order dominated g. The function D is useful for testing statistical significance of first order dominance relations.

[5] See Allison and Foster (2004) for a detailed discussion of how this test can be nicely visualized.

[6] Cf. Lehmann (1955) and Theorem 1 in Kamae, Krenge and O'Brien (1977).

4.2 Finding inequality relations

We proceed next to present necessary and sufficient conditions for f being ordinally more equal than g.

Correlation-increasing switches are median-preserving, so a necessary condition for the statement 'f is more equal than g' to be true is that the two distributions have the same median.[7] We can therefore rely on considering in turn each of four possible cases of common median, and proceed as described below.

Proposition 1 *Let $X = \{0,1\} \times \{0,1\}$ and let f and g be two distributions on X. Then g is more unequal than f if and only if one of the following cases holds:*

Case 1a: $m(f) = m(g) = (1,1)$, *and* $g(0,0) \geq f(0,0), g(0,0) + g(0,1) \geq f(0,0)+f(0,1), g(0,0)+g(1,0) \geq f(0,0)+f(1,0)$, $g(0,0)+g(1,0)+g(0,1) \geq f(0,0)+f(1,0)+f(0,1)$.

Case 1b: $m(f) = m(g) = (1,1)$, *and* $f(1,0) - g(1,0) \geq 0, f(0,1) - g(0,1) \geq 0$, $g(1,1) - f(1,1) \leq \min\{f(1,0) - g(1,0), f(0,1) - g(0,1)\}$.

Case 2: $m(f) = m(g) = (1,0)$, *and* $g(1,0) \leq f(1,0)$, $g(0,1) \leq f(0,1)$, $g(1,1) \geq f(1,1)$, $g(0,0) \geq f(0,0)$, $f(0,1)-g(0,1) \leq \min\{g(1,1)-f(1,1), g(0,0)-f(0,0)\}$, $f(1,0) - g(1,0) \geq f(0,1) - g(0,1)$.

Case 3: $m(f) = m(g) = (0,1)$, *and* $g(0,1) \leq f(0,1)$, $g(1,0) \leq f(1,0)$, $g(1,1) \geq f(1,1)$, $g(0,0) \geq f(0,0)$, $f(1,0)-g(1,0) \leq \min\{g(1,1)-f(1,1), g(0,0)-f(0,0)\}$, $f(0,1) - g(0,1) \geq f(1,0) - g(1,0)$.

Case 4a: $m(f) = m(g) = (0,0)$, *and* $f(0,0) \geq g(0,0), f(0,0) + f(0,1) \geq g(0,0)+g(0,1), f(0,0)+f(1,0) \geq g(0,0)+g(1,0)$, $f(0,0)+f(1,0)+f(0,1) \geq g(0,0) + g(1,0) + g(0,1)$.

Case 4b: $m(f) = m(g) = (0,0)$, *and* $f(1,0) - g(1,0) \geq 0, f(0,1) - g(0,1) \geq 0$, $g(0,0) - f(0,0) \leq \min\{f(1,0) - g(1,0), f(0,1) - g(0,1)\}$.

[7] Suppose that g is derived from f by some correlation-increasing switch. The correlation-increasing switch is *non-trivial* if $f \neq g$. For the case $X = \{0,1\} \times \{0,1\}$, any non-trivial correlation-increasing switch can be conducted by means of two bilateral transfers (of the same amount of mass) from $(0,1)$ and $(1,0)$ to the extreme outcomes $(0,0)$ and $(1,1)$.

The proof of Proposition 1 is given in Appendix A. Note that the inequalities in Case 1a mean that f first order dominates g, and those in 4a that g first order dominates f. The intuitive meaning and the derivation of the inequalities in the other cases are discussed in the Appendix.

5 Empirical illustration

In Mozambique as in most other developing countries, rising incomes and investment in schooling, health, and sanitation have increased the level of human capital and indices of human development. While this development has impacted on living-standards of both adults and children, its impact on children is of particular interest. The acquisition of human capital in early childhood is imperative for future learning, earnings and health status (UNICEF 2006). Moreover, large gaps persist between rich and poor, between rural and urban areas and between boys and girls. This tends to widen the variation in acquisition of human capital, productivity and living standards as summarized in surveys by Strauss and Thomas (1995) and Orazem and King (2007).

To address the above challenges some governments have initiated voucher or cash transfer programmes targeted at disadvantaged children.[8] A general problem with such government transfer programmes is to make sure that transfers are directed at the most disadvantaged children. Judging by observable characteristics, such as gender or household characteristics, some children may not appear to be disadvantaged (to the administrator) while in fact they are. These children are particularly vulnerable. Their group characteristics hide their true state, and the opposite may also be true. Avoiding inefficient loss of resources in transfer programmes, requires that governments can recognize people with high dispersion in well-being indicators and have at their disposal a set of analytical tools to help guide such allocations. Our theoretical framework is such a tool and in this sec-

[8] The most famous of these initiatives is probably Mexico's PROGRESA/Oportunidades programme, which aims at increasing children's school attendance among poor families, by awarding grants to mothers conditional on school enrolment. See Parker, Rubalcava and Teruel (2007) for further discussion and examples.

tion we illustrate how it can be applied in examining the living-standards of Mozambican children.

To measure living standard we use a set of seven UNICEF well-being indicators, the so-called Bristol Indicators, including severe childhood deprivations related to nutrition, water, sanitation, health care, shelter, education and information. Appendix B lists underlying definitions. To illustrate our 2x2 ordinal inequality concept we combine these seven indicators into two-dimensional measures. Such combinations are arbitrary so we chose, initially, to form all possible combinations – 19 in all. Some indicators exclude others. The nutrition and health indicators are only for pre-school children. This excludes associations with the education and information indicators. Among the 19 possible combinations, we focus in what follows on three combinations which illustrate well the key characteristics of our theoretical approach.[9]

Several household- and child characteristics have in the past been used to target government transfer programmes to poor and disadvantaged groups of households in developing countries. Here, we include three such characteristics, rural-urban area of residence, gender of head of household, and gender of the child.[10] This results in a total of eight categories of children, which will be compared with each other. If any of these groups has lower standards of living or is more vulnerable than others, then characteristics of that group should be considered as a particularly important criterion for targeting government transfers.

[9] Results for the remaining pair-wise comparisons are available from the authors upon request.

[10] Urban-rural area of residence is likely to have a significant impact on living standards mainly due to the low population density of rural areas, which makes supply of high quality public services more costly. Children living in female headed households are more likely than other children to fall below the poverty line primarily because woman's wages and education tend to be lower than men's . Buvinić and Gupta (1997) review literature relating female headship and poverty. However, as Handa (1996) observes, female headed households are also likely to spend a larger share of their income on improving children's human capital. Finally, households may discriminate based on gender of the child. For example in Mozambique, it is not uncommon for especially rural families to invest more in the education of boys as compared to girls (UNICEF, 2006).

5.1 Data

Data for our empirical analysis was obtained from the Mozambican Demographics and Health Survey (DHS 2003).[11] Data was collected by the National Institute of Statistics (INE) and the Ministry of Health (MISAU), and the survey sample is representative at national and provincial level and by area of residence (urban/rural). The purpose of the survey was to provide information on health and nutrition, in particular on fertility and on maternal- and child health. Focus was on women and children welfare and respondents were all females between 15 and 49 in the household. Data includes information such as vaccinations, childhood illness (diarrhoea and respiratory infections), nutritional status of children, access to water, sanitation facilities, housing, possessions, education and school attendance, employment, and wealth quintiles.

The survey was sampled based on results of the 2002/3 Mozambican National Household Survey (IAF 2002/3), and the sample strategy resulted in a stratified random sample collected in three stages. In the first stage, Primary Sampling Units (UPA's) or clusters were selected. This was followed by the identification of the Areas of Enumeration (AE) in each UPA, and in the third stage a total of 24 households in each AE were interviewed. All households were asked to answer the questionnaire on household characteristics and in households with young children (less than five years old) additional health data was collected on immunization coverage, vitamin A supplementation, treatment of childhood diseases, and recent occurrences of diarrhoea, fever, or coughing. Overall, 56 UPAs and an equal number of AEs were included in each province except in the provinces of Nampula and Zambezia.[12] Data collection began in August 2003 and ended in December 2003.

Among the 12,315 households selected for interview 13,657 women were

[11] Lindelow (2006) studies socioeconomic health inequalities in Mozambique using the concentration index. His study is based on income and health data from the 1996-1997 household survey.

[12] Due to the population weight of these two provinces, 68 UPAs and AEs were included. Survey weights are supplied with the data.

identified for interview and of these 12,418 were actually interviewed giving a response rate of 91 percent. Information is available for 33,058 children of less than 18 years of age. The exact number of observations for each individual deprivation indicator is given in Table 1, averages are listed in UNICEF (2006), and here we will now investigate two-dimensional combinations.

5.2 Results

Table 1 presents results for the three selected attribute combinations and the eight categories of children referred to above (weighted by survey sample weights). The top panel shows the proportions of children in each category with access to adequate sanitation in combination with vaccine or diarrhoea treatment, while the middle panel shows access to adequate sanitation and adequate housing, and the bottom panel access to adequate sanitation and school attendance. Accordingly, the first row of Table 1 shows the proportion of girls in male headed rural households with access to sanitation and adequate health treatment. Note that 18.8 percent of these children had no access to sanitation in combination with no health treatment. Some 44.4 percent did not have access to sanitation but did receive health treatment, while only 4.8 percent had access to good sanitary conditions but did not receive adequate health treatment. Finally 32 percent of this group of children had access to both sanitation and health treatment.

Area, Sex of head of household, Sex of child	*(0,0)*	*(0,1)*	*(1,0)*	*(1,1)*	*# of obs.*
		(Sanitation deprivation, Health deprivation)			
Rural, Male, Girl	18.8	44.4	4.8	32.0	2262
Rural, Male, Boy	19.2	44.8	4.7	31.3	2288
Rural, Female, Girl	13.9	47.9	4.7	33.6	580
Rural, Female, Boy	15.7	44.9	3.7	35.6	598
Urban, Male, Girl	2.6	18.8	7.6	71.0	1215
Urban, Male, Boy	2.9	18.2	8.1	70.9	1156
Urban, Female, Girl	2.0	19.5	3.2	75.3	382
Urban, Female, Boy	3.7	16.7	8.5	71.1	341
Total	13.9	37.6	5.4	43.1	8822
		(Sanitation deprivation, Shelter deprivation)			
Rural, Male, Girl	58.9	3.5	31.6	6.0	7437
Rural, Male, Boy	56.9	3.5	33.3	6.3	7682
Rural, Female, Girl	54.1	5.3	29.9	10.7	2212
Rural, Female, Boy	55.6	5.1	26.3	13.0	2356
Urban, Male, Girl	13.4	4.3	26.4	55.9	4986
Urban, Male, Boy	13.6	5.3	25.2	55.9	4933
Urban, Female, Girl	13.5	4.5	20.3	61.6	1812
Urban, Female, Boy	13.4	4.8	23.2	58.6	1640
Total	43.0	4.2	29.3	23.5	33058
		(Sanitation deprivation, Education deprivation)			
Rural, Male, Girl	27.7	34.5	10.3	27.6	3716
Rural, Male, Boy	16.3	41.3	9.4	33.0	4010
Rural, Female, Girl	21.6	38.4	8.7	31.2	1223
Rural, Female, Boy	19.2	41.0	8.1	31.7	1348
Urban, Male, Girl	6.0	9.9	7.2	76.9	2858
Urban, Male, Boy	5.0	13.1	5.3	76.6	2912
Urban, Female, Girl	8.2	9.0	5.3	77.5	1140
Urban, Female, Boy	7.2	11.2	4.2	77.4	1025
Total	16.0	28.8	8.2	47.0	18232

Note: The first element, *i*, in vector (*i,j*) indicate sanitation deprivation. The second element, *j*, indicate health deprivation (in the top panel), shelter deprivation (in the middle panel), and education deprivation (in the lower panel). *i,j* = 0 is deprivation, *i,j* = 1 is no deprivation.

Source: Authors' calculations from DHS 2003.

Table 1: Percentages of children's two-dimensional living standards.

Turning to first order dominance and ordinal inequality relations, the three matrices in Table 2 present our results calculated from Table 1. The numbers 1 and 0 represent the presence of first order dominance and no first order dominance, respectively, while the capital letters A, B, C, and D refer to four types of ordinal inequality relations (explained below). Numbers and letters are organized so the entry 1 indices that the row group first order dominates the column group. We test a null-hypothesis of equality of the two distributions using a bootstrap procedure.[13] In the case of first order dominance, the test statistic is the minimum function D in section 4.1. Following common convention, the null-distribution is generated by merging the observations from the two groups. From the null-distribution, two new samples are generated (drawing randomly with replacement) corresponding in size to the original two samples, and the test statistic D is calculated. Repeating this procedure 1000 times, we obtain a distribution over the test statistic consistent with the null-hypothesis - which can then be compared with the test statistic for the original sample. Asterisks in Table 2 indicate significance at the five percent level, meaning that the observed value of D is larger than the 95th percentile of its bootstrapped distribution (indicating that the two groups are genuinely distinct).[14]

[13] We refer to Efron and Tibshirani (1993, ch. 16) for a general discussion of the bootstrap approach to hypothesis testing.

[14] Robertson, Wright and Dykstra (1988) and Bhattacharya and Dykstra (1994) develop a test for equality of distributions against an alternative of first order dominance. We do not discuss this approach here. For continuous-variable models, methods for testing multidimensional first- and higher order dominance have been developed by Crawford (2005), Duclos et al. (2006), McCaig and Yatchew (2007) and Anderson (2008).

(Sanitation deprivation, Health deprivation)

Area, Sex of head of household, Sex of child	*Median*	Rural, Male, Girl	Rural, Male, Boy	Rural, Female, Girl	Rural, Female, Boy	Urban, Male, Girl	Urban, Male, Boy	Urban, Female, Girl	Urban, Female, Boy
Rural, Male, Girl	*(0,1)*	*1D*	1	0	0	0	0	0	0
Rural, Male, Boy	*(0,1)*	0	*1D*	0	0	0	0	0	0
Rural, Female, Girl	*(0,1)*	1	1*	*1D*	0D*	0	0	0	0
Rural, Female, Boy	*(0,1)*	1*	1*	0	*1D*	0	0	0	0
Urban, Male, Girl	*(1,1)*	1*	1*	1*	1*	*1A*	0	0	0
Urban, Male, Boy	*(1,1)*	1*	1*	1*	1*	0	*1A*	0	0
Urban, Female, Girl	*(1,1)*	1*	1*	1*	1*	0	0	*1A*	0
Urban, Female, Boy	*(1,1)*	1*	1*	1*	1*	0	0	0	*1A*

(Sanitation deprivation, Shelter deprivation)

Area, Sex of head of household, Sex of child	*Median*	Rural, Male, Girl	Rural, Male, Boy	Rural, Female, Girl	Rural, Female, Boy	Urban, Male, Girl	Urban, Male, Boy	Urban, Female, Girl	Urban, Female, Boy
Rural, Male, Girl	*(0,0)*	*1B*	0B	0B*	0B*	0	0	0	0
Rural, Male, Boy	*(0,0)*	1	*1B*	0B*	0	0	0	0	0
Rural, Female, Girl	*(0,0)*	1*	1*	*1B*	0	0	0	0	0
Rural, Female, Boy	*(0,0)*	1*	0	0	*1B*	0	0	0	0
Urban, Male, Girl	*(1,1)*	1*	1*	1*	1*	*1A*	0	0	0
Urban, Male, Boy	*(1,1)*	1*	1*	1*	1*	0	*1A*	0	0
Urban, Female, Girl	*(1,1)*	1*	1*	1*	1*	0	1A	*1A*	0
Urban, Female, Boy	*(1,1)*	1*	1*	1*	1*	0	1A	0	*1A*

(Sanitation deprivation, Education deprivation)

Area, Sex of head of household, Sex of child	*Median*	Rural, Male, Girl	Rural, Male, Boy	Rural, Female, Girl	Rural, Female, Boy	Urban, Male, Girl	Urban, Male, Boy	Urban, Female, Girl	Urban, Female, Boy
Rural, Male, Girl	*(0,1)*	*1D*	0	0	0	0	0	0	0
Rural, Male, Boy	*(0,1)*	1*	*1D*	1*	1*	0	0	0	0
Rural, Female, Girl	*(0,1)*	1*	0	*1D*	0	0	0	0	0
Rural, Female, Boy	*(0,1)*	1*	0	0	*1D*	0	0	0	0
Urban, Male, Girl	*(1,1)*	1*	1*	1*	1*	*1A*	0	0C*	0
Urban, Male, Boy	*(1,1)*	1*	1*	1*	1*	0	*1A*	0	0C*
Urban, Female, Girl	*(1,1)*	1*	1*	1*	1*	0	0	*1A*	0
Urban, Female, Boy	*(1,1)*	1*	1*	1*	1*	0	0	0	*1A*

Note: 1 indicate first order dominance, 0 no first order dominance. A and B indicate ordinal equality when the median is extreme, in (1,1) and (0,0) respectively, and there is first order dominance. C is ordinal equality, also, when the median is extreme but, when there is no first order dominance. D indicate ordinal equality when the median is non-extreme, (0,1) or (1,0). Test of significance was conducted for first order dominances and ordinal equality of the diagonal, using the permutation bootstrap method. * indicate a significant test statistic at the 5% level.

Source: Authors' calculations from DHS 2003.

Table 2: First order dominance and ordinal inequality relations among groups of children.

5.2.1 Evidence of first order dominance relations

Making first order dominance comparisons in Table 2 from the two-dimensional distributions of Table 1 involve the four inequalities of Section 4.1. In the case of ordinal inequality (without the presence of first order dominance) the bootstrapped test statistic is the minimum function over the set of inequalities as specified in Proposition 1 for each relevant case.

To illustrate, compare the distribution of access to adequate sanitation and adequate shelter of girls in rural female headed households to that of girls in rural male headed households, and say we wish to determine whether the former first order dominates the latter. From the four inequalities of Section 4.1 and the distributions of the middle panel of Table 1 we need to evaluate if $58.9 \geq 54.1$, $58.9+3.5 \geq 54.1+5.3$, $58.9+31.6 \geq 54.1+29.9$, and $58.9 + 3.5 + 31.6 \geq 54.1 + 5.3 + 29.9$. All inequalities are fulfilled, so girls in rural female headed households are better-off than girls in male headed households. First order dominance exists, indicated by a 1 in the third row - first column of the middle panel of Table 2. Using the permutation bootstrap this dominance is also significant at the five percent level, indicated with an asterisk.

Table 3 summarizes all 19 first order dominance tables matching all possible combinations of the eight categories of children. Note that matching two groups of children, with distributions f and g, there are three possible outcomes: g first order dominates f (i.e. g wins), neither g nor f dominates (i.e. a draw), or f dominates g (i.e. g looses). Overall, Table 3 shows that urban households dominate rural households with children, particularly girls, of urban male headed households being better-off than other children. Rural areas generally show the opposite pattern. Female headed households dominate male headed households with boys being better-off than girls. Studying in more detail the 19 matrices, it becomes clear that girls of rural male headed households loose because they have poor living standards in four (sets of) well-being indicators: access to safe water, adequate sanitation, sufficient shelter and education. The pattern is less clear for dominance of girls in urban male headed households, but combinations of three indicators (access to safe water, access to information, and better anthropometrics) seem important. A government transfer programme focussing on children who are most severely deprived should therefore target male headed households in rural areas, particularly if the share of girls is large in these households.

Area, Sex of head of household, Sex of child	Rural, Male, Girl	Rural, Male, Boy	Rural, Female, Girl	Rural, Female, Boy	Urban, Male, Girl	Urban, Male, Boy	Urban, Female, Girl	Urban, Female, Boy	*Number of won matches*
Rural, Male, Girl	19	4	0	0	0	0	0	0	*23*
Rural, Male, Boy	6	19	3	4	0	0	0	0	*32*
Rural, Female, Girl	13	8	19	1	0	0	0	0	*41*
Rural, Female, Boy	9	4	6	19	0	0	0	0	*38*
Urban, Male, Girl	19	19	19	19	19	5	6	8	*114*
Urban, Male, Boy	19	19	19	19	2	19	3	8	*108*
Urban, Female, Girl	13	13	19	19	2	5	19	1	*91*
Urban, Female, Boy	14	14	19	18	0	1	1	19	*86*
Number of lost matches	*112*	*100*	*104*	*99*	*23*	*30*	*29*	*36*	

Source: Authors' calculations from DHS 2003.

Table 3: Number of first order dominance relations.

5.2.2 Evidence of inequality relations

Ordinal inequality relations compare within-group inequality between groups; that is, if well-being is more unequally distributed in one group than in another. The ordinal inequality relation is particularly useful in determining which of two incomparable groups (from a first order dominance point of view) is more vulnerable.

The following four examples illustrate how ordinal inequality relations can be used to rank groups of children by their within group dispersion of well-being.

A test of ordinal inequality requires examination of the median and subsequently of a set of case dependent inequalities. From Proposition 1, four basic situations imply that f is more equal than g: (A) the median is extreme in (1,1) and f first order dominates g, directly implies that f is more equal than g from Case 1a of Proposition 1. (B) the median is extreme in

(0,0) and g first order dominates f, implies that f is more equal than g from Case 4a. (C) the median is extreme and we have a first order dominance draw (neither f nor g first order dominates), implies that equality i determined by the three inequalities in Case 1b or 4b. Finally (D) the median is non-extreme implies that equality is determined by the five inequalities in Case 2 or 3. Table 4 illustrates the four types of equality (A to D), by pairwise comparing eight selected groups of children from Table 1.

	UFB	0	1	UMB	0	1
A:	0	13.4	4.8	0	13.6	5.3
	1	23.2	58.6	1	25.2	55.9
	RMG	0	1	RFG	0	1
B:	0	58.9	3.5	0	54.1	5.3
	1	31.6	6	1	29.9	10.7
	UMB	0	1	UFB	0	1
C:	0	5	13.1	0	7.2	11.2
	1	5.3	76.6	1	4.2	77.4
	RFG	0	1	RFB	0	1
D:	0	13.9	47.9	0	15.7	44.9
	1	4.7	33.6	1	3.7	35.6

Table 4: Pairwise comparisons illustrated (figures extracted from Table 2).

Type A, where the shared median is $(1, 1)$ is simple. To see this, note in Table 4A (which compares sanitation and shelter deprivation of urban boys in female headed households (UFB) to that in male headed households (UMB) that the distribution for male headed households can be obtained from that of female headed households by moving density away from the shared median backwards in the distribution to worse outcomes. To be exact, make use of three median-preserving spreads by transferring, respectively, 0.5, 2.0, and 0.2 from $(1, 1)$ to $(0, 1)$, $(1, 0)$, and $(0, 0)$. That is, well-being of urban boys in female headed households is concentrated more densely around the median as compared to that in male headed households. Consequently the distribution in female headed households is also more equal.

Type B, also from the middle panel of Table 1, is illustrated in Table 4B. It compares rural girls in female headed households (RFG) to rural girls in male headed households (RMG). Here, the distribution for female headed households can be obtained from that of male headed households by moving 1.8 mass from $(0,0)$ to $(0,1)$, 3.0 from $(0,0)$ to $(1,1)$ and 1.7 from $(1,0)$ to $(1,1)$. Thus, while female headed households dominate male headed households, male headed households are more equal for this group of children.

Type C, in Table 4C also relates to groups with extreme medians, but none of the groups dominate in this case. This situation occurs in the distribution of sanitation and education deprivation of urban boys in male headed households (UMB) matched against urban boys in female headed households (UFB) in the bottom panel of Table 1 and 2. To determine an ordinal more equal relation we evaluate three inequalities: $13.1 \geq 11.2$, $5.3 \geq 4.2$, and $77.4 - 76.6 \leq \min\{13.1 - 11.2, 5.3 - 4.2\}$. It follows that well-being of boys in male headed households is more equally distributed than well-being of boys in female headed households.[15] To see this, start with a correlation-increasing transfer of 1.1, then do a median-preserving spread of 0.3 and 0.8 from $(1,1)$ and $(0,1)$, respectively, to $(0,0)$. Measured by the two indicators (sanitation and education deprivation), it is not possible to determine whether urban boys in male headed or female headed households are better-off (i.e. whether one group first order dominates the other). However, boys in female headed households are more vulnerable since they are more unequally distributed.

Type D, in Table 4D illustrates a non-extreme median from the top panel of Table 1 and 2. It requires evaluation of the five inequalities in Section 4.2, Case 2. To see this, compare girls of rural female headed households (RFG) to boys of these households (RFB). The five inequalities that need

[15] Because the shared median is in (1,1) the left hand side of the last inequality refers to proportions in (1,1). Had the median been in (0,0) the left hand side would also have referred to proportions in that outcome.

to be evaluated for this match are: $44.9 \leq 47.9$, $3.7 \leq 4.7$, $35.6 \geq 33.6$, $15.7 \geq 13.9$, and $4.7 - 3.7 \leq \min\{35.6 - 33.6, 15.7 - 13.9\}$. Hence, well-being of girls is more equally distributed than that of boys.[16] Again, to show this use a correlation increasing transfer of 0.9 from $(0, 1)$ and $(1, 0)$ to $(1, 1)$ and $(0, 0)$, followed by a median-preserving spread of 0.9 and 1.1 from $(0, 1)$ to $(0, 0)$ and $(1, 1)$, respectively.[17]

The well-being distribution of boys is illuminating. Boys with very good access to household sanitation and medical treatment hide the fact that within this group, some boys lack access to these basic necessities, making the group appear equally well-off to that of girls. Based on estimates of first order dominance, decision makers are therefore equally likely to transfer resources to the two groups, while in fact they may wish to focus more on providing better living-standards for the more vulnerable group of boys than for the group of girls.

Table 5 summarizes results from all the 19 tests of ordinal inequality. We note that one group has well-being more equally distributed than a comparator group if they share the same median and one of the four situations just described is fulfilled. In Table 5 this corresponds to the number of times row f is more equal than column g. So urban households are in general more equally distributed than rural households; male headed households tend to be more equally distributed than female headed households; and boys to be more equal than girls.

[16] The left hand side of the last inequality refers to proportions in (1,0) as the shared median is in (0,1). Had the median been in (1,0) the left hand side would have referred to proportions in (0,1).

[17] Due to rounding errors, it is not possible to create precisely one distribution from the other.

Area, Sex of head of household, Sex of child	Rural, Male, Girl	Rural, Male, Boy	Rural, Female, Girl	Rural, Female, Boy	Urban, Male, Girl	Urban, Male, Boy	Urban, Female, Girl	Urban, Female, Boy	*Total*
Rural, Male, Girl	19	3	3	3	0	0	0	0	*28*
Rural, Male, Boy	1	19	2	1	0	0	0	0	*23*
Rural, Female, Girl	1	1	19	5	0	0	0	0	*26*
Rural, Female, Boy	0	0	0	19	0	0	0	0	*19*
Urban, Male, Girl	4	4	1	1	19	5	7	8	*49*
Urban, Male, Boy	4	4	1	1	2	19	3	9	*43*
Urban, Female, Girl	1	1	1	1	2	5	19	1	*31*
Urban, Female, Boy	2	2	1	1	0	1	1	19	*27*
Total	*32*	*34*	*28*	*32*	*23*	*30*	*30*	*37*	

Source: Authors' calculations from DHS 2003.

Table 5: Number of ordinal inequality relations

6 Conclusion

In this paper we have developed an ordinal concept of multi-dimensional inequality, building on Allison and Foster's (2004) framework for comparing inequalities with one-dimensional categorical data. To illustrate how our model can be applied in the 2x2 case we used DHS data from Mozambique. Such DHS data are available for a large number of countries across the developing world.

It emerged that urban households are better-off than rural households from a first order dominance point of view. This is hardly surprising. However, it also emerged that rural female headed households are better-off than rural male headed households; and girls in urban male headed households are best off, while girls in rural male headed households are worse off (Table 3). These findings deserve attention in policy debates.

We note that the exotic ordinal inequality relations (Type C and D) did not appear frequently in the present sample and for the chosen indicators. Whether this is because these types of inequality relations are indeed rare empirically cannot be established with the data at hand. While the DHS data are widely available across a large number of countries our results highlight that the standard DHS does not produce obvious binary measures of well-being. For example, it would have been revealing to be able to compare nutrition and education of children. However, nutrition (antropometric) variables are only available for non-school children and woman of 15-49 years of age. This suggests that a revision of DHS data collection efforts would be desirable.

Following common convention, we focussed on tests of equality of the two distributions. An alternative null-hypothesis discussed by Dardanoni and Forcina (1999) in the context of one-dimensional dominance of first- and higher order is non-dominance (including exact equality of distributions). That means that dominance is rejected unless there is strong evidence in favor of it. A similar statement applies to a test for inequality. In order to perform such a test in a multidimensional framework, the analyst would have to determine a 'least favorable case'; that is, a null-distribution consistent with the null-hypothesis that makes the observed distributions as plausible as possible. We conjecture that the 'least favorable case' in this situation is, in fact, a case of equal distributions, and that it is valid to interpret our bootstrapping procedure along this line of reasoning.

In sum, we believe to have shown that it is possible to develop a meaningful and intuitive concept of ordinal multidimensional inequality. We have also demonstrated how it can be applied in the 2x2 case. Future research will be required to explore how to deal with more general cases.

A Appendix: Proof of Proposition 1

We will make use of the following lemma.

Lemma A *Suppose that $m(f) = m(g)$ and g is obtained from f by a sequence of $m(f)$-directed bilateral transfers. Then each of these bilateral transfers is inequality-increasing.*

Proof of Lemma A: Define the sets $L(m(f)) = \{x \in X | x \leq m(f)\}$ and $U(m(f)) = \{x \in X | m(f) \leq x\}$. Then g is obtained from f by a sequence of bilateral transfers of the following four kinds: from $m(f)$ to outcomes in the sets $L(m(f)) \backslash \{m(f)\}$ and $U(m(f)) \backslash \{m(f)\}$ respectively, and $m(f)$-directed bilateral transfers within the sets $L(m(f))\backslash\{m(f)\}$ and $U(m(f))\backslash\{m(f)\}$ respectively.

Consider an ordering of the bilateral transfers, where the bilateral transfers are numbered $1, 2, ...$ etc., such that we first have the $m(f)$-directed bilateral transfer within $L(m(f))\backslash\{m(f)\}$, second the $m(f)$-directed bilateral transfers within $U(m(f))\backslash\{m(f)\}$, third the $m(f)$-directed bilateral transfers from $m(f)$ to outcomes in $L(m(f))\backslash\{m(f)\}$ and fourth the $m(f)$-directed bilateral transfers from $m(f)$ to $U(m(f))\backslash\{m(f)\}$.

Suppose that the bilateral transfers $1, 2, ..., h-1$ are median-preserving, but bilateral transfer h fails to be median-preserving. Let $\overline{m}$ denote the new median following bilateral transfer h. It is clear that the median-directed bilateral transfers of the first two types in the ordering (that is, the median-directed bilateral transfers within $L(m(f))\backslash\{m(f)\}$ and the median-directed bilateral transfers within $U(m(f))\backslash\{m(f)\}$) are median-preserving, hence we consider the case where bilateral transfer h is either from $m(f)$ to an outcome in $L(m(f))\backslash\{m(f)\}$ or from $m(f)$ to an outcome in $U(m(f))\backslash\{m(f)\}$.

If bilateral transfer h is from $m(f)$ to an outcome in $L(m(f))\backslash\{m(f)\}$, we must have $\overline{m} < m(f)$. Hence, for the median $\widetilde{m}$ resulting after the last bilateral transfer from $m(f)$ to outcomes in $L(m(f))\backslash\{m(f)\}$ we also have $\widetilde{m} < m(f)$. This implies that the last bilateral transfers from $m(f)$ to $U(m(f))\backslash\{m(f)\}$ will not further change the median relative to $\widetilde{m}$, contradicting $m(f) = m(g)$.

If bilateral transfer h is from $m(f)$ to an outcome in $U(m(f))\backslash\{m(f)\}$, for the new median $\overline{m}$ we have $m(f) < \overline{m}$. Hence, for the median $\widetilde{m}$ resulting after the last bilateral transfer from $m(f)$ to outcomes in $U(m(f))\backslash\{m(f)\}$ we also have $m(f) < \widetilde{m}$, contradicting $m(f) = m(g)$. $\square$

We are now ready to prove Proposition 1. Case 3 is similar to Case 2, and Case 4 is similar to Case 1. Thus, we focus below on Cases 1 and 2.

Case 1: $m(f) = m(g) = (1, 1)$.

Suppose that f first order dominates g. That is, it is possible to go from f to g by a finite sequence of diminishing bilateral transfers. By Lemma A, each such bilateral transfer is median-preserving and inequality-increasing. Thus, g is more unequal than f.

Suppose that f does not first order dominate g, and g does not first-order dominate f. Then, it is impossible to go from f to g (or vice versa) without making use of at least one correlation-increasing switch. This implies that, if g is more unequal than f, $f(1,0) - g(1,0) > 0$ and $f(0,1) - g(0,1) > 0$, since if either $f(1,0) - g(1,0) \leq 0$ or $f(1,0) - g(1,0) \leq 0$ it would be possible to go from f to g without any correlation-increasing switches. On the other hand, these two conditions are not sufficient (for g being more unequal than f). Roughly speaking, we need a condition ensuring that all density that is going to be transferred to $(1,1)$ can be transferred from $(0,1)$ or $(1,0)$ in connection with a correlation-increasing switch.

Now, we claim that if g is more unequal than f (and g is not first order dominated by f) then it is possible to obtain g from f from a sequence (of correlation-increasing switches and median-directed bilateral transfers) that involves only a single correlation-increasing transfer and no transfers from the outcome $(1,1)$ to other outcomes.

We first prove the latter, i.e. we prove that no transfers from $(1,1)$ to other outcomes are required. For this, consider a given sequence (leading from f to g) that contains a transfer of the amount β from $(1,1)$ to another outcome z. Assume, without loss of generality that $z = (0,1)$. (If $z = (0,0)$ then split the bilateral transfer up into two nested bilateral transfers, one from $(1,1)$ to $(0,1)$ and one from $(0,1)$ to $(0,0)$; the case $z = (1,0)$ is symmetric to the one treated here and hence can be omitted).

As noted earlier, we know that the sequence contains at least one correlation-increasing switch (since if otherwise f would first order dominate g). Now, pick an arbitrary correlation-increasing switch from the sequence, and let α denote the amount of density moved from each of the outcomes $(0,1)$ and $(1,0)$ (to $(0,0)$ and $(1,1)$ respectively). We can then decompose this correlation-increasing switch into two bilateral transfers: a bilateral transfer of the amount α from $(0,1)$ to $(1,1)$ and a bilateral transfer of the amount α from $(1,0)$ to $(0,0)$. We consider two cases: (a) $\alpha \geq \beta$, and (b) $\alpha < \beta$.

(a) Replace the bilateral transfer from $(1,1)$ to $(0,1)$ of the amount β

with a bilateral transfer of the amount β from $(1,0)$ to $(0,0)$, and reduce the amount of density transferred between each pair of outcomes from α to $\alpha-\beta$. Note that the amount of density eventually allocated to each outcome remains the same.

(b) Replace the correlation-increasing switch (which moves the amount α between each pair of outcomes) with a bilateral transfer of the amount α from $(1,0)$ to $(0,0)$, and reduce the size of the bilateral transfer from $(1,1)$ to $(0,1)$ to $\beta-\alpha$. Again, note that the amount of density eventually allocated to each outcome remains the same.

Proceeding in this way until no transfers from $(1,1)$ to other outcomes remain, we can eliminate all transfers from $(1,1)$ to other outcomes. Note that we have not shown (and it is not needed for our argument) that after each elimination of some bilateral transfer from $(1,1)$ to another outcome, the resulting sequence of pseudo-distributions consists entirely of distributions. It is sufficient to observe that when all bilateral transfers from $(1,1)$ to other outcomes have been eliminated, what remains is a sequence of correlation-increasing transfers and/or bilateral transfers from $(0,1)$ to $(0,0)$ and from $(1,0)$ to $(0,0)$. For this sequence, it is clear that each intermediate pseudo-distribution is a distribution. In particular, the operations (i.e. correlation-increasing transfers and/or bilateral transfers from $(0,1)$ to $(0,0)$ and from $(1,0)$ to $(0,0)$) can be arranged in an arbitrary order and we can obtain g from f at single operation of each the three types.

From these observations we get the following: Suppose that f does not first order dominate g. Then g is more unequal than f if and only if the following 3 inequalities are satisfied: $f(1,0)-g(1,0)\geq 0$, $f(0,1)-g(0,1)\geq 0$ and $g(1,1)-f(1,1)\leq \min\{f(1,0)-g(1,0), f(0,1)-g(0,1)\}$.

Note that in conjunction with the assumption that f does not first order dominate g, the 3 inequalities imply that $g(0,0)-f(0,0)>0$. From this observation it follows that the 3 inequalities are both necessary and sufficient: The three inequalities are necessary, since if one of them were violated, clearly we could not get g from f by a single correlation-increasing transfer and/or bilateral transfers from $(0,1)$ and $(1,0)$ to $(0,0)$. To verify that the conditions are sufficient, we give the following constructive argument: Suppose that the conditions are satisfied. Let $\alpha = g(1,1) - f(1,1)$. Let $\widehat{f}$ be the distribution obtained from a correlation-increasing transfer of the amount α (where α is transferred from $(0,1)$ to $(0,0)$ and α is transferred from $(1,0)$ to $(1,1)$). Thus, $\widehat{f}(1,1) = g(1,1)$, $\widehat{f}(0,1) \geq g(0,1), \widehat{f}(1,0) \geq g(1,0)$. This means that g can be obtained from $\widehat{f}$ by diminishing bilateral transfers from $(0,1)$ and/or $(0,1)$ to $(0,0)$, and we are done.

Case 2: $m(f) = m(g) = (1,0)$.

Note that if $m(f) = m(g) = (1,0)$ and if g is more unequal than f, then g can be obtained from f by a finite number of correlation-increasing switches (from $(1,0)$ and $(0,1)$ to $(1,1)$ and $(0,0)$) and bilateral transfers from $(1,0)$ to the extreme outcomes $(1,1)$ and $(0,0)$. Regardless of how these correlation-increasing switches and bilateral transfers are ordered, each intermediate pseudo-distribution is a distribution. Thus, a single correlation-increasing switch is enough (since all correlation-increasing switches can be amalgamated into a single correlation-increasing switch and still each intermediate pseudo-distribution is a distribution). In particular, we can obtain g from f in three steps, ordered as follows: (1) a correlation-increasing transfer, (2) a bilateral transfer from $(1,0)$ to $(0,0)$, and (3) a bilateral transfer from $(1,0)$ to $(1,1)$.

From these observations, we can show that g is more unequal that f if and only if the following 5 inequalities hold: $g(1,0) \leq f(1,0)$, $g(0,1) \leq f(0,1)$, $g(1,1) \geq f(1,1)$, $g(0,0) \geq f(0,0)$, $f(0,1) - g(0,1) \leq \min\{g(1,1) - f(1,1), g(0,0) - f(0,0)\}$ and $f(1,0) - g(1,0) \geq f(0,1) - g(0,1)$.

Necessity of the first four inequalities is straightforward. The fifth inequality $f(1,0) - g(1,0) \geq f(0,1) - g(0,1)$ must hold since the only way that density can be transferred away from $(0,1)$ is by means of a correlation-increasing transfer and thus it must be the case that at least the same amount is going to be transferred away from $(1,0)$. For sufficiency, we give the following constructive argument.

Suppose that the 5 inequalities hold. Let $\lambda = f(0,1) - g(0,1)$. Define the distribution $\widehat{f}$ by $\widehat{f}(0,1) = f(0,1) - \lambda$, $\widehat{f}(1,0) = f(1,0) - \lambda, \widehat{f}(0,0) = f(0,0) + \lambda$, and $\widehat{f}(1,1) = f(1,1) + \lambda$. Then $\widehat{f}(0,1) = g(0,1)$, $\widehat{f}(0,0) \geq g(0,0)$

and $\widehat{f}(0,0) \geq g(0,0)$. Thus, g can be obtained from $\widehat{f}$ by bilateral transfers from $(1,0)$ to $(0,0)$ and $(1,1)$ and we are done.

B Appendix: Deprivation indicators

Welfare indicator	Description	Number of observations
Severe nutrition deprivation	Children under five years of age whose heights and weights for their age are more than -3 standard deviations below the median of the international reference population, i.e. severe anthropometric failure	8,001
Severe water deprivation	Children under 18 years of age who only have access to surface water (e.g. rivers) for drinking or who live in households where the nearest source of water is more than 15 minutes away	33,058
Severe sanitation deprivation	Children under 18 years of age who have no access to a toilet of any kind in the vicinity of their dwelling, including communal toilets or latrines	33,058
Severe health deprivation	Children under five years of age that have never been immunised against any diseases or young children who have had a resent illness involving diarrhoea and did not receive any medical advise or treatment	8,822
Severe shelter deprivation	Children under 18 years of age living in dwellings with more than five people per room (severe overcrowding) or with no flooring material (e.g. mud floor)	33,058
Severe education deprivation	Children aged between 7 and 18 who have never been to school and are not currently attending school	18,232
Severe information deprivation	Children aged between 5 and 18 with no possession of and access to radio, television or telephone at home	26,619

Source: UNICEF (2006), Annex I.

References

[1] Allison RA, Foster JE, 2004. Measuring health inequality using qualitative data. Journal of Health Economics 23, 505-524.

[2] Anderson G, 2008. The empirical assessment of multidimensional welfare, inequality and poverty: sample weighted multivariate generalizations of the Kolmogorov-Smirnov two sample test for stochastic dominance. Journal of Economic Inequality 6, 73-87.

[3] Atkinson AB, Bourguignon F, 1982. The comparison of multi-dimensioned distributions of economic status. Review of Economics Studies 12, 183-201.

[4] Atkinson AB, 2003. Multidimensional deprivation: contrasting social welfare and counting approaches. Journal of Economic Inequality 1, 53-65.

[5] Bhattacharya B, Dykstra RL, 1994. Statistical Inference for Stochastic Ordering. In: M Shaked M, JG Shanthikumar (Eds.), Stochastic orders and their applications, Academic Press.

[6] Boland PJ, Proschan F, 1988. Multivariate arrangement increasing functions with applications in probability and statistics. Journal of Multivariate Analysis, 25, 286-298.

[7] Bourguignon F, Chakravarty S, 2003. The measurement of multidimensional poverty. Journal of Economic Inequality 1, 25-49.

[8] Buvinic M, Gupta GR, 1997. *Female-Headed Households and Female-Maintained Families_ Are They Worth Targeting to Reduce Poverty in Developing Countries.* Economic Development and Culyural Change. Vol. 45, No. 2, pp. 259-280.

[9] Crawford I, 2005. A nonparametric test of stochastic dominance in multivariate distributions. DP 12/05, Department of Economics, University of Surrey.

[10] Dardanoni V and Forcina A, 1999, Inference for Lorenz curve orderings. Econometrics Journal 2, 49-75.

[11] DHS. Demographic and Health Survey. Macro International Inc. United States. Email: archive@orcmacro.com. Web: http://www.measuredhs.com.

[12] Duclos J-Y, Sahn DE, Younger SD, 2006. Robust multidimensional poverty comparisons. Economic Journal 116, 943-968.

[13] Efron B, Tibshirani RJ, 1993. An Introduction to the Bootstrap. Chapman & Hall.

[14] Epstein L, Tanny SM, 1980. Increasing generalized correlation: a definition and some economic consequences. Canadian Journal of Economics 1980.

[15] Gravel N, Moyes P, 2006. Ethically robust comparisons of distributions of two individual attributes. Working paper 0605, IDEP.

[16] Gravel N, Moyes P, Tarroux, 2008. Robust International Comparisons of Distributions of Disposable Income and Regional Public Goods. Economica, *forthcoming.*

[17] Handa S, 1996. *Expenditure behavior and children's welfare_ An analysis of female headed households in Jamaica.* Journal of Development Economics. Vol. 50, pp 165-187.

[18] IAF, 2002. Inquérito aos Agregados Familiares (Household Survey). Web: http://www.ine.gov.mz/inqueritos_dir/iaf/.

[19] INE. Instituto Nacional de Estatística (National Institut of Statistics). Maputo, Mozambique. Web: www.ine.gov.mz.

[20] Kakwani NC, Wagstaff A, van Doorslaer E, 1997. Socioeconomic inequalities in health: measurement, computation and statistical inference. Journal of Econometrics 77, 87-104.

[21] Kamae T, Krengel U, O'Brien GL, 1977. Stochastic inequalities on partially ordered spaces. The Annals of Probability 5, 899-912.

[22] Lindelow M, 2006. Sometimes more equal than others: how health inequalities depend on the choice of welfare indicator. Health Economics 15, 263-279.

[23] Lehmann EL, 1955. Ordered families of distributions. The Annals of Mathematical Statistics 26, 399-419.

[24] Maasoumi E, 1999. Multidimensioned approaches to welfare analysis. In: J Silber (Ed.), Handbook of Income Inequality Measurement, Kluwer.

[25] McCaig B, Yatchew A, 2007. International welfare comparisons and nonparametric testing of multivariate stochastic dominance. Journal of Applied Econometrics 22, 951-969.

[26] MISAU. Ministério da Saúde (Ministry of Health). Maputo, Mozambique. Web: http://www.misau.gov.mz/.

[27] Müller A, Stoyan D, 2002. Comparison methods for stochastic models and risks. John Wiley and Sons, New York, NY.

[28] Orazem PF, King OM, 2007. *Schooling in Developing Countries: The Roles of Supply, Demand and Government Policy*. In: Handbook of Development Economics, vol. 4. Elsevier Science B.V., Amsterdam.

[29] Parker S, Rubalcava L, Teruel G, 2007. *Evaluating Conditional Schooling and Health Programs*. In: Handbook of Development Economics, vol. 4. Elsevier Science B.V., Amsterdam.

[30] Quirk JD, Saposnik R, 1962. Admissibility and measurable utility functions. Review of Economic Studies 29, 140-146.

[31] Robertson T, Wright F, Dykstra RL, 1988. Order Restricted Statistical Inference. John Wiley, New York, NY.

[32] Shaked M, Shanthikumar JG, 1994. Stochastic Orders and Their Applications. Probability and Mathematical Statistics.

[33] Strauss J, Thomas D, 1995. *Human resources: Empirical modeling of household and family decisions.* In: Srinivasan, T.N., Behrman, J.R. (Eds.), Handbook of Development Economics, vol. 3A. North-Holland, Amsterdam, pp. 1883–2023.

[34] Tsui K, 1999. Multidimensional inequality and multidimensional generalized entropy measures: an axiomatic derivation. Social Choice and Welfare 16, 145-157.

[35] UNICEF, 2006. *Childhood Poverty in Mozambique: A Situation and Trends Analysis.* United Nations, Mozambique. Available at: http://www.eldis.org/fulltext/childpoverty_Moz.pdf.

[36] Wagstaff A, van Doorslaer E, Paci P, 1989. Equity in the finance and delivery of health care: some tentative cross-country comparisons, Oxford Review of Economic Policy 5, 89-112.

[37] Wagstaff A, Paci P, van Doorslaer E, 1991, On the measurement of inequalities in health, Social Science and Medicine 33, 545-557.

[38] Weymark JA, 2006, The normative approach to measurement of multidimensional inequality. In: Inequality and Economic Integration. Editors: F Farina, E Savaglio, Routledge.